Case Study
Multiple Internal Rates of Return for a Stadium Project

This case involves a hypothetical example in which there are multiple internal rates of return. The case is structured as the kind of situation that could face a young engineering economist trying to rank alternative projects for senior management. If the analysis is done using net present value, then the potential benefits of the project are clear. But when senior management insists upon using the internal rate of return, all kinds of problems emerge.

You have accepted an internship position that gives you a chance to do what you always wanted to do – work in the front office of a major league team. With your background in project evaluation, you have been asked to calculate the financial potential of the proposed new stadium. At a meeting with the president, the general manager, the VP finance, and several consultants, you learn the general plan. The stadium will be constructed over two years, with a total cost of $250 million. Constructing the stadium will lead to two very large immediate payoffs from selling 20-year leases on corporate boxes and selling real estate around the stadium to various hotel and restaurant chains. Revenue will be about $8 million per year in the first 10 years, after which a major rehabilitation and expansion is planned at cost of $50 million. With a somewhat larger stadium, revenues are expected to rise. Management plans to use all of that revenue plus some additional funds for further development of the area; thus cash flows are expected to be negative for several years before there is another big payout in year 20 when they are able to complete and sell more of the real estate. Table 1 summarizes the expected cash flows.

Table 1 Expected Cash Flows for Stadium Project

Year	Investment	Revenue	Cost	Cash Flow
0	100	0		-100
1	150	10		-140
2		150	2	148
3		150	2	148
4		10	2	8
5		10	2	8
6		10	2	8
7		10	2	8
8		10	3	7
9		10	3	7
10		0	50	-50
11		30	20	10
12		30	25	5
13		30	30	0
14		30	35	-5
15		30	40	-10
16		30	50	-20
17		30	60	-30
18		30	70	-40
19		30	90	-60
20		100	10	90

You create a spreadsheet with all of this information and then set out to determine the internal rate of return. First you try 8%, but that gives a positive NPV of $2.8 million; then you try 12%, but that gives a negative NPV of $4.8 million. You figure the NPV will be zero somewhere in the middle of these two, and you eventually determine that the NPV is zero with a discount rate of 9.85%. You therefore prepare a presentation that concludes that the stadium project has

an internal rate of return of "just under 10%". You're quite proud of your work and rush into show your boss your result (Table 2).

Table 2 IRR for Stadium Project is Nearly 10%

Year	Cash Flow	8%	12%	9.85%
0	-$100.0	-$100.0	-$100.0	-$100.0
1	-$140.0	-$129.6	-$125.0	-$127.4
2	$148.0	$126.9	$118.0	$122.6
3	$148.0	$117.5	$105.3	$111.7
4	$8.0	$5.9	$5.1	$5.5
5	$8.0	$5.4	$4.5	$5.0
6	$8.0	$5.0	$4.1	$4.6
7	$8.0	$4.7	$3.6	$4.1
8	$7.0	$3.8	$2.8	$3.3
9	$7.0	$3.5	$2.5	$3.0
10	-$50.0	-$23.2	-$16.1	-$19.5
11	$10.0	$4.3	$2.9	$3.6
12	$5.0	$2.0	$1.3	$1.6
13	$0.0	$0.0	$0.0	$0.0
14	-$5.0	-$1.7	-$1.0	-$1.3
15	-$10.0	-$3.2	-$1.8	-$2.4
16	-$20.0	-$5.8	-$3.3	-$4.4
17	-$30.0	-$8.1	-$4.4	-$6.1
18	-$40.0	-$10.0	-$5.2	-$7.4
19	-$60.0	-$13.9	-$7.0	-$10.1
20	$90.0	$19.3	$9.3	$13.7
Total		**$2.8**	**-$4.3**	**$0.0**

Your boss is rather more subdued that you were, as he understands that 10% is no great rate of return for the crafty men and women who run the major league team. He also notes that the cash flows are weird, with several shifts between positive to negative (Figure 1). He fears that there might be a problem with the IRR that you calculated. He therefore runs the cash flows through his program and quickly obtains what is shown as Table 3. A quick look at the bottom row of this table indicates that the NPV is zero when the discount rate is 2%, suggesting that the IRR is a dismal 2%. On the other hand, the table also supports your calculation, as the NPV is also zero for something a little less than 10%.

