4. QED

4.4 Photons
Quantum Electrodynamics

Relativistic quantum field theory of electrodynamics describing how light and matter interacts.

QED describes all phenomena involving electrically charged particles interacting by photon exchange.

Photon is an elementary particle, the quantum of the electromagnetic field.

QED can be described as a perturbation theory and provides extremely accurate predictions. It’s “our pride and joy!”
Classical Electrodynamics: Maxwell's equations

\[ \nabla \cdot \vec{E} = \rho \]  
\[ \nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j} \]  
\[ \nabla \cdot \vec{B} = 0 \]  
\[ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \]  

\[ \vec{B} = \nabla \times \vec{A}, \quad \vec{E} = -\nabla \Phi - \frac{\partial \vec{A}}{\partial t}, \]  

(Gauss),  
(Ampère),  
(Gauss),  
(Faraday).
Maxwell’s equation

\[ A^\mu = (\Phi, \vec{A}) \quad \text{and} \quad j^\mu = (\rho, \vec{j}) \]

\[ \Box \equiv \partial^\mu \partial_\mu = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \]

\[ \Box A^\mu - \partial^\mu (\partial_\nu A^\nu) = j^\mu \]

\[ \partial_\mu F^{\mu \nu} = j^\nu, \quad \text{with} \quad F_{\mu \nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \]

\[ F^{\mu \nu} = \begin{pmatrix}
0 & -E_x & -E_y & -E_z \\
E_x & 0 & -B_z & B_y \\
E_y & B_z & 0 & -B_x \\
E_z & -B_y & B_x & 0 
\end{pmatrix} \]
Gauge

\[ \square A^\mu - \partial^\mu (\partial_\nu A^\nu) = j^\mu \]

\[ A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \chi; \]

Coulomb gauge

\[ \square A^\mu = j^\mu \]
Aμ becomes the wave function of the photon satisfying □Aμ = 0

\[ A^\mu(x) = ae^{-\frac{i}{\hbar}p \cdot x} \epsilon^\mu(p) \]

εμ is the polarization vector and a a normalization factor.

We find

\[ p^\mu p_\mu = 0, \quad \text{or} \quad E = \lvert p \rvert c \]
Polarization vector

The choice $A^0 = 0$ and $\nabla \cdot A = 0$ of gauge requires that

\[ \epsilon^0 = 0, \quad \epsilon \cdot p = 0 \]

i.e. the three-vector is perpendicular to the direction of propagation $\rightarrow$ a free photon is transversely polarized.

Photons have two independent solutions (polarization states) for a given momentum.