Introduction to Nuclear and Particle Physics - 8.701
Lecture 2
SM processes and Relativistic Kinematics

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Lecture 2: Standard Model Interactions and Relativistic Kinematics

Overview:

1. Standard Model Interactions:
   a) Four Forces
   b) QED
   c) QCD
   d) Weak Interactions

2. Relativistic Kinematics:
   a) Lorentz Transformation
   b) Consequences of Lorentz Transformation
   c) Four Vectors
   d) Energy + Momentum
   e) Conservation Laws
1. Standard Model Interactions:

a) Four forces:

As far as we know, there are just four fundamental forces in nature:

- **Strong**, electromagnetic, weak and gravitational

The **Standard Model** refers to three of these fundamental forces!

Properties:

<table>
<thead>
<tr>
<th>Mediator</th>
<th>Strong</th>
<th>Electromag.</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>gluon ($m=0$)</td>
<td>$g=2$</td>
<td>$e^{-1} m=0$</td>
<td>4</td>
</tr>
<tr>
<td>Acts on</td>
<td>color charge</td>
<td>el. charge</td>
<td>flavor</td>
</tr>
<tr>
<td>Particles experiencing it</td>
<td>gluons, quarks</td>
<td>charged particles</td>
<td>leptons, quarks</td>
</tr>
<tr>
<td>Range</td>
<td>$\frac{\hbar}{m_{\pi} c}$</td>
<td>$\sim 10^{-14}$ F</td>
<td>$\frac{\hbar}{m_{\rho} c}$</td>
</tr>
<tr>
<td>Typical lifetime</td>
<td>$10^{-23}$ s</td>
<td>$10^{-16}$ s</td>
<td>$10^{-12}$ s</td>
</tr>
<tr>
<td>Typical cross-section</td>
<td>$10^{-6}$ mb</td>
<td>$10^{-2}$ mb</td>
<td>$10^{-6}$ mb</td>
</tr>
<tr>
<td>Typical coupling</td>
<td>(\alpha_S)</td>
<td>(\alpha_e)</td>
<td>(\alpha_W)</td>
</tr>
</tbody>
</table>

\(\alpha_S\): strong interaction

\(\alpha_e\): electromagnetic interaction

\(\alpha_W\): weak interaction
6. QED

All electromagnetic phenomena are ultimately reducible to the following elementary coupling:

\[ u(1) \] gauge theory

- two elements: charged particle, photon
- meaning: charged particle \( e \) enters, emits (or absorbs) a photon \( \gamma \) and emits \( e \)

1. Example of a complete process: \( \text{Møller scattering} \)

\[ e^- e^+ \rightarrow e^- e^+ \]

- meaning: Interaction of two electrons which is mediated by a photon
- classical case: \( \text{Coulomb repulsion} \)

2. Example of a complete process: \( \text{Shabba scattering} \)

\[ e^- e^+ \rightarrow e^- e^+ \]
Rule: Particles which are running "backward in time" are to be interpreted as the corresponding anti-particle!

Bhabha scattering is related to Upleler scattering by a general principle which is known as crossing symmetry:

\[ A + \beta \rightarrow C + D \]

Rule: Any particle can be "crossed" over the other side of the equation, provided it turns into its anti-particle:

\[ A \rightarrow \beta + C + D \]
\[ A + \bar{C} \rightarrow \bar{B} + D \]
\[ C + \bar{D} \rightarrow \bar{A} + \bar{B} \]

These processes are dynamically allowed, but not necessarily kinematically!

(e.g. if \( A \) weighs less than \( \beta, C \) and \( D \), then this process is kinematically not allowed.)

Bhabha scattering and Upleler scattering are related by crossing symmetry!
3. Example of complete process:

Pair annihilation
\[ e^- e^+ \rightarrow \sigma + \bar{\sigma} \]

4. Example of complete process:

Pair production
\[ \sigma + \bar{\sigma} \rightarrow e^- e^+ \]

5. Example of complete process:

Compton scattering
\[ e + \sigma \rightarrow e + \gamma \]
Remarks on Feynman diagrams:

1. Feynman diagrams are purely symbolic, they do not represent particle trajectories.

2. Example: "Components" of a Feynman diagram

3. Each Feynman diagram represents a matrix, \( M \), to account for a particular process.

4. Feynman rules enforce conservation of energy and momentum at each vertex, and hence for the diagram as a whole.

5. Procedure:
   a) Draw all diagrams that have the appropriate external lines.
   b) Sum of all diagrams with the given external lines represents the actual physical process.