Now you and your boss can't go to the CEO and the stadium committee and say that the project is perhaps OK, with an IRR of nearly 10%, except that it may be dismal, with an IRR of only 2%. You need to fix this problem – and you need to fix it fast!

Figure 1 Expected cash flows for the stadium project

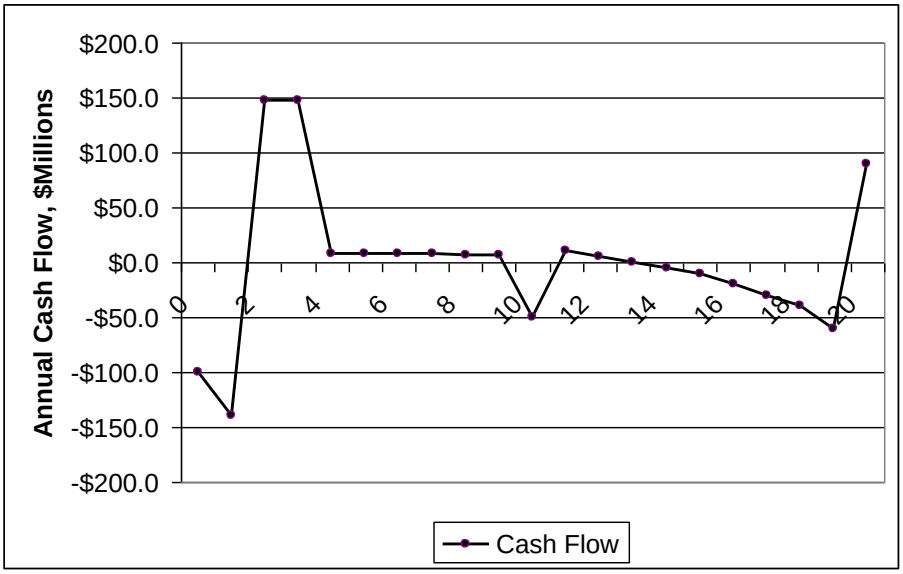

Table 3 NPV of stadium cash flows, showing sensitivity of discounted cash flows to discount rate

Year	Cash Flow	Discounted Cash Flows for the Given Discount Rate								
		1%	2%	4%	6%	8%	10%	12%	14%	16%
0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0
1	-140.0	-138.6	-137.3	-134.6	-132.1	-129.6	-127.3	-125.0	-122.8	-120.7
2	148.0	145.1	142.4	136.8	131.7	126.9	122.3	118.0	113.9	110.0
3	148.0	143.6	139.7	131.6	124.3	117.5	111.2	105.3	99.9	94.8
4	8.0	7.7	7.4	6.8	6.3	5.9	5.5	5.1	4.7	4.4
5	8.0	7.6	7.3	6.6	6.0	5.4	5.0	4.5	4.2	3.8
6	8.0	7.5	7.1	6.3	5.6	5.0	4.5	4.1	3.6	3.3
7	8.0	7.5	7.0	6.1	5.3	4.7	4.1	3.6	3.2	2.8
8	7.0	6.5	6.0	5.1	4.4	3.8	3.3	2.8	2.5	2.1
9	7.0	6.4	5.9	4.9	4.1	3.5	3.0	2.5	2.2	1.8
10	-50.0	-45.3	-41.2	-33.8	-27.9	-23.2	-19.3	-16.1	-13.5	-11.3
11	10.0	9.0	8.1	6.5	5.3	4.3	3.5	2.9	2.4	2.0
12	5.0	4.4	4.0	3.1	2.5	2.0	1.6	1.3	1.0	0.8
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	-5.0	-4.3	-3.8	-2.9	-2.2	-1.7	-1.3	-1.0	-0.8	-0.6
15	-10.0	-8.6	-7.5	-5.6	-4.2	-3.2	-2.4	-1.8	-1.4	-1.1
16	-20.0	-17.1	-14.7	-10.7	-7.9	-5.8	-4.4	-3.3	-2.5	-1.9
17	-30.0	-25.3	-21.6	-15.4	-11.1	-8.1	-5.9	-4.4	-3.2	-2.4
18	-40.0	-33.4	-28.3	-19.7	-14.0	-10.0	-7.2	-5.2	-3.8	-2.8
19	-60.0	-49.7	-41.6	-28.5	-19.8	-13.9	-9.8	-7.0	-5.0	-3.6
20	90.0	73.8	61.2	41.1	28.1	19.3	13.4	9.3	6.5	4.6
Total	**-$8.0**	**-$3.3**	**$0.0**	**$3.8**	**$4.4**	**$2.8**	**-$0.3**	**-$4.3**	**-$8.9**	**-$13.8**

