# Equivalence of Cash Flows

*"What it comes down to is pieces of paper, numbers, internal rate of return, the net present value, discounted cash flows – that's what it's all about. ... Sure, we want to build quality and we want to build something that is going to be a statement, but if you can't do that and still have it financed and make a return, then why are we doing it?"*

Terry Soderberg[1]

## Introduction

Many processes are needed to assess the performance of infrastructure projects and to evaluate alternatives for improving performance. Inevitably, there will be many aspects of performance to consider and many possible impacts on society or the environment that must be minimized or mitigated. Financial analysis, which is concerned with the cash flows directly related to a project, will be critical, but so will economic analysis, which also includes the impacts of a project on the overall economy. Both financial and economic impacts can be measured in monetary terms; which types of impacts are considered will depend upon who is doing the analysis. Owners, developers and users will largely be interested in financial matters; public agencies that must approve projects are concerned with broader economic matters, such as job creation and regional prosperity.

Financial impacts concern the cash flows that are directly related to the project; these cash flows are what will determine the profitability of the project and the returns to those who invest in the project. If there is not enough cash to support construction, then a project will not be completed; if it appears that revenues will be insufficient to provide a desirable rate of return for investors, then they will not invest in the project. In Panama, the French effort to build the canal suffered from the extraordinary loss of life, but the project failed because the interest charges on the construction debt rose higher than predicted revenues – and the canal was nowhere near to being finished.

Economic impacts are broader than financial impacts, as they may include costs and benefits related to such things as consumer surplus, multiplier effects of construction, or safety, all of which are important factors to the public, even though they may not show up as revenues or expenses related to the investment in or operation of the project. Financial and economic impacts together are likely to motivate many projects, but there will also be social and environmental objectives and constraints that must be considered in proposing and evaluating projects that are aimed at enhancing the sustainability of infrastructure-based systems. The Panama Canal was eventually completed only when the U.S. government, which had strategic interests in a shorter all-water link between its east and west coasts, was able to invest what was needed to complete a less ambitious project, namely a canal with locks rather than a sea-level canal.

In this essay, the focus is on economic and financial aspects of projects, not because these are the most important measures of success, but because there are well-defined techniques that are commonly used to evaluate financial feasibility and economic desirability of a project. The concept of **net present value** (NPV) provides a very useful method for determining whether the predicted future benefits of projects justify the investment. NPV also provides a useful way for comparing different alternatives that may be proposed for a project.

When seeking ways to improve infrastructure performance, there will always be many alternatives to consider. It may be possible to improve performance by investment in markedly different types of infrastructure, by regulating land use or development or by subsidizing certain types of activities. To determine which is best, from an economic or financial perspective, it will be necessary to compare costs and benefits over a long time period. Calculating the NPV for each option provides a convenient way to make such a comparison.

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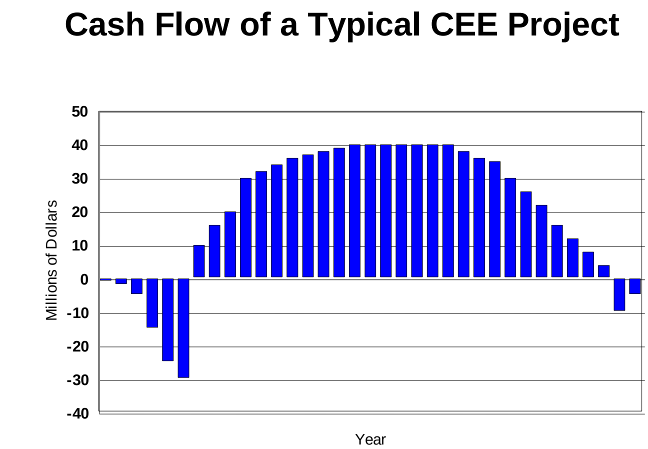
[1] Terry Soderberg was in charge of leasing a 50-story office tower for the Worldwide Plaza, quoted by Karl Sabbagh in **Skyscraper: The Making of a Building**, Penguin Books, NY, NY, 1991, p. 377

To simplify the presentation, let's focus on financial matters.[2] For each major alternative, it is necessary to predict cash flows over a long time horizon, taking into account the costs of construction, the continuing costs of maintenance, and the costs and revenues related to operations. A typical proposal will have cash flows similar to those shown in Figure 1, which shows the net annual cash flows over the life of a hypothetical project. Net cash flows are the sum of revenues, subsidies and any other source of income minus investment, operating, maintenance and any other type of expense. In a typical proposal, cash flows are negative at the outset, because of the expenses related to planning, site acquisition, and construct ... ıg costs of operation and maintenance. Eventually ... net cash flows begin to decline. At the end of the ... .




The alternatives that must be investigated may have sizable differences in terms of investments, construction costs, performance capabilities, and projected operating costs and revenue potential. Comparisons among options with markedly different cash flows will be difficult. For example, how should a low-cost option be compared with an option that requires much higher investment, but offers a chance to earn more money over a longer time frame? To answer questions like this, it is necessary to understand a basic concept of engineering economics, namely the **equivalence of cash flows**. If someone – an individual, a public agency or a company – is indifferent between two projected streams of cash flows, then those cash flows can be viewed as equivalent for that person, agency, or company. It is particularly useful to be able to take the complex flows of a typical project and compare them to something that is equivalent, but easier to understand. One obvious possibility would be to determine the amount of cash – the net present value – that would be equivalent to each projected stream of cash flows. The projected cash flows for each alternative could then be reduced to something as simple as a deposit to or a withdrawal from a bank account, either today or at some point in the future. Comparing alternatives would then be trivial, at least in terms of financial matters: the bigger the deposit the better, any deposit is better than any withdrawal, and if the best option is equivalent to a withdrawal then it is clear that there better be some non-financial objective for pursuing the project! This is why net present value is such a widely used measure of financial performance.

The next section goes into more detail concerning the time value of money, the need to discount future cash flows, the concept of a discount rate, and the concept of equivalence. Equivalence relationships can most easily be understood in the context of fixed interest payments, where there is a well-defined relationship between money invested today and the interest that will be earned over time

[2] Discounting and NPV analysis can be applied to any stream of costs and benefits that can be expressed in monetary terms. It is convenient to begin by focusing on the cash flows that are directly related to the project, as the cash flows are well-defined and easily understood. Moreover, for most investors, private companies, and entrepreneurs, the analysis of cash flows dominates their concerns.

**Time Value of Money**

Cash today is worth more than a promise that you will receive the same amount of cash in the future. There are several reasons why this is so, including the opportunities for investing the cash today, the likelihood of inflation, and the risk that the promised cash will not materialize. If the money is invested in a low-risk investment, such as savings bonds or a savings account at a bank, then the money will earn interest and the total amount available will be greater in the future. If the money is invested in stocks, bonds, or real estate, there could be even greater returns. Thus, there is an **opportunity cost** if money is only available in the future rather than being available today.

The second major reason for preferring money today rather than in the future is that **inflation** will generally reduce what can be bought with a given amount of money. Having the same amount of money in the future will not be as good as having the money today because it will not purchase as many goods and services.

