8.701

Introduction to Nuclear and Particle Physics

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0. Introduction

0.8 Relativistic Kinematics

Relativistic Kinematics

Often deal with particles traveling close to the speed of light.

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}, \qquad |\boldsymbol{\beta}| < 1$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \gamma \geq 1$$

Total energy of particle with non-zero mass

$$E = \gamma m c^2$$

and momentum $$\boldsymbol{p} = \gamma m \boldsymbol{v} = \gamma m c \boldsymbol{\beta}$$

Relativistic Kinematics

Total energy squared

$$E^2 = p^2c^2 + m^2c^4$$

Consider m=0 or p=0!

Lorentz-transformation along x-direction

$$t' = \gamma(v)\left(t - \frac{vx}{c^2}\right), \qquad E' = \gamma(v)\left(E - vp_x\right)$$

$$x' = \gamma(v)(x - vt), \qquad p'_x = \gamma(v)\left(p_x - \frac{vE}{c^2}\right)$$

$$y' = y, \qquad p'_y = p_y$$

$$z' = z, \qquad p'_z = p_z$$

Example: Relativistic Kinematics

Lorentz-transformation (boost) in z-direction by v_b

$$E' = \gamma_b \left(E - \frac{v_b}{c} p_z c \right), \qquad p'_z c = \gamma_b \left(p_z c - \frac{v_b}{c} E \right)$$

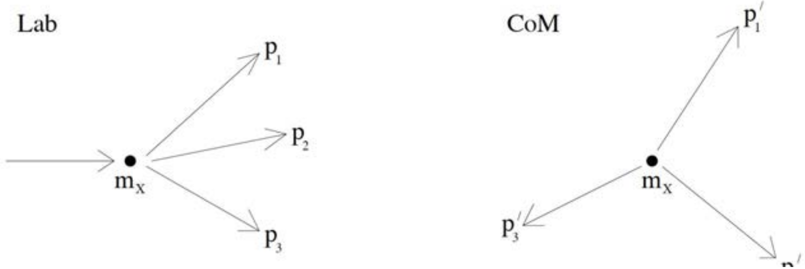
How does m'^2c^4 transform?

Multiparticle systems

In collisions or decays, more than one particle is involved. Total energie $\sum_i E_i$ and total momentum $\sum_i \boldsymbol{p}_i$ are always conserved (not invariant). Frame indepent is the property

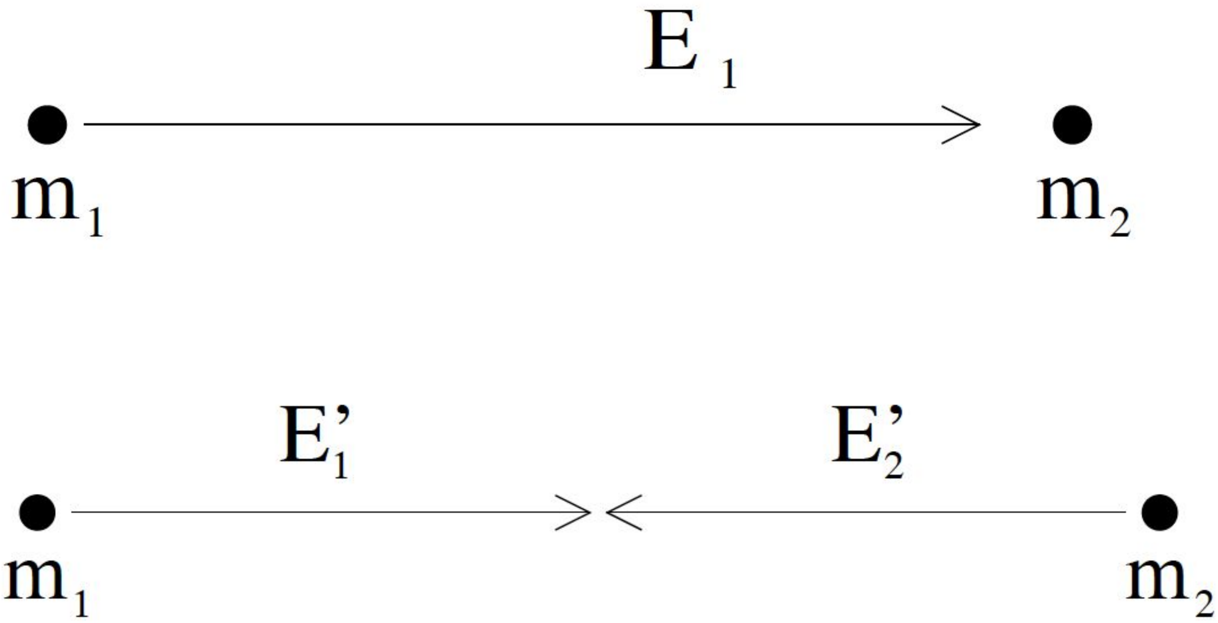
$$m_T^2 c^4 = E_T^2 - p_T^2 c^2$$

Consider the case of a particle decay to three daughter particles



mT = mx, hence the particle can be identified from its decay products.

