1 Morrow 4.11 (pp.107-8)

Note that the ideal point of the median voter is \( y_n \), and that the mid-point between \( x_1 \) and \( x_2 \) is \( (x_1 + x_2)/2 \).

a) Partition the set of ideal points and call \( n_l = |\{i|y_i < (x_1 + x_2)/2\}| \), \( n_r = |\{i|y_i > (x_1 + x_2)/2\}| \), \( n_c = |\{i|y_i = (x_1 + x_2)/2\}| \).

Voters at the midpoint vote for candidate 1 with probability 1/2, and for candidate 2 with probability 1/2.

Let \( v_j \) be the expected number of votes for party \( j \); \( v_1 = n_l + \frac{n_r}{2}; v_1 = n_r + \frac{n_l}{2} \).

Let \( u_j \) be the utility of party \( j \). Then \( u_1 = v_1 - v_2 = n_l - n_r; u_2 = -u_1 = n_r - n_l \).

b) If \( i < n \), candidate 1 should choose \( x_1 \) such that \( x_2 < x_1 < 2y_{i+1} - x_2 \).

If \( i \geq n \) and \( x_2 > y_i \), candidate 1 should choose \( x_1 \) such that \( 2y_i - x_2 < x_1 < x_2 \).

If \( i > n \) and \( x_2 = y_i \), candidate 1 should choose \( x_1 \) such that \( 2y_{i-1} - x_2 < x_1 < x_2 \).

If \( i = n \) and \( x_2 = y_i \), candidate 1 should choose \( x_1 = x_2 \).

Both candidates choose the ideal point of the median voter, in which case both get utility of 0. If any candidate chooses a position marginally to the left of the right of the median voter, then that candidate would lose and get negative utility. So we get convergence at the median.