The third major reason for preferring money today is **financial risk**: something could go wrong, causing the future payment to be smaller or later than expected. If the money is coming from the anticipated sale of property, there could be less than expected if there is a decline in the housing market. If the money is coming from the repayment of a loan, perhaps the borrower will be unable to make the payment. If the money is linked to some sort of international deal, perhaps a change in government will reduce the revenue from the deal.

These and other factors all affect the time value of money. Since people have different needs and expectations about the future, they will vary in their perceptions of investment opportunities, inflation, and financial risk, and different people and different organizations will put different relative values on current and future sums of money. For now, suffice it to say that there are several major reasons why money in the future is less valuable than money at hand in the present. Therefore, it is necessary to **discount** future cash flows. "Discount" comes from a Latin term that means "count for less," so discounting future cash flows means that they will count for less when evaluating a project. The **discount rate** is defined as the annual percentage by which future cash flows (a **future value**) must be reduced (discounted) to a **present value** for comparison with current cash.

The simplest way to understand discounting is to consider the benefits of investing in something safe that earns a respectable, steady i% interest per year. After one year, the money will have increased by a factor of (1+i%). If the same interest rate is maintained for t years, the money will have grown by a factor of $(1+i\%)^t$. In other words, assuming an interest rate of i%, M dollars today is equivalent to $M(1+i\%)^t$ dollars in the future. To look at this same situation from the perspective of the future, M dollars in the future would be equivalent to $M/(1+i\%)^t$ dollars today if those M dollars were invested and earning i% per year.

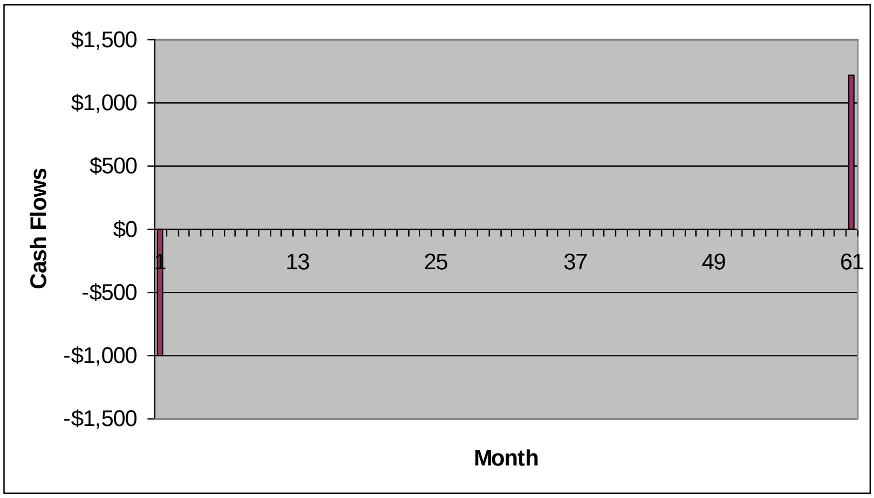
For example, suppose you deposit $1,000 in a bank that pays 4% interest at the end of each year. How much will you be able to withdraw at the end of five years? To determine the future value of your deposit, it is necessary to begin by calculating the annual interest that will be received at the end of the first year and adding this interest to your account. If the interest rate is 4%, then the value of the account will increase by 4% at the end of the year. The same procedure can be repeated for four more years. The results will be as shown in Table 1; the value at the end of one year equals the value at the beginning of the next year, and the value at the end of five years will be $1,216.65. This result could also be obtained directly as $\$1000 * (1.04)^5 = \$1,216.65$.

**Table 1 Future Value of Money Deposited in a Bank Account**

| Year | Value at Beginning of Year | Interest Rate | Interest Received at End of Year | Value at End of Year |
|---|---|---|---|---|
| 1 | $1,000 | 4% | $40 | $1,040.00 |
| 2 | $1,040 | 4% | $41.60 | $1,081.60 |
| 3 | $1,081.60 | 4% | $43.26 | $1,124.86 |
| 4 | $1,124.86 | 4% | $44.99 | $1,169.86 |
| 5 | $1,169.86 | 4% | $46.79 | $1,216.65 |

It is often useful to prepare a **cash flow diagram** that depicts exactly what is being analyzed. In such a diagram, it is important to know whether cash flows occur at the beginning, middle, or end of the period. The end of one period can be assumed to equal the beginning of the next period. Figure 2 shows the cash flow diagram for Table 1. In this diagram, it is assumed that cash flows occur at the beginning of the period. From your perspective, there are just two cash transactions: a deposit of $1000 deposit is made today and a withdrawal of $1216.65 is made five years later. The annual interest payments will be added to your account and will not be taken as cash; they therefore are not shown on this chart. The chart shows the deposit at the beginning of month 1 and the withdrawal five years later at the beginning of month 61.

**Figure 2 Cash Flow Diagram for Table 1**




In financial terms, the $1,000 that is invested at the beginning of the first month is equivalent to the $1,216.65 that will be available at the end of the last month. The same chart could be used to illustrate the amount of money that must be invested today to grow to $1,216.65 in 60 months.

The choice of a discount rate is a very important issue in project evaluation, because large investments in the near future must be justified by benefits that are achieved over a long time period. The more that those benefits are discounted, the harder it is to justify the investment. Table 2 illustrates the present value of $100 received five to 100 years in the future and discounted at a rate of 1 to 20%. The higher the discount rate or the longer the period, the lower the present value.

**Table 2 Present Value of $100 Received in 5, 10, 20 or 50 Years**

| Discount Rate | 5 Years | 10 Years | 20 Years | 50 Years |
|---|---|---|---|---|
| **1%** | $95 | $91 | $82 | $61 |
| **5%** | 78 | 61 | 38 | 8.80 |
| **10%** | 62 | 38 | 15 | 0.90 |
| **20%** | 40 | 16 | 2.60 | 0.11 |

## Equivalence Relationships

Equivalence is a key concept in project evaluation and project finance. Two projected streams of cash flows are equivalent for someone if they are equally acceptable, i.e. if the individual is indifferent to receiving one or the other. As noted in the introduction, our goal is to transform an arbitrary stream of cash flows into an equivalent cash flow that is easily understood, such as present value, future value, or uniform annual value.[3] Given a discount rate, it is possible to calculate these three measures so that each is equivalent to a given stream of cash flows:

1. Present value or present worth (P): the equivalent present value of the cash flows (what is the projected stream of cash flows worth today?)
2. Future value or future worth (F): the equivalent future value of the cash flows (what is it worth at a specified time in the future?)
3. Annual value or annuity worth (A): the equivalent annuity amount (what is it worth in terms of receiving a uniform cash flow of A at the end of each period for a specified number of periods).

P, the present value of the cash flows, is clearly the easiest to understand. If we have a choice among various alternatives, each of which is equivalent to receiving a lump sum of money today, then the larger the present value the better (assuming for now that money is our chief object in life and that the money related to the various projects is indeed legally acquired!).

The other two measures are also easy to understand. If our time of reference is some time in the future, we will presumably want the alternative with the maximum future value. If we are more comfortable dealing with monthly or annual cash flows, then we can express options in terms of an annuity and choose the one that pays the most per period.

