**INSTRUCTOR:** Hi. Today we're going to conclude our study of learning our differential equations. And we're going to use the occasion to introduce the concept of a Laplace transform.

I should point out that the Laplace transform has much greater application than just two linear differential equations. It's related to the Fourier transform. It has applications in the subject known as the convolution integral-- many applications, which we'll talk about during the learning exercises in this unit.

But within the framework of learning our differential equations, I thought this would be a good time to introduce this rather important concept. Consequently, our topic for today is called, quite simply, the Laplace transform. And it hinges on the fact that e to the t goes to infinity-- as t approaches infinity, you see-- much more rapidly than most functions of t.

And by the way, I say t here instead of x, because the Laplace transform was introduced primarily for differential equations in which the independent variable was time. So traditionally, one usually talks about f of t when one is talking about Laplace transform concepts.

But the entire idea is this. Remember, as we saw in part 1 of our course, that e to the t goes to infinity much faster than t to the n for any integer n. Consequently, what this means is, since most functions can be represented in the power series, one would expect that if one were to take a function like f of t and multiply it by e to the minus st where s was a constant greater than 0, one would expect that e to the minus st f of t would approach 0 rapidly as t approaches infinity. Now, this may not happen. We'll talk about that in more detail later.

But at any rate, let's assume that we're dealing with a particular function f of t where not only does e to the minus f of t go to 0 rapidly for some value of s as t approaches infinity, but so rapidly that the integral of e to the minus st f of t dt from 0 to infinity converges. Let's just keep that in mind for the time being. And with this in mind, one defines the Laplace transform of a function f of t. Namely, given f of t, the Laplace transform of f of t, written l of f of t, is defined to be the integral from 0 to infinity e to the minus st f of t dt, provided that this integral converges.

Notice, by the way, that since t is the variable of integration, when we evaluate this integral at t equals 0 and t equals infinity, the resulting function becomes a function of s alone. In other words, we evaluate this. As t goes from 0 to infinity, the resulting integral is a function of s alone. Consequently, one often-- to identify the function with this Laplace transform-- indicates that, if the function was f of t, the Laplace transform is f bar of s.

And notice, by the way, what this thing means pictorially. This is an area under a curve. And what you're really saying, if the Laplace transform of a function exists, is that this area under the curve as t goes from 0 to infinity is, indeed, finite. By the way, this should also lead you to believe that if this is finite for one value with of s, if s is replaced by something larger, this goes to 0 even faster. And consequently, the area under the curve will exist for all values of s beyond a certain value once the area exists for that one value.

Maybe the easiest way to say that is in terms of a picture. Here, I've drawn two representative functions y equals f of t, one case being a pulsation type of thing where you have a irregular pulse repeated here, the other being a polynomial y equals t squared. What we're saying is that, if I multiply f of t by e to the minus st-- remember, e to the minus st for positive values of s is a curve that goes to 0 very rapidly. And what this tends to do is to pull this thing down so that you go to 0 very rapidly. And the fact that the integral exists says not only does this go to 0 very rapidly, but it goes to zero so fast that the area under the curve stays finite.

Remember, that was our version of infinity times 0. It's not enough that this goes to 0, that this approaches the t axis. It has to approach fast enough so that the area under the curve is finite. And all we're saying is that if you can find one value of s that makes the area under the curve finite, any larger value of s will certainly make the area under the curve finite, because this will be pulled down even faster. And perhaps you can begin to see intuitively that, as I let s get larger and larger, not only does the area under the curve stay finite, but one would expect that it would also approach 0 because, for large values of s, e to the minus st goes to 0 so rapidly that even large values up here would be pulled down very rapidly.

At any rate, let me illustrate this by means of an example. Suppose I pick, for f of t, something as simple as e to the at where a is a given constant. What I mean by "simple" here is recalling that the Laplace transform involves multiplying f of t by e to the minus st. This will give me a simple algebraic manipulation to perform.

By definition, how do I form the Laplace transform of e to the at? What I do is I multiply e to the at by e to the minus st and integrate that from 0 to infinity. And if that integral exists-- meaning if the integral converges, if this limit is finite-- I call that the Laplace transform.

At any rate, just substituting n here then, f of t becomes e to the at. This says integral from 0 to infinity e to the a minus s times t power dt. Since a and s are constants, the integral e to the a minus st comes out to be 1 over a minus s, e to the a minus st, evaluated as t goes from 0 to infinity.

Now, the key point to notice here is that, as soon as s is greater than a, this gives me a negative exponent-- as soon as s is greater than a. Consequently, as t goes to infinity, this essentially behaves like one over e to the infinity, which is 0. In other words, once s is greater than a, the upper limit drops out. My lower limit would just be e to the zero, you see. And I'm subtracting that.

At any rate, to make a long story short and to summarize what we're saying, simply observe that, if s is greater than 0, the limit as t approaches infinity e to the a minus st is 0 and that, therefore, the Laplace transform of e to the at is simply 1 over s minus a, provided s is greater than a. In other words, again, just looking back here-- see-- I subtract the lower limit. When t is 0, this is 1. But I'm subtracting. It makes it minus 1. And minus 1 over a minus s is the same as 1 over s minus a.

To state this then in other words, if f of t is e to the at, f bar of s is 1 over s minus a where the domain of f bar is a set of all s, such that s is greater than a. And what does this mean, geometrically? This is the area under the curve e to the minus st-- e to the at-- as t goes from 0 to infinity. That area will be finite as soon as s is greater than a.

