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Hello, and welcome back to 8.701. In this class, we'll talk about spin. If you remember the discussion on relativistic kinematics we had last week, you noticed that I discussed the decay of a pion, say a neutral pion, into an electron and-- a positron and an electron. And we were able to calculate the velocity of those two particles, the electron and the positron, quite easily by knowing the mass of the pion and the masses of the electron and the positron, which is served into the rest frame of the pion, and we can calculate our velocity.

I also told you that this decay is highly suppressed because of the spins of the particles involved. The pion has spin zero, and the electron and the positron have spin $1/2$, but it is not easily possible to align the electron and the positron such that the spins align to 0. So therefore, this decay is not usually possible.

Now, let's dive a little bit into this. In quantum mechanics, the spin of a particle with a vector is quantized, and in terms of its length and its components. You calculate the length of the spin vector [S] You find that it's square root of f times s plus 1 in units of \hbar . The components, and along any axis, actually-- and in this case here, the z -axis-- have eigenvalues, and they are listed here. And we find that there is $2s$ plus 1 possible values.

So I'll pick here, just arbitrarily, the z axis. But the question-- it's an obvious question-- which axis is a sensible choice for this problem? So I want you to actually stop here and think about this. What are sensible options? If you want to get an eigenvalue with the physical state of particles, which axis are the right ones to choose-- or, sensible? There's no right and wrong in this discussion.

Let me motivate this. If you look at the orbital momentum of a particle, that's given by $\mathbf{r} \times \mathbf{p}$, where \mathbf{p} is the momentum vector of the particle. So now, if you're looking at the total momentum, we have to look add the angular momentum and the spin of the particle together.

So as shown in this picture here, you see that the parallel component-- the component in parallel to the slide direction-- is 0 by definition, because the cross-product is defined-- the angular momentum is defined as a cross-product with the momentum. So this is a nice choice of coordinate system or of axis-- namely, that is the choice of the momentum of the particle.

So you find that the total momentum perpendicular is the spin of the particle perpendicular, and the transverse component is its angular momentum, the orbital angular momentum, and the spin of the particle in the transverse direction. This, then, immediately gets us to a new definition, then of helicity. You can define the helicity of a particle as the spin of the particle dotted with the momentum of the particle, and then normalized by the momentum.

So basically, for a fermion, which has a spin $1/2$, you get plus $1/2$ if the spin points in the momentum direction and minus $1/2$ if it points in the opposite direction. So now, if you go back to our particle here, off by here, decaying into an electron and a positron, spin is at $1/2$, but you find that an electron is a left-handed particle, so its helicity will point in this direction. And for the positron-- sorry-- for the positron, it points in the same connection.

So the pion here is spin 0, and if you discuss this in the rest frame of the pion, the electron and the positron fly off in opposite directions, which means that the spin doesn't align to 0. So that's why this is highly suppressed. It's not 0, because you can find-- you can put [INAUDIBLE] into the rest frame, where you're just basically looking at both particles from one side that was coming to you, and in that case, it's allowed. But the spintronic configuration is highly [INAUDIBLE].