The basic tasks in financial analysis of a project can therefore be summarized as follows:

- Predict the cash flows over the life of the project
- Estimate the net present value of the cash flows
- Calculate the equivalent future value or annuity, if desired
- Rank projects by P, F, or A (since they are equivalent measures, the ranking will be the same)

Let's assume that we are a private company considering various projects that we can finance by borrowing from a bank at an interest rate of i%. For simplicity, assume that we can also put money into a savings account at the bank and also receive i% interest. Our choices therefore are either to invest in more projects or to put money into our bank account. If a project earns more than i%, then we will borrow money from the bank; if it earns less than i%, we will be better off putting money into our bank account.

It is useful to introduce some notation for discounting. [F/P,i,N] can be used to denote the factor that calculates the future value F as a fraction of the present value P, assuming a discount rate of i% for N periods. [P/F,i,N] denotes the factor that is used to determine the present value P given the future value F, again assuming a discount rate of i% for N periods. The two factors are used as follows to compare future and present values:

(Eq. 1) $$F = P * [F/P,i,N] = P * (1+i)^N$$

(Eq. 2) $$P = F * [P/F,i,N] = F / (1+i)^N$$

It is a small step from discounting one future payment to discounting all anticipated cash flows over a project's time horizon. If you have a spreadsheet, you can use these equations repeatedly to convert an arbitrary cash flow into either a present or a future value. The present value of the entire stream of cash flows will be the sum of the individual discounted cash flows.

[3] Present value, future value, and equivalent uniform annual value are the terms most commonly used in general business. Present worth PW, future worth FW, and annuity worth AW are encountered in many engineering economics textbooks. This text uses the two sets interchangeably; the very common use of net present value may make the use of present and future value more desirable.

The term **net present value** is commonly used to denote the present value of a stream of cash flows that includes both costs C(t) and benefits B(t) over a designated time horizon. Given a discount rate i, it is straightforward to calculate the net benefits during any period, the present value of those benefits, and the net present value (NPV) of the entire project:

(Eq. 3) Net benefits during period t = B(t) – C(t)

(Eq. 4) Present value of net benefits in period t = NPV(t) = $(B(t) - C(t))/(1 + i)^t$

(Eq. 5) NPV(project) = $\Sigma((B(t) - C(t))/(1 + i)^t)$ for the life of the project

Sometimes it is desirable to consider an annuity rather than a present or future value. An annuity can be compared to other measures reported annually, such as revenue, operating costs, or profitability. There are multiple ways to find the equivalent annuity. Since an annuity of A per period is certainly one possible cash flow, you can find the annuity that is equivalent to either a present or future value using the above equations. If you make interest rate, annuity amount, and the number of periods a variable, you can easily find the annuity amount that is equivalent to any present or future value. However, it can be more elegant (and less time-consuming) to use algebraic expressions to convert P or F into annuities (or to convert annuities into P or F).

The equivalent future value F of an annuity A is the amount that would be accumulated by the end of the last payment assuming that all payments were invested so as to grow at a rate equal to the discount rate i. As was the case in calculating the relationships between P and F, there will be an equivalence factor that can be used to find F as a function of A. This factor depends upon the discount rate per period and the number of periods. It is denoted [F/A,i,N], and it is called the **uniform series compound amount factor**. It is assumed that a payment of A is made at the **end** of each period and that each payment is invested at i% per period for the remaining periods.

Given i and N, the future value will be proportional to A, and [F/A,i,N] will be the proportionality factor:

(Eq. 6) F = A * [F/A,i,N]

There is a simple algebraic expression for [F/A,i,N]:

(Eq. 7) $[F/A,i,N] = [(1+i)^N-1]/i$

Values for the expression can be found in tables, and spreadsheets have functions that will calculate the expression for you.

Derivation of the Uniform Series Compound Amount Factor [F/A,i,N]

The derivation of this expression for [F/A,i,N] is based upon well-known relationships for geometric sequences. First, we can calculate F just by converting each payment A(t) to a value at the end of the Nth period. The payment at the end of period t will earn interest for another N-t periods, so its future value will be:

(Eq. 8) $A(t) = A(1+i)^{N-t}$

Summing over the entire N periods:

(Eq. 9) $F(N) = A ((1+i)^{N-1} + (1+i)^{N-2} + ... +(1+i)^{N-t} + ... + (1+i)^0 )$

If we let b = 1+i, this is equivalent to a simple geometric sequence:

(Eq. 10) $F(N) = A ( b^{N-1} + b^{N-2} + ... + b^{N-t} + ... + b^0 )$

If we rearrange the terms, then

(Eq. 11) $F(N) = A (1 + b + ... + b^{N-t} + ... + b^{N-1} ) = A [F/A,i,N]$

And

(Eq. 12) $[F/A,i,N] = (1 + b + ... + b^{N-t} + ... + b^{N-1} )$

Now use a mathematical trick: multiply this by one expressed as (1-b)/(1-b) to get a more elegant result:

(Eq. 13) $[F/A,i,N] = [1/(1-b)] [(1 + b + ... + b^{N-1} ) - (b + b^2 + ... + b^{N-t+1} + ... + b^N )]$

(Eq. 14) $[F/A,I,N] = [1/(1-b)] [1-b^N] = [1-b^N] / (1-b)$

Substitute (1+i) = b and rearrange terms to get the uniform series compound amount factor:

(Eq. 15) $[F/A,i,N] = [(1+i)^N-1]/i$

Expressions for the Other Equivalence Factors

Expressions for the other factors can readily be found. Since F(N) = A * [F/A,i,N] we can invert the above to get [A/F,i,N], which is known as the **sinking fund factor**:

(Eq. 16) $[A/F,i,N] = i/[(1+i)^N-1]$

A sinking fund may be established by a local government or a company as a means of paying off a future debt. An amount is paid each year into the sinking fund, which could be a savings account or another safe investment. Each year, the sinking fund would earn interest, and at the end of the time period, enough would have accumulated to pay off the debt (or fix the roof on the town hall or deal with whatever problem the sinking fund was established to solve). The relevant question is the size of the annuity.

A related question involves the present worth of an annuity. This amount of money that will be available in the future if a specified amount is invested each period for n periods at interest rate i. This amount can be calculated using what is known as the **uniform series present worth factor** and symbolized as [P/A,i,N]. This factor represents the present value of the annuity, and the annuity has an equivalent future value represented by [F/A,i,N] (Equation 15). Taking the present value of that future value will produce the desired uniform series present worth factor:

(Eq. 17) $[P/A,i,N] = [F/A,i,N] / (1+i)^N$

The uniform series present worth factor can be obtained by substituting for [F/A,i,N] using equation 15:

(Eq. 18) $[P/A,i,N] = [(1+i)^N-1]/[i (1+i)^N]$

The inverse of this expression will give the **capital recovery factor**, which can be used to determine the size of an annuity that is required to recover an initial capital investment:

(Eq. 19) $[A/P,i,N] = [i (1+i)^N]/[(1+i)^N-1]$

These expressions may not seem too elegant, nor are they easy to remember. However, note that when N gets large, they become very clear and simple:

(Eq. 20) [P/A,i,N] = 1/i

(Eq. 21) [A/P,i,N] = i

These approximate expressions for the sinking fund and the capital recovery factors can be very useful in estimating the present value of a long-term annuity or in estimating the present value of a long-term annuity. For example, suppose a toll road generates $1 million per year in profit. What would that be worth to a potential purchaser with a discount rate of 10%? We could approach this by assuming a life of 30, 40 or 100 years and looking up the values for [P/A,10%,30], [P/A,10%,40] and [P/A,10%,100]. If we did this, we would find the following:

($1 million) [P/A,10%,30] = $1 million (9.4269) = $9.4 million

($1 million) [P/A,10%,40] = $1 million (9.7791) = $9.8 million

($1 million) [P/A,10%,100] = $1 million (9.9993) =$10 million

If we had just used the approximation, we would immediately have said that

($1 million) (1/0.1) = $10 million

This result is very close for 40 years and almost exact for 100 years. In many analyses, particularly preliminary analyses where few if any of the numbers are precise, the approximation will be quite adequate.

