Problem 1: $\gamma$-matrices

By considering the three cases $\mu = \nu = 0$, $\mu = \nu \neq 0$, and $\mu \neq \nu$ show that $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$.

• Just consider the cases, $\mu = \nu = 0$, $\mu = \nu = k = 1, 2, 3$ and $\mu \neq \nu$ and use the commutation relations.

Problem 2: Negative energy solutions

Consider the $e^+e^- \rightarrow \gamma \rightarrow e^+e^-$ annihilation process in the center-of-mass frame where the energy of the photon is $2E$. Discuss energy and charge conservation for the two cases where:

(a) the negative energy solutions of the Dirac equation are interpreted as negative energy particles propagating backwards in time;
(b) the negative energy solutions of the Dirac equation are interpreted as positive energy antiparticles propagating forwards in time.

• (a) In the first interpretation (left diagram), the intial-state positive $e^-$ of energy $+E$ emits a photon of energy $2E$. To conserve energy it is now a negative energy $e^-$ and therefore propagates backwards in time. At the other vertex, the photon interacts with a negative energy $e^-$, which is propagating backwards in time and scattering results in a positive energy $e^-$.  


(b) In the Feynman-Stückelberg interpretation (right diagram), the initial-state positive $e^-$ of energy $+E$ annihilates with a positive energy $e^+$ to produce a photon of energy $2E$. At the second vertex the photon produces an $e^+e^-$ pair. All particles propagate forwards in time.