1. Colonel Blotto has 3 divisions with which to defend 2 mountain passes. His opponent also has 3 divisions. Blotto successfully defends a pass if and only if he allocates an equal or greater number of divisions to it as his opponent. Blotto wins if he successfully defends both passes, and loses otherwise. Blotto’s payoff is 1 if he wins, -1 if he loses; the opponent’s payoff is 1 if he wins, and -1 if he loses.

a) Find a mixed-strategy Nash Equilibrium for the game.
b) If it is unique, prove that it is so. If not, find another Nash Equilibrium.

2. A set of \( n \) firms seek to bribe the government into granting them a monopoly. The monopoly is worth \( W_i \) to each firm \( i = 1, \ldots, n \), where \( W_i = W \) for all \( i \). Firm \( i \) offers bribes of \( x_i \). Given the bribe vector \( (x_1, \ldots, x_n) \), the probability the government grants the monopoly to firm \( i \) is

\[
p_i(x_1, \ldots, x_n) = \frac{\alpha_i x_i^r}{\sum_{j=1}^{n} \alpha_j x_j^r}
\]

Firm \( i \)'s expected payoff is

\[
\pi_i(x_1, \ldots, x_n) = p_i(x_1, \ldots, x_n)W - x_i
\]

In solving the model, make any assumption you need on \( W \).

(a) Suppose \( n = 3, \alpha_1 = \alpha_2 = 2, \alpha_3 = 1, \) and \( r = 1 \). Find a pure-strategy Nash equilibrium.

(b) Suppose \( n = 3, \alpha_i = 1 \) for all \( i \), and \( r = 2 \). Find a pure-strategy Nash equilibrium.