In going from present value to annuities, we find a similar result. In this case, the question would concern the annual profit that would be required to justify an investment of $10 million in a turnpike. The approximation says that the long-term annuity would be approximately $1 million multiplied by the discount rate of 10% or $1 million per year. The more precise calculations would call for somewhat higher returns, but nothing markedly greater than the quick estimate of $1 million:

$10 million * [A/P,10%,30] = $10 million (.1061) = $1.06 million

$10 million * [A/P,10%,40] = $10 million (.1023) = $1.02 million

Using these approximations is sometimes called the **capital worth method**.

Figure 3 summarizes the concept of equivalence. The chart in the upper left shows a typical stream of cash flows for a project. There are substantial investments during the first four years, profitable operations beginning in year five, a dip in earnings midway through the life of the project reflecting the need to expand or rehabilitate the project, and ultimately a decline in profitability and a decommissioning expense. It is impossible to tell for sure how good this project simply by looking these cash flows. The other three charts show the equivalent present value, annuity, and future value, each of which is easy to understand. Since the present value is positive, then investing in this project is better than investing in something that earns interest equal to the discount rate. If the present value is negative, then the project is not as good as investing in something that earns interest equal to the discount rate.

# Equivalence of Cash Flows

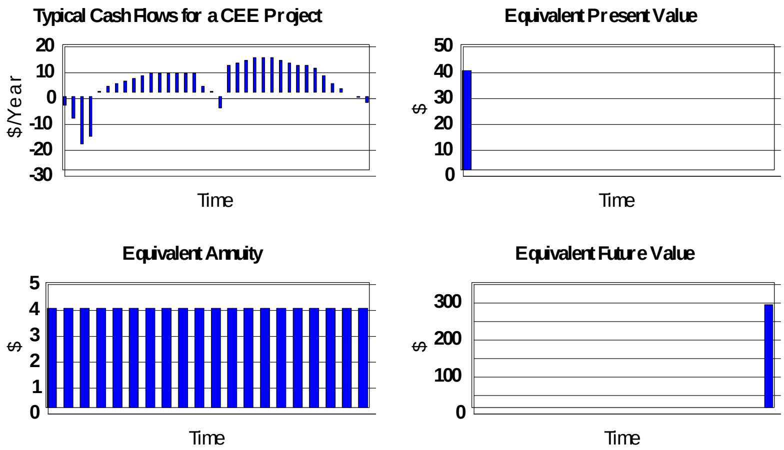



Table 3 summarizes the six factors derived above. F refers to the future value, P to the present value, and A to the equivalent annuity amount. The equations are all functions of the discount rate i and the number of periods N. The final two rows of this table highlight the two easily remembered factors known as the "Capital Worth Method." When N is large, these factors provide an easy way to get a quick estimate of the value of an annuity (A/i) or the annuity that is equivalent to any present amount (Pi).

**Table 3 Summary of Equivalence Factors, Discrete Compounding**

| Symbol | Name | Comment | Value |
|---|---|---|---|
| [F/P,I,N] | Future value given present value | How much growth can be expected | $(1+i)^N$ |
| [P/F,I,N] | Present value given future value | Discounted value of a future amount | $1/(1+i)^N$ |
| [F/A,I,N] | Uniform series compound amount factor | If I save some each period, how much will I accumulate? | $[(1+i)^N-1]/i$ |
| [A/F,I,N] | Sinking fund payment | How much must I save each period to meet my retirement goals? | $i/[(1+i)^N-1]$ |
| [A/P,I,N] | Capital recovery factor | What will my mortgage payment be? | $[i\ (1+i)^N]/[(1+i)^N-1]$ |
| [P/A,I,N] | Uniform series present worth factor | If I can pay A per month, how large a mortgage can I afford? | $[(1+i)^N-1]/[i\ (1+i)^N]$ |
| [A/P,I,infinity] | Capital recovery factor for very long time periods | Capital Worth Method | i |
| [P/A,I,infinity] | Uniform series present worth factor for very long time periods | Capital Worth Method | 1/i |

The next two figures illustrate the relationship between annuities and present or future worth. Figure 4 shows the "uniform series, compound amount factor", which is the amount by which the future value exceeds the annual amount of an annuity depending upon the length of time and the earnings rate. This is the factor that is useful in determining how much should be invested each year toward retirement. Figure 5 shows the annuity that is equivalent to a present

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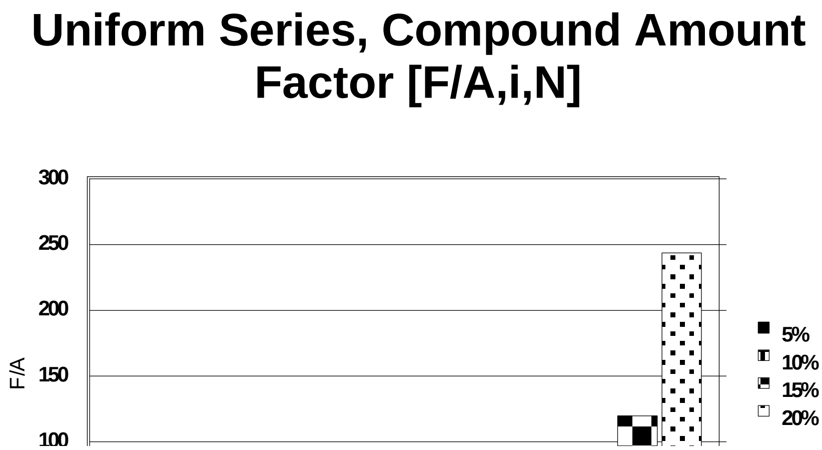