Problem: There are infinite many Feynman diagrams?
Example: Møller scattering

Besides the above 2-vertex Feynman diagrams for Møller scattering, we have with 4 vertices:

Calculations get very complicated!

BUT: Each vertex contributes a factor $\alpha^2$.

$\alpha$: QED fine-structure constant (general: QED coupling constant)

Higher order terms are suppressed since $\alpha \ll 1$ (perturbation theory).

What matters 'mainly' is the leading order contribution!

However: There are many examples where measurements are so precise that higher order terms have to be taken into account! $\rightarrow$ Test of QED

Example: Measure monopole anom. mag. moment of muon.
AED vacuum polarization:

In AED, the vacuum behaves like a dielectric, creating electron pairs out of the vacuum:

\[
\begin{array}{c}
\begin{array}{c}
\text{or view it like this:} \\
\end{array}
\end{array}
\]

"bare" electron charge is screened eff. charge \ll bare charge

\[
\alpha = \frac{1}{\text{distance}}
\]
In Quantum Chromodynamics: $\mathfrak{su}(3)$ gauge theory

- $2$ charges of bosons (gluons)

Color plays the role of charge and the fundamental process is: quark $\rightarrow$ quark + gluon

1. Fundamental vertex:

```
        u(r)
        /\6------------------\6
       /                \  g(b, \bar{r})
      /                    \\
     u(b)
```

color is always conserved:
$\rightarrow$ gluon is carrying away the difference

2. Types of gluons: $9$ "possibilities"

- $r\bar{r}$, $r\bar{b}$, $r\bar{g}$, $b\bar{r}$, $b\bar{b}$, $b\bar{g}$, $g\bar{r}$, $g\bar{b}$, $g\bar{g}$

```
1\alpha = \frac{1}{\sqrt{2}}(r\bar{b} + b\bar{r})
1\beta = \frac{1}{\sqrt{2}}(r\bar{g} - g\bar{r})
1\gamma = \frac{1}{\sqrt{2}}(r\bar{g} + g\bar{r})
1\delta = \frac{1}{\sqrt{2}}(b\bar{g} - g\bar{b})
```

$1\gamma = \frac{1}{\sqrt{2}}(r\bar{b} + b\bar{r} + g\bar{g})$

"no net color"

... like a photon...
Note:

Continuum requires that all naturally occurring particles be color singlets. If it is a color singlet state, it must exist as a mediator, it should also occur as a free particle. As such a mediator: Exchange between color singlet particles, e.g., a proton and neutron $\rightarrow$ long-range force with strong coupling.

However: Strong force is of very short range. So: Experiments tell us that there are only 8 gluons: color octet: $SU(3)$

Because gluons themselves carry color (in contrast to the photon which is electrically neutral): Gluons couple directly to one another:

3 gluon vertices:

4 gluon vertices:

Calculations: Apply QCD Feynman rules to calculate various processes!

How does the QCD coupling constant behave compared to the QED coupling constant?
Clear difference between QED and QCD:

- Self-coupling of gluons which is absent in case of QED.

QCD vacuum polarization diagrams:

\[ \text{Quark polarization} \quad \text{and} \quad \text{Gluon polarization} \]

\[ \rightarrow \text{Behavior is opposite for } \alpha_s : \]

- Quark polarization: \( \alpha_s \) is large at short distances
- Gluon polarization: \( \alpha_s \) is small at short distances

A priori not clear, who wins!
The winner depends on the relative number of flavors ($f$) and colors ($n$):

Critical parameter: $a = 2f - 11n$

Standard model: $f = 6$, $n = 3$ \Rightarrow $a = -21$

\rightarrow QCD coupling constant decreases at short distances!

Another "part picture":

"asymptotic freedom" quarks are quasi-free + small coupling

Distance from bare quark color charge
Weak interactions:

- $SU(2)$ gauge theory: 3 mediators: $\omega^0$, $\omega^\pm$
- Quarks and leptons take part in weak interactions

2 types of interactions:

a) charged current: $W^\pm$

b) neutral current: $\omega^0$

1. leptons:

Example:

\[ \mu^- \rightarrow e^- + \nu_e + \bar{\nu}_e \]

\[ \bar{\nu}_e + e^- \rightarrow \nu_e + e^- \]

First "picture" of neutral weak process discovered at CERN in 1973.
The Glashow–Weinberg–Salam (GWS) model includes neutral weak processes as an essential ingredient. Their existence was confirmed experimentally at CERN in 1973.

