Problem 1) Renormalization of $c_F(\mu)$

Draw the diagrams needed to compute the anomalous dimension of the coefficient $c_F(\mu)$ which appears in the $\mathcal{O}(1/m_Q)$ magnetic moment HQET Lagrangian $\mathcal{L}_F^{(1)}$. Discuss whether the kinetic energy Lagrangian $\mathcal{L}_K^{(1)}$ mixes with $\mathcal{L}_F^{(1)}$ under renormalization. Argue that the anomalous dimension vanish in the abelian case (and therefore is proportional to the adjoint Casimir $C_A$) without computing any integrals. (Hint: think about Coulomb gauge.)

Problem 2) Heavy-to-Light Currents in HQET

Consider the $\mathcal{O}(1/m_Q)$ heavy-to-light vector currents

\begin{align*}
O_1 &= \bar{q} \gamma^\mu i\not{D}Q_v, \\
O_2 &= \bar{q} v^\mu i\not{D}Q_v, \\
O_3 &= \bar{q} iD^\mu Q_v, \\
O_4 &= \bar{q} (-i v \cdot \not{D}) \gamma^\mu Q_v, \\
O_5 &= \bar{q} (-i v \cdot \not{D}) v^\mu Q_v, \\
O_6 &= \bar{q} (-i \not{D}^\mu) Q_v, \\
\end{align*}

(1)

with coefficients $B_1$ to $B_6$. Using reparameterization invariance determine which of these coefficients are fixed by the coefficients $C_1$ and $C_2$ of the leading order vector heavy-to-light currents $\bar{q}\gamma^\mu Q_v$ and $\bar{q} v^\mu Q_v$.

Problem 3) OPE for $B \to X_c e \bar{\nu}$

Use the results of Sec. 6.2 of your text to derive the $\lambda_2$ terms appearing in the double differential decay rate in Eq. (6.57). Explain why the $\lambda_1$ terms in Eq. (6.57) include one proportional to $\delta'(z)$ while the $\lambda_2$ terms do not. (You are not being asked to derive the $\lambda_1$ coefficients explicitly, just those for $\lambda_2$.)