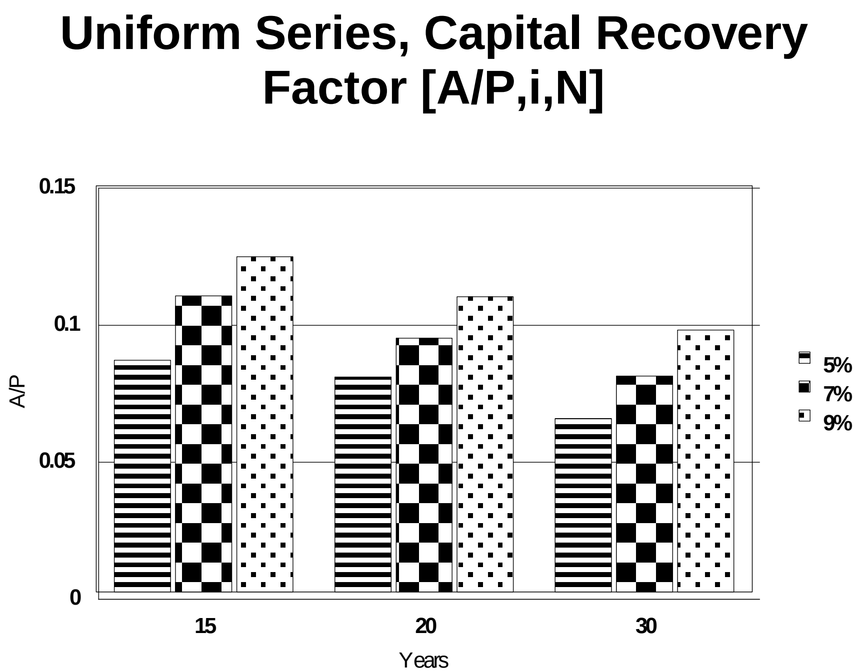


**Continuous Compounding: Nominal vs. Effective Interest Rates**

Interest rates are normally expressed in terms of annual returns, and the **nominal rate of interest** is the interest rate that you would receive if interest were compounded annually. If interest is compounded more frequently, then there will be a higher **effective rate of interest.** With a 12% nominal rate, an investment of $1000 on January 1 would earn $120 interest on December 31. However, if you compound interest semiannually, then you would get more interest. First after 6 months, you would earn 6% or $60 on your initial investment. Then, at the end of the year, you would earn another $60 on the initial investment plus $3.60 on the interest that you received at the end of June. For the year, you therefore earned an effective rate of 12.36% interest on your investment, even though the nominal rate was 12%.

More rapid compounding gives a bit higher effective rate, but there are diminishing returns:

- Quarterly rate is 12.55%

- Bimonthly rate is 12.62%
- Monthly rate is 12.68%
- Daily rate is 12.75%

Clearly, this is approaching a limit as interest is compounded more frequently. It turns out that all of the formulas for discrete cash flows can be revised into expressions for continuous compounding by substituting $e^{rN}$ for $(1 + i)^N$, where r is the nominal rate of interest and i is the effective rate (Table 4). For example, if the nominal rate is 12%, as in the example we have been discussing, then $e^{rN} \sim 2.718^{0.12} = 1.1275$ which equals 1 plus the effective daily rate of 12.75% that we just calculated.

The nuances of nominal versus effective rates will be important in certain areas, notably in banking and finance, where contracts will specify interest rates and compounding periods. If you are buying a car or making a deposit to a savings account, these details will affect the size of your payments or your earnings. In project evaluation – especially during the early stages of the process – the differences between discrete and continuous compounding will be minor compared to the uncertainties in estimating costs, revenues, time periods, and other factors that will affect the outcome of the analysis.

**Table 4 Summary of Equivalence Factors, Continuous Compounding**

| Symbol | Name | Comment | Value |
|---|---|---|---|
| [F/P,I,N] | Future value given present value | How much growth can be expected | $e^{rN}$ |
| [P/F,I,N] | Present value given future value | Discounted value of a future amount | $1/ e^{rN}$ |
| [F/A,I,N] | Uniform series compound amount factor | If I save some each period, how much will I accumulate? | $[e^{rN} -1]/r$ |
| [A/F,I,N] | Sinking fund payment | How much must I save each period to meet my retirement goals? | $r/[ e^{rN} -1]$ |
| [A/P,I,N] | Capital recovery factor | What will my mortgage payment be? | $[r (e^{rN} )]/[ e^{rN} -1]$ |
| [P/A,I,N] | Uniform series present worth factor | If I can pay A per month, how large a mortgage can I afford? | $[e^{rN} -1]/[r (e^{rN} )]$ |
| [A/P,I,infinity] | Capital recovery factor for very long time periods | Capital Worth Method | R |
| [P/A,I,infinity] | Uniform series present worth factor for very long time periods | Capital Worth Method | 1/r |

Some Useful Approximations

The continuous compounding formulations for [F/P,r,N] and [P/F,r,N] are very useful because they are easy to remember and can readily be used for quick approximations. Since [F/P,r,N] = $e^{rN}$ and [P/F,r,N] = $1/ e^{rN}$, it can be very helpful to remember a few useful results:

- If rN = 1, $e^{rN} = 2.718...$
- If rN = 0.7, $e^{rN} = 2.013...$, approximately 2
- If rN = 1.1, $e^{rN} = 3.004...$, approximately 3
- If rN = 1.4, $e^{rN} = 4.055...$, approximately 4

You can use these relationships to figure out how long it will take to double, triple or quadruple your money. If $rN = 2$, then money invested at r% for N years will double in value. If the nominal interest rate is 10%/year, then it will take seven years to double your money; if the nominal rate is 5%, then it will take 14 years to double your money. Likewise, if $rN = 1.1$, then $e^{rN} \sim 3$ and the future value will be about three times the present value. Thus, if you earn 10% per year for eleven years, you can triple your money. You can also use the inverse relationship to calculate present values. For example, if the nominal interest rate is 7%, then the present value of something received in ten years will be half its future value.

In short, it is possible to make mental estimates of quite complex functions made up of incomprehensible expressions such as $(1+i)^n$. Mental math has been an increasingly undervalued skill since the invention of the electronic calculator, but you will certainly find it useful to use the above relationships if you are ever involved in face-to-face discussions, negotiations, or debates related to project evaluation when it would be inconvenient or inappropriate to use your calculator or computer. You <u>can</u> do present value analysis in your head - and that <u>will</u> give you an advantage in negotiation!

<u>Proof that $e^{rN} = (1 + i)^N$</u>

To prove this relationship, we can begin by writing an algebraic expression for what we are doing as we compound more frequently. If r is the nominal interest rate, but we compound M times per year, then the effective rate will be

(Eq. 22) $$i = [1 + (r/M)]^M - 1$$

and the factor $[F/P,\underline{r}\%,1]$ will be

(Eq. 23) $$[F/P,\underline{r}\%,1] = [1 + (r/M)]^M$$

Note that the term "$\underline{r}$%" represents the nominal interest rate r and the use of continuous compounding.

If we let $p = M/r$ and rewrite this equation, we get

(Eq. 24) $$[F/P,\underline{r}\%,1] = (1 + 1/p)^{rp} = ((1 + 1/p)^p)^r$$

This is a classic relationship, as the limit of $(1+1/p)^p$ as p approaches infinity is e = 2.7128...!
Thus, we have

(Eq. 25) $$[F/p,\underline{r}\%,1] = e^r$$

and therefore

(Eq. 26) $$[F/P,\underline{r}\%,N] = e^{rN}$$

which somewhat unexpectedly gives us a very nice relationship:

(Eq. 27) $$[F/P,\underline{r}\%,N] = e^{rN} = (1 + i)^N$$

where the exponential expression assumes continuous compounding using the nominal rate and the other expression uses the effective rate. Hence, we can revise all of the formulas for discrete cash flows into expressions for continuous compounding by substituting $e^{rN} = (1 + i)^N$.

