Unit 5: Polar Coordinates II

1. Read Thomas, Sections 11.4 and 11.5.

2. Exercises:

2.5.1(L)

The curve C has the polar equation $r = f(\theta)$. Let 4 denote the angle between the tangent line to C at the point (r_0,θ_0) and the vector $\vec{OP}_0$.

a. Use only polar coordinates to develop the fact that

$$\tan\psi = \frac{r_0}{\left(\frac{dr}{d\theta}\right)_{\theta=\theta_0}}$$

b. Derive the same result as in (a) but by using $\frac{dy}{dx}$ expressed in terms of r and θ.

c. Find the angle ψ at the point $P_0(\frac{5}{4},\frac{\pi}{6})$ on the curve C where the polar equation for C is $r = \sin^2\theta + 1$. Use this information to <u>construct</u> the tangent line to C at P_0.

d. With C and P_0 as in (c), use $\frac{dy}{dx}$ to find the tangent line to C at P_0.

2.5.2(L)

Let C be the curve whose polar equation is $r = \sin 2\theta$, and let $P_0(\frac{1}{2}\sqrt{3},\frac{\pi}{6})$ be on C.

a. Use $\frac{dy}{dx}$ to find the slope of C at any point, and in particular, compute the slope of C at P_0.

b. At what point in the first quadrant does C attain its greatest height?

c. At what point(s) in the first quadrant is the line tangent to C parallel to the y-axis?

2.5.3

With C and P_o as in Exercise 2.4.2(L), use the equation for $\tan \psi$ to find the line tangent to C at P_o.

2.5.4

Let C denote the curve whose polar equation is $r = \sin^2\theta$.

a. Compute $\tan \psi$ at the point $P_o(\frac{1}{2},\frac{\pi}{4})$ on C.

b. Use (a) to construct the line tangent to C at P_o.

c. From either (a) or (b), determine the slope of the line tangent to C at P_o.

2.5.5(L)

a. From the fact that

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2},$$

where s denotes arc length, show that if x, y, and r are differentiable functions of θ, then

$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}.$$

b. Use (a) to find the length of the curve C if its polar equation is given by $r = \sec\theta$, $0 \leqslant \theta \leqslant \frac{\pi}{4}$.

2.5.6

Find the length of the curve C if its polar equation is $r = 1 + \cos\theta$, $0 \leqslant \theta \leqslant 2\pi$.

2.5.7

a. If the polar equation for C is $r = \sin\theta$, where $0 \leqslant \theta \leqslant 2\pi$, find the area enclosed by C.

b. C_1 has the polar equation $r = \sin\theta$ while C_2 has the polar equation $r = \cos\theta$. Find the area of the region common to C_1 and C_2.

c. If C is the curve defined by the polar equation $r = \sin\frac{\theta}{4}$, $0 \leqslant \theta \leqslant 4\pi$, then C has two "pieces" in the first quadrant. Find the area of the region bounded between these two pieces and the positive x- and y-axes.