Problem 1: Gamma Factor for LEP Electron

At LEP @ CERN, electrons and positrons were accelerated to 100 GeV. How large was $\gamma$?

- We know that:
  \[ E_e = \gamma m_e c^2 = 100 \text{ GeV}, \]

and:

\[ m_e c^2 \approx 0.5 \text{ MeV} = 5 \times 10^{-4} \text{ GeV}. \]

So,

\[ \gamma = \frac{100 \text{ GeV}}{5 \times 10^{-4} \text{ GeV}} = 2 \times 10^5. \]

Problem 2: Splitting the Deuteron

How much energy do we need to split a proton and neutron (deuteron)?

- In the atomic mass unit, $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, the masses of the relevant particles are:

  \[ m_d = 2.01355 \text{ u}, \]
  \[ m_p = 1.00728 \text{ u}, \]
  \[ m_n = 1.00867 \text{ u}, \]
So, we would need \((m_d - m_p - m_n)c^2 = 0.0025uc^2\) of energy to split the deuteron into a proton and a neutron. That is equivalent to \(2.23\text{MeV}\) of energy. This nuclear process is written as:

\[
\text{n} + \text{p} \leftrightarrow \text{d} + 2.23\text{MeV}
\]

**Problem 3: Photon Reabsorption**

An excited particle emits a photon. Under which condition can this photon be reabsorbed?

- Suppose the excited particle is initially at rest with mass \(m_0\). After emitting a photon with energy \(Q\), it has mass \(m'_0\) and is moving at speed \(v\).

  By conservation of energy and momentum, we have:

\[
\begin{align*}
\frac{m_0c^2}{c^2} &= Q + m'_0\gamma(v)c^2 \\
0 &= \frac{Q}{c} - m'_0\gamma(v)v,
\end{align*}
\]

which means the energy and momentum of the particle, after emitting the photon, are:

\[
\begin{align*}
E' &= m_0c^2 - Q \\
p' &= \frac{Q}{c}.
\end{align*}
\]

Requiring the particle to be on-shell, it gives:

\[
E'^2 - (p'c)^2 = (m_0c^2 - Q)^2 - Q^2 = (m_0c^2)^2 - 2m_0c^2Q = (m'_0c^2)^2.
\]

So, \[
Q = Q_0 \left(1 - \frac{Q_0}{2m_0c^2}\right) < Q_0,
\]

where \(Q_0 \equiv (m_0 - m'_0)c^2\).

The energy conservation in Equation 1 also requires:

\[
m'_0 = m_0 - \frac{Q}{\gamma(v)}
\]
Problem 4: Fixed-Target $\bar{p}$ Production

What is the minimal beam energy in a proton on proton fixed target experiment to produce anti-protons?

- The process can be written as: $p + p \rightarrow p + p + p + \bar{p}$.
  - In the center of mass frame, $E = 2\gamma m_p c^2 = 4m_p c^2 \Rightarrow \gamma = 2; \beta = 0.866$.
  - Switching to the lab frame, $\beta = \frac{2\times0.866}{1+0.866^2} = 0.990 \Rightarrow \gamma = 7$
  - So, we need $E = \gamma m_p c^2 \approx 6.57 \text{ GeV}$ of beam energy in a proton on proton fixed target experiment to produce anti-protons.

Problem 5: Pion Decays

Assume the decay of a pion at rest into an electron and positron. How fast are the decay products?

- By conservation of momentum, the electron and positron must fly out in equal and opposite directions, which means $\gamma_{e^-} = \gamma_{e^+}$, and $E_{e^-} = E_{e^+} = \frac{1}{2}m_{\Pi^0} c^2 = 67.5 \text{ MeV}$.
  - $E_{e^-} = \gamma_{e^-} m_e c^2 = \gamma_{e^-} (0.511 \text{ MeV}) \Rightarrow \gamma_{e^-} = 132 \Rightarrow \nu = 0.99997c$

Problem 6: Fixed-Target Pion Production

What is the minimal beam energy of a proton colliding with a proton at rest to produce a $p + n + \Pi^+$?

- The process can be written as: $p + p \rightarrow p + n + \Pi^+$.
  - In the center of mass frame, $E = 2\gamma m_p c^2 = (m_p + m_n + m_{\Pi^+}) c^2 \Rightarrow \gamma = 938 + 940 = 1402 \times 938 = 1.08; \beta = 0.37$.
  - Switching to the lab frame, $\beta = \frac{2\times0.37}{1+0.37^2} = 0.65 \Rightarrow \gamma = 1.32$
  - So, we need $E = \gamma m_p c^2 \approx 1.24 \text{ GeV}$ of beam energy in a proton on proton fixed target experiment to produce a $p + n + \Pi^+$.

Problem 7: Compton Effect

The energy of a photon is $E = h\nu = \frac{h}{\lambda}$. Calculate the change in the photon’s wavelength.
• Conservation of 4-momenta:

\[ p_\gamma + p_e = p'_\gamma + p'_e \]
\[ (p_\gamma - p'_\gamma)^2 = (p'_e - p_e)^2 \]
\[ p_\gamma^2 + p'_\gamma^2 - 2p_\gamma \cdot p'_\gamma = p_e^2 + p'_e^2 - 2p_e \cdot p'_e \]

We let \( c = 1 \) in the following derivation, and use the fact that \( m_\gamma = p_\gamma^2 = 0 \), \( \vec{p}_e = 0 \) and \( p_\gamma = E_\gamma(1, \hat{r}) \):

\[-2p_\gamma \cdot p'_\gamma = 2m_e^2 - 2p_e \cdot p'_e \]
\[-p_\gamma \cdot p'_\gamma = m_e^2 - p_e \cdot p'_e \]
\[-p_\gamma \cdot p'_\gamma = m_e^2 - m_e E'_e \]
\[ - (E_\gamma E'_\gamma - \vec{p}_\gamma \cdot \vec{p}'_\gamma) = m_e(m_e - E'_e) \]
\[ E_\gamma E'_\gamma(1 - \cos \phi) = m_e(E'_e - m_e) \]
\[ E_\gamma E'_\gamma(1 - \cos \phi) = m_e(E_\gamma - E'_e) \]

Since \( E_\gamma = \frac{\hbar}{\lambda} \),

\[ \frac{\hbar^2}{\lambda \lambda'}(1 - \cos \phi) = m_e \hbar \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) \]
\[ \Delta \lambda = \lambda' - \lambda = \frac{\hbar}{m_e}(1 - \cos \phi) \]
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