**Financing Mechanisms**

Equivalence relationships enable financing of large projects. Those investors or banks that have money to invest are willing to make cash available for implementing a project in return for future interest payments, mortgage payments, or dividends. This section describes financing mechanisms that are used for infrastructure projects.

Mortgages

A mortgage is a loan that is backed by property. If the borrower defaults on a payment, then the lender can seize the property and either use it or sell it to recover their costs. From the lender's perspective, a mortgage is less risky than a long-term unsecured loan, and therefore merits a lower interest rate. A mortgage will typically be limited to about 80% of the assessed value of the property in order to reduce the risk to the lender in case the owner defaults. If the mortgage is less than the value of the property, then the bank will be able to sell the property and regain its investment.

The monthly payments on a mortgage depend on the amount of the loan (the principal amount), the interest rate and the period of the mortgage. If the mortgage principal is PRICE, the annual interest rate is i% and the term is N years, then the monthly payment will be:

(Eq. 28) $M = (\text{PRICE})\ [A/P,i\%,N]$

where $[A/P,i\%,N] = [i\ (1+i)^N]/[(1+i)^N-1]$ as shown in Table 3. This factor can be found in several ways. It can of course be calculated using the formula, but it can also be obtained from a table (e.g. Table 5) and it can be obtained using a function available on many spreadsheets. In Excel, the function PMT(i%,N,P) will give the desired answer.[4]

**Table 5 Capital Recovery Factor [A/P,i%,N] for selected interest rates i% and years N**

| Years | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% |
|---|---|---|---|---|---|---|---|---|
| **5** | 0.2184 | 0.2246 | 0.2310 | 0.2374 | 0.2439 | 0.2505 | 0.2571 | 0.2638 |
| **10** | 0.1172 | 0.1233 | 0.1295 | 0.1359 | 0.1424 | 0.1490 | 0.1558 | 0.1627 |
| **15** | 0.0838 | 0.0899 | 0.0963 | 0.1030 | 0.1098 | 0.1168 | 0.1241 | 0.1315 |
| **20** | 0.0672 | 0.0736 | 0.0802 | 0.0872 | 0.0944 | 0.1019 | 0.1095 | 0.1175 |
| **25** | 0.0574 | 0.0640 | 0.0710 | 0.0782 | 0.0858 | 0.0937 | 0.1018 | 0.1102 |
| **30** | 0.0510 | 0.0578 | 0.0651 | 0.0726 | 0.0806 | **0.0888** | 0.0973 | 0.1061 |

For example, let's calculate the annual and monthly payments on a 30-year mortgage for $1,000,000 at 8% interest. The mortgage payment will be the annuity that – for the bank – is equivalent to the principal amount of the loan. To obtain the annual payment, the amount of the loan needs to be multiplied by the capital recovery factor [A/P,i%,N], which Table 5 shows to be 0.0888 for a 30-year mortgage with an 8% interest rate,. The annual payment would therefore be approximately $88,800:

(Eq. 29) PMT = (PRICE) [A/P, 8%, 30] = $1,000,000 (0.0888) = $88,800

If payments were to be made monthly, then the PMT would be approximately $1/12^{th}$ of the annual amount or $7,333.33. These amounts are approximate because the table only shows the capital recovery factor to four significant digits. More accurate answers for annual or monthly mortgage amounts could be obtained by using the formula for the capital recovery factor or more readily by using the PMT function in Excel:

(Eq. 30) Annual payment = PMT(8%, 30, $1,000,000) = $88,827.43

[4] The Excel function is PMT(interest rate, number of periods, present value). Tables such as Table 5 can easily be created in any spreadsheet.

To obtain the monthly payment, it is necessary to use the monthly interest rate and 360 monthly payments. Using the PMT function, the result is:

(Eq. 31) Monthly payment = PMT(8%/12, 360, $1,000,000] = $7337.65

These results are slightly higher than the results obtained using the factor from Table 5. The bank would be sure to use the actual number, but the difference is far too small to make any difference in project evaluation, which naturally must deal with many ill-defined numbers when comparing options. Neither the amount of the loan nor the interest rate would be known with certainty until a particular option has been chosen and the project is almost completed.

After making mortgage payments for several years, an owner may decide to refinance the mortgage. The most likely reasons for refinancing would be to reduce monthly payments or to obtain cash:

- Interest rates may be lower, so that the mortgage payment could be reduced.
- The mortgage could be extended over a longer time period in order to reduce monthly payments.
- It may be possible increase the size of the mortgage and extend the term of the mortgage without increasing the size of the monthly payment.
- If the value of the property has increased, the owner may be able to borrow additional money for another project.

There are three steps in refinancing:

1. Figure out the remaining balance on the initial loan.
2. Negotiate terms for the new loan.
3. Close on the new mortgage:
    a. Use some of the proceeds to pay off the original loan
    b. Give a check to the mortgagee for any additional funds that are borrowed.

The remaining balance on the new loan can be calculated as of a point in time immediately after a regular mortgage payment has been made. The remaining balance could be calculated in multiple ways:

1. Read the statement: the monthly or annual statements will show the amount outstanding as of the previous payment, the amount due for the current payment, and the portion of the payment that goes toward interest and principal. This is what homeowners look at when they are considering refinancing.
2. Calculate the value of all the payments that have been made so far and subtract from the original amount of the loan.
3. Calculate the value of the remaining payments, using the interest rate of the loan as a discount rate. This is equivalent to what must be repaid to the bank.

These calculations will involve multiple applications of various equivalence factors. For example, suppose that payments had been made for 15 years on the 30-year, 8% mortgage for $1 million that was described above. If the borrower has a chance to refinance with a 20-year mortgage carrying a 6% interest rate, what would the new payments be? First, it is necessary to calculate the amount remaining on the mortgage. Assume that the borrower just made the $15^{th}$ payment, so that there would be 15 more payments due on the original 30-year mortgage. Although half the payments have been made, much less than half the loan has been paid off, because most of the payments have gone toward interest.

There are various ways to calculate the amount remaining. One way would be to subtract the amounts paid off each year from the initial purchase price. The first payment of $88,827 included $80,000 interest (8% of $1 million) and therefore a principal payment of $8,827. With a spreadsheet, it is possible to continue the analysis, reducing the remaining principal by the proper amount after each payment as shown in Table 6. The amount remaining after the 15th year is the amount shown at the beginning of year 16, namely $760,317.