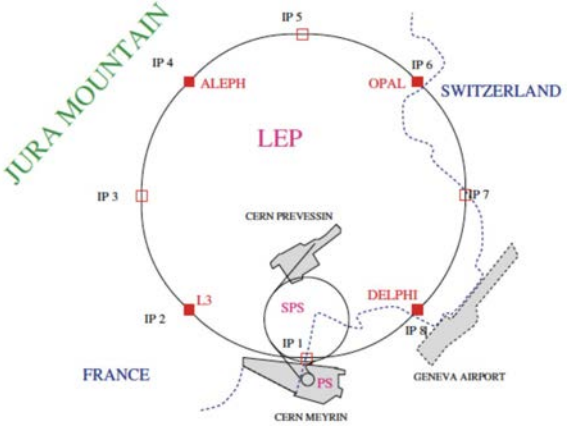
Fixed target or colliding beams



Exercise: Fixed target or colliding beams

To make a Z boson of mass 91 GeV by colliding a positron with an electron, both with mass 0.511 MeV we need Ecm = $\sqrt{s}$ = 91 GeV. The beam energy needed is 45.5 GeV. However, of the positron collided with a fixed target of stationary electrons, what is the minimal positron beam energy to produce Z bosons?

More implications of E = mc^2



1) $E = K + m_0c^2 = \gamma m_0c^2$

   At LEP @ CERN, electrons and positron were accelerated to 100 GeV. How large was γ?

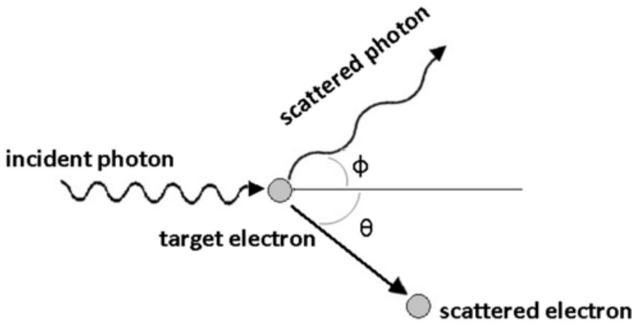
2) How much energy do we need to split a proton and neutron (deuteron)?
3) An excited particle emits a photon. Under which condition can this photon be reabsorbed?
4) What is the minimal beam energy in a proton on proton fixed target experiment to produce anti-protons?

More implications of $E = mc^2$

4) Assume the decay of a pion at rest into an electron and positron. How fast are the decay products?

5) What is the minimal energy of a proton colliding with a proton at rest to produce a $p+n+\pi^+$?

6) Compton effect. The energy of a photon is $E = h\nu = h/\lambda$. Calculate the change in the photons wavelength.



Exercise: Fixed target or colliding beams (solution)

$$s = m_T^2 c^4 = E_T^2 - p_T^2 c^2 = E_1^2 + 2E_1 m_2 c^2 + m_2^2 c^4 - E_1^2 + m_1^2 c^4 = 2E_1 m_2 c^2 + m_1^2 c^4 + m_2^2 c^4$$

$$E_1 = \frac{s - m_1^2 c^4 - m_2^2 c^4}{2 m_2 c^2}$$

$E_1 \approx s/2m_e c^2 = 8.1$ **PeV = 8100000 GeV**

Example: Relativistic Kinematics (solution)

Lorentz-transformation (boost) in z-direction by v_b

$$E' = \gamma_b \left(E - \frac{v_b}{c} p_z c \right), \qquad p'_z c = \gamma_b \left(p_z c - \frac{v_b}{c} E \right)$$

How does m'^2c^4 transform? It is invariant!

$$\begin{aligned} m'^2 c^4 &= E'^2 - p'^2 c^2 \\ &= \gamma_b^2 \left(E^2 - 2E v_b p_z + v_b^2 p_z^2 \right) - p_x^2 c^2 - p_y^2 c^2 - \gamma_b^2 \left(p_z^2 c^2 - 2E v_b p_z + \frac{v_b^2 E^2}{c^2} \right) \\ &= \gamma_b^2 \left(E^2 - p_z^2 c^2 \right) \left(1 - \frac{v_b^2}{c^2} \right) - p_x^2 c^2 - p_y^2 c^2 \\ &= E^2 - p_x^2 c^2 - p_y^2 c^2 - p_z^2 c^2 = m^2 c^4 \end{aligned}$$

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