Problem 1: Decay rate

Calculate the lifetime of $A$, the particle in our toy experiment.

- The lowest-order contribution to $A \rightarrow B + C$ is shown in Figure 1. There is no internal line and one vertex. Following the Feynman rules we find a factor $-ig$ and a delta function which we have to drop and replace by an $i$. Thus, we get $M = g$.

![Figure 1](image-url)
The decay is then $\Gamma = \frac{g^2 |p|}{8\pi\hbar m_A^2 c}$ with the magnitude of the outgoing particles $|p| = \frac{c}{2m_A} \sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$. The lifetime for $A$ is then 
\[ \tau = \frac{1}{\Gamma} = \frac{8\pi\hbar m_A^2 c}{g^2 |p|} \]
Problem 2: Scattering cross section

Calculate the differential cross section for the process \( A + A \rightarrow B + B \)

- The scattering process at lowest order is shown in Figure 2.

![Figure 2: Lowest-order contribution to \( A + A \rightarrow B + B \).](image)

We have two vertices and one internal line with the propagator \( \frac{i}{q^2 - m^2_{C^2}} \), two delta functions \( 2\pi \delta^4(p_1 - p_3 - q) \) and \( 2\pi \delta^4(p_2 + q - p_4) \) and have one integration \( \frac{1}{(2\pi)^4} d^4q \).

This yields \(-i(2\pi)^4 g^2 \int \frac{1}{q^2 - m^2_{C^2}} \delta^4(p_1 - p_3 - q) \delta^4(p_2 + q - p_4) d^4q\).

Integrating and replacing the remaining delta function result in \( M = \frac{g^2}{(p_4 - p_2)^2 - m^2_{C^2}} \).

There is a second Feynman diagram (see Fig. 3) of the same order with \( p_3 \) and \( p_4 \) interchanged.

Looking at the specific example of \( m_A = m_B = m \) and \( m_C = 0 \) we find \( M = -\frac{g^2}{p^2 \sin^2 \theta} \) and for the differential cross section \( \frac{d\sigma}{d\Omega} = \frac{1}{2} \left( \frac{h c g^2}{16\pi E p^2 \sin^2 \theta} \right)^2 \).
Figure 3: Lowest-order contribution to $A + A \rightarrow B + B$. 