0. Introduction

0.8 Relativistic Kinematics
Relativistic Kinematics

Often deal with particles traveling close to the speed of light.

\[ \beta = \frac{v}{c}, \quad |\beta| < 1 \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \gamma \geq 1 \]

Total energy of particle with non-zero mass

\[ E = \gamma m c^2 \]

and momentum

\[ p = \gamma m v = \gamma m c \beta \]
Relativistic Kinematics

Total energy squared

\[ E^2 = p^2 c^2 + m^2 c^4 \]

Consider \( m=0 \) or \( p=0 \)!

Lorentz-transformation along x-direction

\[
\begin{align*}
t' &= \gamma(v) \left( t - \frac{vx}{c^2} \right), \\
x' &= \gamma(v)(x - vt), \\
y' &= y, \\
z' &= z, \\
E' &= \gamma(v) \left( E - vp_x \right), \\
px' &= \gamma(v) \left( px - \frac{vE}{c^2} \right), \\
py' &= py, \\
pz' &= pz
\end{align*}
\]
**Example: Relativistic Kinematics**

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Lorentz-transformation (boost) in z-direction by $v_b$

$$E' = \gamma_b \left( E - \frac{v_b}{c} p_z c \right), \quad p'_z c = \gamma_b \left( p_z c - \frac{v_b}{c} E \right)$$

How does $m'^2 c^4$ transform?
Multiparticle systems

In collisions or decays, more than one particle is involved. Total energy $\sum_i E_i$ and total momentum $\sum_i p_i$ are always conserved (not invariant). Frame independent is the property

$$m_T^2 c^4 = E_T^2 - p_T^2 c^2$$

Consider the case of a particle decay to three daughter particles

$m_T = m_x$, hence the particle can be identified from its decay products.
Fixed target or colliding beams
Exercise: Fixed target or colliding beams

To make a Z boson of mass 91 GeV by colliding a positron with an electron, both with mass 0.511 MeV we need $E_{cm} = \sqrt{s} = 91$ GeV. The beam energy needed is 45.5 GeV. However, of the positron collided with a fixed target of stationary electrons, what is the minimal positron beam energy to produce Z bosons?
More implications of $E = mc^2$

1) $E = K + m_0c^2 = \gamma m_0c^2$

At LEP @ CERN, electrons and positron were accelerated to 100 GeV. How large was $\gamma$?

2) How much energy do we need to split a proton and neutron (deuteron)?

3) An excited particle emits a photon. Under which condition can this photon be reabsorbed?

4) What is the minimal beam energy in a proton on proton fixed target experiment to produce anti-protons?
More implications of $E = mc^2$

4) Assume the decay of a piion at rest into an electron and positron. How fast are the decay products?

5) What is the minimal energy of a proton colliding with a proton at rest to produce a $p+n+n^+$?

6) Compton effect. The energy of a photon is $E = hv = h/\lambda$. Calculate the change in the photons wavelength.
Exercise: Fixed target or colliding beams (solution)

\[ s = m_1^2c^4 = E_1^2 - p_T^2c^2 = E_1^2 + 2E_1m_2c^2 + m_2^2c^4 - E_1^2 + m_1^2c^4 = 2E_1m_2c^2 + m_1^2c^4 + m_2^2c^4 \]

\[ E_1 = \frac{s - m_1^2c^4 - m_2^2c^4}{2m_2c^2} \]

\[ E_1 \approx \frac{s}{2m_ec^2} = 8.1 \text{ PeV} = 8100000 \text{ GeV} \]
Example: Relativistic Kinematics (solution)

Lorentz-transformation (boost) in z-direction by $v_b$

\[
E' = \gamma_b \left( E - \frac{v_b}{c} p_z c \right), \quad p'_z c = \gamma_b \left( p_z c - \frac{v_b}{c} E \right)
\]

How does $m'^2 c^4$ transform? It is invariant!

\[
m'^2 c^4 = E'^2 - p'^2 c^2
\]

\[
= \gamma_b^2 \left( E^2 - 2E v_b p_z + v_b^2 p_z^2 \right) - p_x^2 c^2 - p_y c^2 - \gamma_b^2 \left( p_z^2 c^2 - 2E v_b p_z + \frac{v_b^2 E^2}{c^2} \right)
\]

\[
= \gamma_b^2 \left( E^2 - p_z^2 c^2 \right) \left( 1 - \frac{v_b^2}{c^2} \right) - p_x^2 c^2 - p_y^2 c^2
\]

\[
= E^2 - p_x^2 c^2 - p_y^2 c^2 - p_z^2 c^2 = m^2 c^4
\]