8.871

Solutions to problem set #7

1.
a, b) There are two possible enhanced simple gauge groups (rank 17)
i) $SU(18)$

If $\frac{1}{gs} = 0$ at the position of an $08^-$, we can 'extract' a D8 brane. The $08^-$ becomes an $08^*$, with charge $-9$.

The equation for $\frac{1}{gs} \equiv \lambda$ is:

$$\frac{d^2 \lambda}{dx^2} = -\sum_i \frac{\lambda}{i} \delta(x-x_i) \quad (1)$$

Where $i$ runs over the objects in the interval, $x$ is the coordinate along the interval (0 $\leq x \leq R$) and $\lambda, x_i$ are the charges and locations of the objects.

We can integrate eq. (1):
\[ \frac{d\lambda}{dx} = -\frac{1}{2} \sum_i Q_i \Theta(x-x_i) \quad (2) \quad (\Theta(x) = \theta(x) - \theta(-x)) \]

Here we omit the integration constant, which is putting the background cosmological constant to zero.

And:

\[ \lambda(x) = -\frac{1}{2} \sum_i Q_i |x-x_i| + \lambda_0, \quad (\lambda_0 = \lambda(x=0)) \quad (3) \]

To get SU(18) we have to stack the D8 branes together, let's say at \( x = a \).

Eq. (3) becomes:

\[ \lambda(x) = \frac{1}{2} \left( 91x^2 - 18x^2 + 91x - 91 \right) + 9a = \frac{9}{2} R \]

We need to have \( \lambda(R) = 0 \) in order to have an O8* there. This gives:

\[ a = \frac{R}{2} \]

The plot of \( \frac{1}{y_s} \) is:

Moving the stack of D8 branes from \( B_2 \) will change \( \frac{1}{y_s} \) at the endpoints and spoil the enhancement. The stack must be fixed, and that's why we get SU(18) instead of U(18).
ii) $SO(34)$

\[ \chi(x) = \frac{1}{2} \left( 91x - 171x - R1 + 81x - R1 \right) + R \]

The plot of $\frac{1}{g^2}$ is:

Again, the D8 branes are stuck at $x = R$, otherwise the enhancement at $x = 0$ is broken.

(c, d) There are two possible enhanced gauge groups of the form $G_1 \times G_2$ with $G_1, G_2$ simple.

i) $E_{n+1} \times SU(17-n)$, $n = 0, 1, \ldots, 7$

If the stack of branes in the bulk is
Located at \( x = a \) the solution for \( \phi \) is:

\[
\lambda(x) = \frac{1}{2} \left[ (8-n) 1x1 - (17-n) 1x-a1 + 91x-R1 \right] + \lambda_0
\]

Demanding \( \lambda(0) = \lambda(R) = 0 \) gives:

\[
\begin{aligned}
\lambda_0 &= 0 \\
\frac{1}{2} = \frac{9}{17-n} \quad \Rightarrow \quad a = \frac{9}{17-n} R
\end{aligned}
\]

Note:

\[
\frac{R}{2} < a < R
\]

\[\text{ii) } E_{\text{type}} \times SO(2(16-n)) \quad \text{for } n = 0, 1, \ldots, 7
\]

\[
\lambda(x) = \frac{1}{2} \left[ (8-n) 1x1 - (8-n) 1x-R1 \right] + \frac{1}{2} (8-n) R
\]
2. a) We can read off the extra W-boson state from the $U(1)$ charge of the states under the decomposition $E_n \supset SO(2(n-1)) \times U(1)$.

This $U(1)$ quantum number is the quantised D0 brane charge and reveals the number of D0 (or D0̅) branes in the marginal massless bound state that is the W-boson for the enhanced gauge symmetry.

So, for $E_8$ and $E_7$ we have:

$$E_8 \supset SO(14) \times U(1)$$

$$248 \rightarrow \begin{array}{c} \downarrow \downarrow \\ 9_{10} + 14_2 + 14_{-2} + 1_0 + 6_{4,1} + 6_{4,-1} \\ 2D0 \quad 2D0̅ \\ b.s. \quad b.s. \end{array}$$

$$E_7 \supset SO(12) \times U(1)$$

$$133 \rightarrow \begin{array}{c} \downarrow \downarrow \downarrow \downarrow \\ 66_1 + 32_{1} + 32_{-1} + 1_2 + 1_{-2} + 1_0 \\ D0 \quad D0̅ \quad 2D0 \quad 2D0̅ \\ b.s. \quad b.s. \quad b.s. \quad b.s. \end{array}$$