The multiple estimates of IRR occur because the cash flows shift several times between positive and negative. By using the external rate of return method, it is possible to get a better estimate of the return on investment for the stadium project. The costs are all discounted to the present using a rate of 8%, which is the MARR for the stadium owners. The future value of annual revenues is calculated as of year 20 (using the same rate of 8%). The NPV of the costs is $399 million, while the future value FV of the benefits is $1,873 million, as shown in Table 4. The return on investment calculated with the ERR method is the annual rate of return at which $399 million would grow to $1,873 million in 20 years. The answer can be gained by solving the following equation for i%.

$$FV = NPV\ (1+ i\%)^{20}$$

Trial and error on a spreadsheet shows the answer to be 8.04%, so the report to the CEO and the committee could indicate that the project has an expected return on investment of 8%.

Table 4 Calculating the External Rate of Return for the Stadium Project

Year	Investment	Revenue	Cost	Cash Flow	Cost	NPV of Costs	FV of Benefits
0	$100	$0		-$100.0	-$100.0	-$100.0	$0.0
1	150	10		-140.0	-150.0	-138.9	43.2
2		150	2	148.0	-2.0	-1.7	599.4
3		150	2	148.0	-2.0	-1.6	555.0
4		10	2	8.0	-2.0	-1.5	34.3
5		10	2	8.0	-2.0	-1.4	31.7
6		10	2	8.0	-2.0	-1.3	29.4
7		10	2	8.0	-2.0	-1.2	27.2
8		10	3	7.0	-3.0	-1.6	25.2
9		10	3	7.0	-3.0	-1.5	23.3
10		0	50	-50.0	-50.0	-23.2	0.0
11		30	20	10.0	-20.0	-8.6	60.0
12		30	25	5.0	-25.0	-9.9	55.5
13		30	30	0.0	-30.0	-11.0	51.4
14		30	35	-5.0	-35.0	-11.9	47.6
15		30	40	-10.0	-40.0	-12.6	44.1
16		30	50	-20.0	-50.0	-14.6	40.8
17		30	60	-30.0	-60.0	-16.2	37.8
18		30	70	-40.0	-70.0	-17.5	35.0
19		30	90	-60.0	-90.0	-20.9	32.4
20		100	10	90.0	-10.0	-2.1	100.0
Total				-$8.0		-$399.1	$1,873.2

The external rate of return approach is favored by academics, as it avoids the necessity of implying unreasonable returns for reinvesting profits, and it provides a reasonable means of dealing with future periods with negative cash flow. However, this approach is unlikely to be encountered outside of textbooks. Public agencies are apt to consider ratios of benefits to cost rather than ROI, while private companies use the internal rate of return as an easier and apparently more objective result.

Resource: Project Evaluation: Essays and Case Studies

Carl D. Martland