Production of $Z^0$ at LEP: $\max E_e^+ E_{e^-}$ at $m_{Z^0}$

\[
\text{production of } Z^0 \text{ at LEP: max. } E_{e^+} E_{e^-} \text{ at } m_{Z^0}
\]

\[
\begin{array}{c}
e^+ \\
e^-
\end{array} \quad \rightarrow \quad \begin{array}{c}
f^+ \\
f^-
\end{array} \quad \rightarrow \quad \begin{array}{c}
\ell^+ \mu^+ \tau^+ \bar{\nu}_\ell \bar{\nu}_\mu \bar{\nu}_\tau \\
u \bar{d} \bar{s} \bar{c} \bar{b}
\end{array}
\]

\[
\begin{array}{c}
e^- \mu^- \tau^- \nu_\ell \nu_\mu \nu_\tau \\
u \bar{d} \bar{s} \bar{c} \bar{b}
\end{array}
\]

---

Established 3 families in nature!
2. Quarks:

**Fundamental Vertex:**  \( W^\pm \)

\[\begin{array}{c}
\sqrt{\frac{2}{3}} \text{q} \rightarrow W^- \rightarrow W^+ \\
\text{q} \rightarrow W^-
\end{array}\]

**Note:** A quark of charge \(-\frac{1}{3}\) (\(d, s, b\)) converts into the corresponding quark with charge \(+\frac{2}{3}\) (\(u, c, t\)) with the emission of \(W^-\) (vice versa for \(W^+\)).

*Flavor is not conserved in weak interactions!*

Since the quark flavor changes at a weak vertex \((W^\pm)\), as a quark color changes at a strong vertex, weak interactions are sometimes called *Parradyne.*

**Example:** \( \bar{\nu}_e \rightarrow e^- + \nu_e \)

\[\begin{array}{c}
\bar{\nu}_e \rightarrow W^- \rightarrow e^- + \nu_e
\end{array}\]
Beta-decay of the neutron:

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

**Fundamental vertex:** $Z^0$

\[ \nu_e \rightarrow e^- \text{ at } Z^0 \]

*Note: quark flavor is not changed!*

**Example:**

\[ \nu \rightarrow \nu + p \]

\[ \nu + p \rightarrow \nu + p \]
Flavor changing reactions: \( W^\pm \)

In the spirit of the charged weak coupling with respect to leptons which yield only changes within each family, i.e. \( e^- \leftrightarrow \nu e \); \( \mu^- \leftrightarrow \nu \mu \); \( \tau^- \leftrightarrow \nu \tau \), one might assume that this also holds for quarks:

\[
\begin{pmatrix}
    u \\
    d \\
    s \\
    b
\end{pmatrix}
\]

However:

The observed decay \( \Lambda \rightarrow \rho^- \pi^+ \) or \( \Xi^- \rightarrow \Lambda^- \pi^- \) involve the conversion of a strange quark into an up-quark:

**Note:** Flavor changes do occur not only within one family!
Therefore:

The flavor eigenstate $|u>$ is not the partner to the flavor eigenstate $|d>$, but to a linear combination of $d$, $s$, and $b$:

$$\begin{pmatrix} u' \\ d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c \\ t \end{pmatrix}$$

$|d'>$ can be expanded as a linear combination of $|d>$, $|s>$, and $|b>$:

$$\begin{pmatrix} |d'> \\ |s'> \\ |b'> \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} |d> \\ |s> \\ |b> \end{pmatrix}$$

Note:

The matrix is called after their "inventors":

3x3 Kobayashi - Maskawa matrix! The coefficients are sometimes expressed as cosine and sine values of an angle: Cabibbo angle $\theta_c$.

Earlier scheme (2 families): $|d'> = |d> \cos \theta_c + |s> \sin \theta_c$

$|s'> = |d> \sin \theta_c + |s> \cos \theta_c$
Experimental data:

\[ V_{ij} = \begin{pmatrix}
0.1741 & -0.0248 & -0.0012 \\
-0.0248 & 0.1728 & 0.0004 \\
-0.0012 & 0.0004 & 0.1674
\end{pmatrix} \]


Self coupling of weak bosons and photon:

Best remark:

\[ \alpha_i \]

\[ SU(3) \otimes SU(2) \otimes U(1) \]

Grand Unified Theory (GUT) "one force"
2. Relativistic Kinematics:

a) Lorentz Transformations:

Given two inertial frames S and S', with S' moving at uniform speed v with respect to S:

\[ \begin{align*}
x' &= \gamma (x - vt) \\
y' &= y \\
z' &= z \\
t' &= \gamma (t - \frac{v}{c^2} x)
\end{align*} \]

\[ \begin{align*}
x &= \gamma (x' + vt') \\
y &= y' \\
z &= z' \\
t &= \gamma (t' + \frac{v}{c^2} x')
\end{align*} \]

Events are related in space-time as follows (Lorentz transformations):

with: \[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \beta = \frac{v}{c} \]
Consequences:

1. Relativity of Simultaneity:
   If two events occur at the same time in $S$, but at different locations, then they do not occur at the same time in $S'$: With $t_1 = t_2$:
   \[
   t'_1 = t'_2 + \frac{v}{c^2} (x_2 - x_1)
   \]

2. Lorentz Contract'ın:
   A moving object is shortened by a factor $\gamma$:
   \[
   L = L' \cdot \gamma^{-1}
   \]

3. Time Dilation:
   Moving clocks run slow:
   \[
   \tau = \tau' \cdot \gamma
   \]
4. Velocity addition:

Particle moves with speed \( u \) with respect to \( S' \). What is the speed \( v' \) with respect to \( S \)? Note: \( S' \) moves with speed \( w' \) with respect to \( S \):

\[
\nu = \frac{\nu' + \nu}{1 + \frac{(\nu' \cdot \nu)}{c^2}}
\]

If \( \nu' = c \rightarrow \nu = c \)

C) Four vectors:

\[
\begin{align*}
x^0 &= c\cdot t ; \quad x^1 = x ; \quad x^2, \quad x^3 = z
\end{align*}
\]

Transformation:

\[
\begin{align*}
x'^0 &= \gamma (x^0 - \beta x^1) \\
x'^1 &= \gamma (x^1 - \beta x^0) \\
x'^2 &= x^2 \\
x'^3 &= x^3
\end{align*}
\]

\( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \)

Compact:

\[
\lambda = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} \cdot x^\nu
\]

\[
\Lambda = \begin{pmatrix}
\delta & -\gamma & 0 & 0 \\
-\gamma & \delta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
\[ x^\mu = \Lambda^\mu_{\nu} \cdot x^\nu \]

\textit{contravariant by Einstein: Repeated indices are to be summed!}

\textit{invariant:}

\[ I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x^0)^2 - (x^1)^2 + (x^2)^2 + (x^3)^2 \]

\[ \rightarrow \text{same in any inertial system!} \]

\textit{Different notation:}

\[ g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \]

\[ I = g_{\mu\nu} \cdot x^\mu \cdot x^\nu \]

\textit{covariant four vector:} \[ x_\mu = g_{\mu\nu} \cdot x^\nu \]

\textit{contra variant four vector:} \[ x^\mu \quad (I = x_\mu \cdot x^\mu) \]

\textit{metric tensor:} \[ g_{\mu\nu} \]
1. Any two four vectors:

\[ a^\mu, b_\mu = a_\mu \cdot b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3. \]

2. \( b_\mu - a_\mu \):

\[ a^2 = a \cdot a = (a^0)^2 - (a^i)^2. \]

- \( a^2 > 0 \): timelike
- \( a^2 < 0 \): spacelike
- \( a^2 = 0 \): lightlike

3. Energy + momentum:

\[ \eta^\mu = \gamma \cdot \eta^\nu \] (proper velocity): \( \eta^\mu = \gamma (c, \nu, \nu, \nu) \)

\[ \eta = m \cdot \eta^\mu = \gamma \cdot m \cdot \eta^\nu \] \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \)

\[ E = \gamma m c^2 = \frac{m c^2}{\sqrt{1 - \beta^2}} \] \( \rho^\mu = (\gamma E, \rho^1, \rho^2, \rho^3) \)

\[ \beta = \frac{\nu}{c} \]
e.) Conservation laws: Analysis of particle

Reaching:

Conservation of:

1. Energy and momentum
2. Angular momentum
3. Electric charge
4. Color charge
5. Baryon number: \( A = 1 \) for baryons; \( A = -1 \) for anti-baryons. \( A = 0 \) for non-baryons.

6. Lepton number:
   - Particles at each generation (leptons) are conserved: \( e^-, \mu^-, \nu_e \)
   - \( L = +1 \): particle
   - \( L = -1 \): anti-particle
   - Example: \( \bar{\nu}_e \to \mu^- + \bar{\nu}_\mu \)

7. Flavor is conserved in strong and electromagnetic interactions, but not in weak interactions (\( W^\pm \)).