PROFESSOR: So while I've argued that there are many ways in which a room can become well mixed, hopefully well enough to apply our well mixed criterion for airborne transmission through long range aerosols, there is one very important way in which the transmission problem is never well mixed.

And that is taking into account the source of the particles, which is the exhaling of an infected individual which leads to a very high concentration of particles leaving the mouth, which then ends up being dispersed throughout the room.

And there is a sort of space and time dependence of that process which we must consider in looking at other possibilities of transmission than just the well mixed background air.

So in particular, I've already indicated that the flows generated by breathing tend to be at higher enough Reynolds number to generate turbulence.

And in particular, they generate a turbulent jet which ends up taking the form of a cone.

Not perfectly but approximately a cone, which means that the radius of the jet is alpha times the position of the jet here.

So if I write a coordinate system, a cylindrical coordinate system where z is the direction of the flow and r is the radial coordinate, then that's the cone and the cone angle is alpha.

The term alpha also has a physical interpretation as the air entrainment coefficient.

And for respiratory jets in the air, this coefficient is usually around 0.1 to 0.15.

So that gives you basically the opening angle of the cone.

So I will come to explaining and deriving why the jet has the shape of a cone.

But let's just assume that.

And let's continue with the assumption that the flow is at high Reynolds number which means it's dominated by inertia, by kinetic energy, and by the tendency for the fluid to keep wanting to move in a new direction.

Now if the fluid coming out of the mouth were just being spit out at a very high rate, you might imagine a very narrow kind of stream of air.

But the reason it's widening is that air is actually being entrained, is that some of the ambient air is being sucked in.

And this is making the jet have more and more fluid in it.

But then that fluid is sharing the momentum.

It's also spreading and diffusing the particles.

And the wider it gets, the more it slows down because now that momentum is being shared through the sort of turbulent exchange going on.

So let's do that calculation.

So we have-- so just remind you that we are in a situation of very high Reynolds number typically.

And we will assume that there is roughly-- and it's actually a good assumption-- roughly a constant momentum flux.

What that means if I take a slice here and I look at, essentially, the kinetic energy density or the momentum per time that is crossing a slice of the jet, that that actually should be conserved.

And so if I write that momentum flux as capital K is the area of the cross section pi r^2, if I look at a given position r here.

And then I have the momentum is the density of the fluid, which is the air, times the velocity field, the velocity.

And so that's momentum.

Momentum flux would be momentum times velocity or rho v^2, which is also kinetic energy density.

This quantity should be roughly constant.

So we can now solve for the average velocity, v bar, which would be square root of K over pi rho_a times 1/r, after we take the square root, but then because we have a cone r is alpha z So this is square root of K over pi a 1 over alpha z.

So we can see the velocity is decaying like 1 over distance from the mouth.

So the jet is slowing down.

But it's still, of course, continued to advance as the momentum is being shared across a larger and larger area of entrained, turbulent flow.

So we can now use this result to figure out what is the rate of progress of the front.

So if I call this z of the front, so let's say when I first start exhaling it is kind of a wall of droplets and I can sketch that this flow is actually full of droplets that we're interested in tracking, I'd like to know how those are first leaving the mouth.

Well, I can write that this velocity is, at a given position corresponding to the front, is dzf dt.

And if this is the f, so I apply this at the front, then I can put the zf on the other side and I have zf dzf dt is equal to square root of K over pi rho_a.

Then I've got, I guess also, an alpha or 1 over alpha.

And then this expression here can be written as 1/2 times the derivative of zf squared.

So I can then solve for the position of the front and I find zf is equal to-- well, let's see what I get.

I put the 2 on the other side.

I take a square root.

So I get 2 over alpha to the 1/2, I get K over pi rho_a was a square root.

And then I take another square root so I get a 1/4.

And then I get t to the 1/2.

Because when I integrate this equation, I get zf squared is all this stuff times t, starting from t equals 0.

So we find is that, initially, the jet starts to progress like square root of time.

And so this coefficient here is some kind of-- has the units of a diffusivity.

So it kind of like appears that the jet is sort of diffusing.

You could call this D_effective, sort of for the front of the jet.

But that doesn't last forever because some point the person closes their mouth and starts breathing in again, and maybe pulls the fluid back a little bit.

But not too much.

Because it has momentum and it keeps moving forward.

So when somebody breathes, it starts out looking like that.

But then at a later time if I draw this same person again, then-- in fact, actually, let me draw him with a closed mouth.

Because he's just finished, let's say, exhaling is getting ready for his next breath.

So now this blob of fluid has kind of worked its way out and it's now somewhere out here.

And this is what we call a puff.

So if you're smoking, for example, a cigarette you know that you can create a puff of smoke by just releasing a finite amount of fluid.

And then it kind of goes out and it makes some interesting patterns and usually is very turbulent unless you're very careful in trying to control it.

And of course this contains lots of these droplets that we're interested in.

And so then now we can briefly ask, how fast does the puff move.

Well, you see here, the fluid keeps moving because it's constantly being given momentum.

So if it's a steady jet as you're breathing, you're pushing, pushing, pushing and so this thing keeps moving on the square root of t, it keeps entraining more.

But as soon as you stop giving it more momentum, you give it just a finite amount of momentum, then the puff actually slows down.

It doesn't keep pushing ahead as quickly.

It also doesn't entrain more air as quickly either.

And we can do a very simple argument to see what happens to the scaling.

So this momentum flux here was a constant momentum flux in the case of a jet.

But here in the case of a puff, we could, maybe as a very crude estimate, just say that the momentum flux is something now replaced by some kind of constant value that is maintained only for a time, tb, which is the time of the breath, and then averaged over a longer period times.

If you think of this as kind of like an average momentum flux, we've injected some momentum flux but then we took it away.

And so if I want to find out the average over a period of time t I have to divide by t.

So you see if I do that then, I arrive that in the puff case that the position of the front or the puff is a lot of these same constants here, but is now scaling as t to the 1/4 because my K has a 1 over t and it's raised to 1/4 power, then I end up with zf goes like t to 1/4.

So it slows down.

So first it was sort of square root of time.

Now it's become more [INAUDIBLE] time.

And it's almost just sort of sitting there, it's not progressing that much.

And then a new breath comes.

And so that's what happens next.

And especially if one is speaking or singing, then the exhaling is a much longer and more continuous process than the inhaling which is very sudden.

So you talk and take a breath, and you talk.

And so there's a lot more of this going on than this, even in normal speech and in normal breathing to some extent as well, but especially when one is speaking.

And so an interesting development then is that if we have a person who is speaking, then we can generate what has been called a puff train.

So we have here is the most recent new breath getting exhaled and then the previous one was here, still kind of floating around and the one ahead of that is dispersed a bit more.

But it hasn't progressed as far.

And so it's a little bit thinner.

And if we kind of color each breath a different way, then what we're left with is something that actually looks an awful lot like a continuous cone again.

So this is a puff train.

This could be, for example, from speaking or singing.

And overall, the behavior is quite similar to a jet just where the K is replaced by the time average momentum flux.

So I should mention that this way of thinking here was recently introduced and verified experimentally in a paper by Abkarian and Howard Stone and collaborators, and also introduced this notion of the scaling and showed that actually the puff train has a scaling which is very similar to the initial jet with the square root of time.

And so that's an important concept come to now.

Because we want to ask now, what if somebody is speaking or even just breathing continuously in a certain direction and generating aerosol particles, how does their concentration evolve in this sort of a respiratory plume or jet.