Problem 1: \( A \rightarrow B + B \)

- Is \( A \rightarrow B + B \) a possible process in the ABC theory?

- Suppose a diagram has \( n_A \) external \( A \) lines, \( n_B \) external \( B \) lines, and \( n_C \) external \( C \) lines. Develop a simple criterion for determining whether it is an allowed reaction.

- Assuming \( A \) is sufficiently heavy, what is the most likely decay mode, after \( A \rightarrow B + C \)? Draw a Feynman diagram for each decay.

1) No. The process is not possible.

2) Allowed if (and only if) \( n_A, n_B, \) and \( n_C \) are either all even or all odd.

Take the allowed diagram and snip every internal line. We now have \( n'_A = n'_B = n'_C = N \) 'external' lines, where \( N \) is the number of vertices. When we now reconnect the internal lines, each join removes two 'external' lines of one species. Thus when they are all back together, we have \( n_A = N - 2I_A, n_B = N - 2I_B, \) and \( n_C = N - 2I_C, \) where \( I_A \) is the number of internal \( A \) lines, and so on. Clearly, they’re all even, or all odd, depending on the number of vertices.
Given $n_A$, $n_B$, and $n_C$, pick the largest of them (say, $n_A$) and draw that number of vertices, with A, B, C as 'external' lines on each one. Now just connect up B lines in pairs (converting two 'external' lines into one internal line, each time you do so), until you're down to $n_B$ – as long as $n_A$ and $n_B$ are either both even or both odd, you will obviously be able to do so. Now do the same for $n_C$. We have constructed a diagram, then, with $n_A$ external A lines, $n_B$ external B lines, and $n_C$ external C lines.

![Figure 1: Answer.](image_url)