**Table 6 Payments of Principal and Interest and Remaining Balance Over the Life of a 30-year Mortgage for $1 million at 8% Interest**

| Year | Mortgage balance | Payment | Interest | Principal |
|---|---|---|---|---|
| 0 | $1,000,000 | $88,827 | $0 | $0 |
| 1 | $1,000,000 | $88,827 | $80,000 | $8,827 |
| 2 | $991,173 | $88,827 | $79,294 | $9,534 |
| 3 | $981,639 | $88,827 | $78,531 | $10,296 |
| 4 | $971,343 | $88,827 | $77,707 | $11,120 |
| 5 | $960,223 | $88,827 | $76,818 | $12,010 |
| 6 | $948,213 | $88,827 | $75,857 | $12,970 |
| 7 | $935,243 | $88,827 | $74,819 | $14,008 |
| 8 | $921,235 | $88,827 | $73,699 | $15,129 |
| 9 | $906,106 | $88,827 | $72,488 | $16,339 |
| 10 | $889,767 | $88,827 | $71,181 | $17,646 |
| 11 | $872,121 | $88,827 | $69,770 | $19,058 |
| 12 | $853,063 | $88,827 | $68,245 | $20,582 |
| 13 | $832,481 | $88,827 | $66,598 | $22,229 |
| 14 | $810,252 | $88,827 | $64,820 | $24,007 |
| 15 | $786,244 | $88,827 | $62,900 | $25,928 |
| 16 | $760,317 | $88,827 | $60,825 | $28,002 |
| 17 | $732,314 | $88,827 | $58,585 | $30,242 |
| 18 | $702,072 | $88,827 | $56,166 | $32,662 |
| 19 | $669,410 | $88,827 | $53,553 | $35,275 |
| 20 | $634,136 | $88,827 | $50,731 | $38,097 |
| 21 | $596,039 | $88,827 | $47,683 | $41,144 |
| 22 | $554,895 | $88,827 | $44,392 | $44,436 |
| 23 | $510,459 | $88,827 | $40,837 | $47,991 |
| 24 | $462,468 | $88,827 | $36,997 | $51,830 |
| 25 | $410,639 | $88,827 | $32,851 | $55,976 |
| 26 | $354,662 | $88,827 | $28,373 | $60,454 |
| 27 | $294,208 | $88,827 | $23,537 | $65,291 |
| 28 | $228,917 | $88,827 | $18,313 | $70,514 |
| 29 | $158,403 | $88,827 | $12,672 | $76,155 |
| 30 | $82,248 | $88,827 | $6,580 | $82,248 |
| 31 | $0 | | | |

A more elegant approach is to think about the value of the remaining payments rather than worrying about the contributions so far to principal and interest. From the lender's perspective, they are receiving 15 more payments at the original 8% interest rate. The value of this annuity, discounted at 8%, will also give the amount remaining on the mortgage:

(Eq. 32) Outstanding amount = $88,000 * [P/A, i%,15] = $88,800 * 8.5595 = $760,084

This amount is slightly less than the balance of $760,317 shown in the table due to rounding errors. With either answer, we can express the outstanding amount as $761 thousand and estimate the new mortgage payment as just over $66 thousand:

(Eq. 33) New payment = $761,000 * [A/P, 6%, 20] = 761,000 * .0872 = $66,359

Thus, by refinancing, the borrower can reduce the annual mortgage payment by $22 thousand. The annual payments are lower because of the lower interest rate and also because the repayment period has been extended by 5 years (15 remaining years on the original mortgage vs. 20 years on the new mortgage).

This example illustrates how multiple paths may be used to reach the same answer. The equivalence relationships can be used repeatedly, in different ways, and so long as the logic is correct along each path, they will each reach the correct destination.

<u>Bonds</u>

Bonds provide a way for companies or agencies to raise money. A bond is offered with a face value V, a life of N years, and annual interest of i%. Interest payments are made at the end of each year for N-1 years. At the end of the $N^{th}$ year, the owner receives the final interest payment and the bond is redeemed for its original face value. Bonds are commonly sold in denominations of $1000 with a 30-year term, but other options are available. Bonds can be bought and sold over their lifetime, so it is possible to buy a 30-year bond that will become due in less than 30 years. Bonds can even be offered as **zero coupon bonds**, in which the seller pays no interest but promises to pay the face value at the end of the term; for these bonds, the purchase price will be much less than the face value.

Bonds are supported by the credit of the issuing agency or company; if the company fails to pay interest when it is due, then the bond-holders can force the company into bankruptcy. If the company declares bankruptcy, then the assets of the company are divided up among the creditors, including the bondholders. Various credit agencies rate the quality of bonds, so that investors have a reasonable idea of the risks involved in buying the bonds. The higher the perceived risks of the issuing agency or company, the lower the credit rating on the bonds, and the higher the interest rates that must be offered to attract investors.

Three interesting questions are 1) the value of the bond to an investor and 2) the change in the value of the bond as interest rates change, and 3) the change in the value of the bond as perceived risks associated with the issuing agency or company change. The value of the bond to an investor depends upon the investor's perception of the risks involved, the investor's discount rate for bonds with such risks, and the interest rates offered for the bond. If the investor's discount rate is lower than the interest rate that is offered, the investor will consider buying the bond. Market forces related to the supply and demand for fixed interest securities will determine what interest rates are actually required to sell bonds.

Assume that the bond is sold at time 0. The seller agrees to pay interest of i% per year for N years and, at the end of N years, to pay back the face value of the bond. The seller offers the bonds to the marketplace, and potential purchasers decide whether or not they want to buy the bonds. The purchasers may plan to hold the bonds until maturity, or they may merely view the bonds as a short- or medium-term investment. The value of a high quality bond to an investor can be calculated as follows:

(Eq. 34) Value = Annual Interest [P/A, i%, N] + Face Value [P/F, i%, N]

The first term is the present worth of the N annual interest payments and the second term is the present worth of the final redemption of the bond at its original face value. It is crucial to recognize that the investor's discount rate of i% can be higher or lower than the interest rate on the bond. If there is a possibility that the bond will default, both terms could be reduced by a factor representing the probability that interest or the final redemption would not be made.

Assuming that the probability of default is close to zero, the value of a bond to an investor will depend upon the face value, the years to maturity, the interest rates, and the discount rate of the investor. If the face value is $1,000 and the interest rate is 6% for a 30-year bond, then the value to a potential purchaser who also has a 6% discount rate will be exactly $1,000:

(Eq. 35) Value = ($1,000) (0.06) [P/A, 6%, 30] + $1,000 [P/F, 6%, 30]
= $60 (13.7648) + $1,000 (0.1741)
= $826 + $174 = $1,000

For someone with a discount rate of 5%, the bond will be worth much more:

(Eq. 36) Value = $60 (15.3725) + $1,000 (0.2314)

= $922 + $231 = $1,153

Likewise, someone with a discount rate of greater than 6% would value the bond at less than $1,000. In each case the equivalence factors were obtained from a table of equivalence factors.

The same type of calculations can be used to show how the value of a bond could change if interest rates change. For example, suppose that 20 years have gone by and interest rates on similar bonds have fallen to 5%. The 6% bond is therefore paying more interest than a bond with similar risk would have to pay today; hence, investors would find that bond more appealing. Someone with a 5% discount rate would be willing to pay $1,000 for the bond paying 5%, but would pay more for the 6% bond with 10 years until maturity:

(Eq. 37) Value = $60 [P/A, 5%, 10] + $1000 [P/F, 5%, 10]

= $60 (7.7217) + $1,000 (0.6139)

= $463 + $614 = $1077

Thus, when interest rates fall, bond values rise. In this instance, note that the redemption value is now much greater than the value of the interest payments. As the bond approaches maturity, the redemption value dominates.