For $E_6$ we have:

$$E_6 \supset SO(10) \times U(1)$$

$$78 \rightarrow 1_0 + 45_0 + 16_{-3} + 16_3$$

The same pattern persists for all lower
$E_n$, i.e. we get a sinlet, the adjoint of $SO(2(n-1))$, and two spinors with $U(1)$ charges $+r$ and 
$-r$. This seems to imply that we have bound 
states of $r$ $D0$'s and bound states of $r$ $\bar{D0}$'s 
($r=3$ for $E_6$). However it is possible that these 
$U(1)$ charges can be consistently renormalized 
to $\pm 1$, and it is not immediately clear what 
the correct choice is.

b) The $D0$-$D8$ strings have a zero point 
energy $\frac{1}{2}$ in the NS sector and $0$ in the 
R sector. The zero modes in the R sector 
carry an index $i$ coming from the D8 
branes. They satisfy a Clifford algebra 

$$\{A^i, A^j\} = \delta^{ij}$$

and so they transform in the spinor of 
the $SO(2n-2)$ part of the gauge group 
associated with the D8 branes.
3.

a) We can think of this background as Type I compactified on $T^3 = S^1 \times S^1 \times S^1$ with a T-duality performed in every compact direction.

The result is 8 f.p. each with an 06- plane spanning the seven non-compact directions and 16 D6 branes at generic points in $T^3/(\mathbb{Z}_2)^3$.

Each 06- has charge -2, for a total charge of -16, cancelled by the 16 D6 branes.
In addition to the 16 v-plets living on the D6 branes, we have 3 v-plets coming from the dimensional reduction of the gravity multiplet.

Each of these v-plets contains 3 scalars. The Narain moduli space is:

$$SO(19,3,\mathbb{R}) \backslash SO(19,3,\mathbb{R}) / SO(19,\mathbb{R}) \times SO(3,\mathbb{R}) , \quad \text{dim} = 19 \times 3.$$

Replacing an 06- with an 06+ (charge +2) increases the orientifold RR charge by four, so we have to decrease the number of D6 branes (and vector multiplets) by four to respect anomaly cancellation.

For $k$ 06+ we have:

$$\begin{align*}
#06^- & = #06^+ & #D6 & \quad \text{scalar manifold} \\
8 - k & = k & 16 - 4k & \quad SO(19 - 4k, 3, \mathbb{R}) \backslash \frac{SO(19 - 4k, 3, \mathbb{R})}{SO(19 - 4k, \mathbb{R}) \times SO(3, \mathbb{R})}
\end{align*}$$

with $k = 1, 2, 3, 4$.
b) The maximal rank $Sp$ gauge group is obtained by putting all the D6 branes in the background on top of a single $06^+$.

For $k$ $06^+$ in the background the maximal $Sp$ gauge symmetry is:

$$Sp(16-4k), \quad k=1,2,3,4.$$  *see last page

4.

a) $W$-boson:

$$\begin{array}{c}
04 \\
\downarrow \\
F1
\end{array}$$

Monopole:

$$\begin{array}{c}
04 \\
\downarrow \\
02
\end{array}$$

Instanton:

$$\begin{array}{c}
04 \\
\downarrow \\
00
\end{array}$$

b) $m_w = \langle \phi \rangle$

$$m_m = \frac{\langle \phi \rangle}{g^2}$$

$$m_i = \frac{1}{g^2}$$

c) $\begin{array}{c}
\alpha \\
04
\end{array}$

$$m_w = \frac{a}{l_s^2} \quad M_m = \frac{a}{g_s l_s^3}$$

$$m_i = \frac{1}{g_s l_s} \quad \left( \frac{1}{g^2} = T_{D4} \cdot l_s^4 \right)$$
d) \( g_s l_s = R \), \( g_s l_s^3 = l_p^3 \)

\[ m_w = \frac{a R}{l_p^3} \], \[ m_m = \frac{a}{l_p^3} \], \[ m_i = \frac{1}{R} \]

e) \( W\)-boson \( \rightarrow \) wrapped M2, stretched between wrapped M5',

monopole \( \rightarrow \) infinite M2, stretched between wrapped M5',

instanton \( \rightarrow \) Kaluza-Klein mode.

* A more careful study of the background in problem 3 will reveal obstructions coming from discrete torsion that limit the number of 06+ orientifolds to 0, 2 or 4. Moreover, there are two inequivalent configurations for 4 06+, depending on whether they are coplanar or not.