Sinking Funds

A sinking fund can be established to meet expected future capital needs, such as paying the principal on bonds when they become due or conducting a major rehabilitation of a factory at some distant point in the future. Companies or agencies can pay a constant amount into a fund that is maintained solely to cover this future capital need. As shown above, the sinking fund factor [A/F,i%,N] can be used to determine the annual amount A to invest at i% to reach the goal of having an amount F at the end of N periods.

The calculations will require another step if the future need is itself an annuity. For example, planning for retirement involves consideration of three questions. First, how much will be needed in annual income after retirement. Second, how much will be needed in savings to produce this level of retirement income. Third, how much will need to be saved each year (e.g. in a sinking fund) in order to have accumulated the money that will be used to purchase the retirement annuity. The first question is a matter of personal needs and desires; in effect, it is necessary to determine how large an annuity will be needed to support your desired lifestyle in retirement. The second question is a matter of equivalence: how much will be needed to purchase the retirement annuity? This is the retirement goal. The third question is a different matter of equivalence: how much do you have to save each year to reach your retirement goal? The relevant equations are as follows:

(Eq. 38) Retirement Goal = Retirement Annuity [P/A,i%,N]

(Eq. 39) Savings Goal = Retirement Goal [A/F,g%, M]

The key unknowns are a) how much will really be needed, b) what interest rate (i%) can you expect for the annuity, and what annual return on investment (g%) can you expect on your savings. Perhaps the most interesting variable for many people will be M, the years until retirement. The more that is saved, the higher the return on investment, and the higher the interest rate on the retirement annuity, the sooner you can retire. The longer you work the more time you will have to reach your retirement goal, and the less you will need to save each year to reach that goal.

Consider a 30-year old couple planning for retirement. They would like to save a constant amount per year in a tax-deferred investment account. After working for another 30 years, they hope to have accumulated enough funds to provide an annuity that will last them until they are 100 years old or older. If they can achieve earnings of 8% per year on their investments, how much should they save each year in order to be able to have $80,000 per year if they retire at age 60 and live forever?

To answer this, assume a discount rate of 8% for all of the calculations. While it is unlikely that they will live forever, that hope at least makes the analysis a bit easier: the anticipated 8% return on their accumulated retirement investments will be $80,000, so they will need to accumulate $80,000/0.08 = $1 million by age 60 (note that this approach uses the simple capital worth method to convert the desired annuity into a required sum). The question therefore is what level of annual investment is needed to accumulate a million dollars in 30 years if returns are 8%?

(Eq. 40) Annual investment = $1,000,000 [A/F, 8%, 30] = $1,000,000 * (.0088) = $8,800

This may seem steep. If they extend their planned retirement ten years to age 70, then they will only need to invest $3,900 per year:

(Eq. 41) Annual investment = $1,000,000 * [A/F, 8%, 40] = $1,000,000 * .0039 = $3,900

Toll-Based Financing

For highways and bridges that carry a lot of traffic, it may be possible to use the projected toll revenues to justify issuing bonds that are sufficient to cover the cost of construction. Some quick estimates may indicate whether or not toll-based financing will work. The interest on the bonds must be compared to the net revenue from the tolls, which is what is left after paying for the annual operating costs of the bridge. If the net toll revenue is well above the anticipated interest payments, then this would be a good candidate for toll-based financing.

Consider a proposal for a bridge that is expected to cost $50 million to construct and $3 million per year for operating and maintenance costs. The bridge is expected to serve five to ten million vehicles per year, and the governor believes that the public would view a toll of $1 to $2 as reasonable. Will the tolls cover the cost of bonds that could be sold to pay the construction costs of the bridge? Interest rates on the bonds are expected to be 4%.

Given the projected traffic volume, the bridge would earn $5 to $20 million per year with a toll of $1 or $2. After deducting $3 million per year for operating expenses, the net revenue would $2 to $17 million. Give the interest rate of 4%, the annual interest on the $50 million investment would be $2 million. Thus, even the lowest estimate of net revenue would cover the annual interest payments on bonds. With a $2 toll, funds would be available for related projects, such as improving access roads or providing support to other transportation projects.

## Summary

When evaluating projects, it will be necessary to compare costs and benefits that are incurred over a period of many years. The costs of the project are generally concentrated at the outset of the project, as benefits do not begin until construction is at least partially completed. One basic question for any proposed project is whether or not the eventual benefits will be sufficient to justify an initial investment in a project. In most situations, there will be multiple alternatives to consider, each with its own investment requirements, time table, and projected stream of future costs and benefits. A second basic question is to determine which project – which projected set of cash flows – is most desirable. While these are not the only questions that must be answered to determine whether a project can be justified, they are questions that will certainly be considered by investors, bankers, and entrepreneurs who are considering participating in such projects. It is therefore essential to understand how such financial comparisons are made.

The first critical concept concerns the **time value of money**. There are several reasons why money in the future is worth less than the same amount of money available in the present. First, if money is available today, there is an **opportunity cost**: the money could be put into a savings account or invested so that it will be worth more in the future. Second, **inflation** is likely to reduce the purchasing power of money, so that today's money will buy more goods and services than the same amount of money would be expected to purchase in the future. Third, there is a **risk** that the actual money that becomes available in the future will be less than was predicted. It is therefore necessary to **discount** future cash flows in order to determine their **present value**. The higher the discount rate that is used, the lower the present value. The greater the potential for growth, the higher the expected rate of inflation, and the greater the risk associated with the proposed investment, the higher the discount rate that will be used.

By discounting costs and benefits over the life of a project, it is possible to determine the **net present value (NPV)** of the project. If the NPV is positive, then the project is worth more than doing nothing; if the NPV is negative, then the project is not worth pursuing, at least from a financial perspective. By estimating the NPV for a set of alternatives, then it is possible to determine which is worth the most financially. Maximizing the net present value of cash flows is a common financial objective for the private sector, although public projects are likely to have more complex objectives.

Discounting provides a means of establishing **equivalence** between two sets of cash flows. If the cash flows are equivalent for a company or an individual, then that company or individual is indifferent between them. The conceptual power of discounting is that it provides a straight-forward methodology for establishing a NPV that is equivalent to any arbitrary set of projected cash flows. In particular, it is possible to establish equivalence between a **present value P** and a **future value F** or an **annuity A**. Factors that relate P, F, and A are functions of the discount rate and time. These equivalence factors make it possible to compare cash flows in various ways, depending upon what is most useful for a particular analysis.

Equivalence relationships are used to determine **mortgage** payments, the **value of bonds**, and other financing mechanisms. A mortgage is a financing mechanism in which a bank or other financial company provides a large payment in return for a series of monthly payments over a period of many years. If the mortgage payments are not made, then the mortgage is in default, and the bank can foreclose on the property. A company or agency can sell bonds to raise money for a project; the bonds are purchased by investors who receive interest payments over the life of the bond and also receive the full value of the bond at the end of life of the bond. If the company or agency fails to make the interest payments or is unable to redeem the bonds as they become due, then the company or agency may be forced into bankruptcy.

The concept of equivalence can be used in cost models, where it is often necessary to consider both investment costs and annual operating costs. The investment cost must be converted to an equivalent annual cost in order to be added to operating cost and used to determine such things as the cost per unit of capacity or the cost per user.

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