By way of illustration, if I replace a by 2, what we're saying is that the Laplace transform of e to the 2t is 1 over s minus 2. Notice that nice polynomial quotient idea, that rather simple expression-- 1 over s minus 2 if s is greater than 2. If we replace a by minus 3 and use this recipe, we have that the Laplace transform of e to the minus 3t is 1 over s minus minus 3, 1 over s plus 3, where S must be greater than minus 3.

Notice also here that, by the comparison test, so to speak-- remember, the same comparison test that we use for infinite series-- if the magnitude of f of t is less than some constant c times e to the at power for all values of t, then the integral from 0 to infinity e to the minus st f of t dt must also converge. In other words, we already know that this integral converges, shall we say, if the integrand were e to the at. Therefore, we're saying it will converge if you multiply that by a constant, because convergence isn't affected by multiplying by a constant. Consequently, if the integrand that we're investigating is smaller than this in magnitude, it must also have a Laplace transform-- that this thing here must converge.

Now, because this is so obvious and so important, one defines a function with this property to have a very special name. Namely, the function f of t is set to have exponential order if and only if there exists constant c and a such that the magnitude of f of t is less than ce to the at for t, all t, greater than 0. What we are saying in summary is that, by this definition of exponential order and the comparison test, all functions of exponential order have Laplace transforms, because that integral from 0 to infinity-- e to the minus st f of t dt-- will converge by comparison with the integral 0 to infinity e to the minus st e to the at dt for s greater than. In other words, this will all converge where the value of s has to be greater than a.

Now, the idea is, what's so important about functions of exponential order? Why do I stress these? And the answer is that, in terms of linear differential equations with constant coefficients, the functions that we tend to deal with all have exponential order. In other words, they will all have Laplace transforms.

For example, e to the at, trivially, is a function that has exponential order. That was our model for the definition. In fact, e to the at is 1e to the at. Simply take a to ba c to b1, and we have the criterion for exponential order being obeyed.

The sine of at where a is any constant has exponential order. Why? Because the magnitude of sine at is certainly no greater than 1. And 1 is certainly no greater than e to the t of any positive value of t. See, e to the 0 is already 1. Therefore, sine at is less than 1e to the 1t if t is greater than 0. So sine at has exponential order.

Finally, we dealt with polynomials t to the n where n was a positive integer. And in part 1 of our course, either using L'Hospital's rule, or power series, or what have you, we showed that for any fixed positive integer n, t to the n times e to the minus t went to 0 in the limit as t went to infinity. Consequently, for large values of t, the magnitude of t to the n is less than e to the t.

I'll leave the details out here. I'm just trying to illustrate what's going on. The important point is that the usual functions encountered in linear differential equations with constant coefficients have Laplace transforms.

And I would like to make a couple of notes about this thing. One I've already said, but I'd like to reinforce it. What we've already seen is that if s has exponential order, f bar of s, the Laplace transform of s must be less than or equal to the Laplace transform of e to the-- well, a constant times e to the at. But the Laplace transforms e to the at is 1 over s minus a so that the Laplace transform of f of t, if f has exponential order, can be no bigger than c over s minus a.

Since c is a constant and a is a constant, as s goes to infinity, f bar of s goes to 0. In other words, for sufficiently large values of s, the Laplace transform, which names an area under a curve, goes to 0 meaning, as we would expect, the area under the curve does go to 0 as s increases without bound. What that means in particular, you see, is that not any function can be a Laplace transform of something. You see, in particular, this says that for f bar to be the Laplace transform of something, f bar of s must go to 0 as s goes to infinity.

For example, if I were to write down f bar of s to be s over s minus 1-- or s plus 1, even, f bar of s is s over s plus 1-- notice that the limit of f bar of s, as s goes to infinity, is 1. Consequently, this f bar cannot be the Laplace transform of any function, at least of exponential order, because we've already seen that the Laplace transform of a function-- at least of exponential order-- must go to 0 as s goes to infinity. So not every function can be a Laplace transform of something.

And secondly, not every function has exponential order. It may be true, as we just showed, that every function that we're dealing with when we're dealing with linear differential equations with constant coefficients-- or most functions that we're dealing with linear differential equations with constant coefficients-- has our functions of exponential order. Not all functions have exponential order.

For example, e to the t squared does not have exponential order-- not order order, order-- since, notice, that e to the t squared divided by ce to the at would be 1 over c, e to the t squared minus at, which is t times t minus a. And notice now that, if t is greater than a, this exponent will be positive. And as t goes to infinity-- you see, once t is greater than a, this thing increases without bound. And consequently, since c is a constant, you essentially have infinity divided by a non-zero constant.

So you have, what? That e to the t squared over ce to the at goes to infinity as t approaches infinity. In particular, this says that e to the t squared dwarfs ce to the at for large values of t. In particular, you cannot have that e to the t squared is less than some constant times e to the at.

Now the question that comes up is, what's so important now about the Laplace transform in terms of linear differential equations? And the answer is that the Laplace transform, amazingly enough, has properties of linearity. What do you mean by properties of linearity? You mean, what? That for a function to be linear, l of a sum must be the sum of the l's. And l of a constant time something must be the constant times l of something.

Well, look at-- when you're dealing with integrals, given two definite integrals which exist, the sum of the integrals is the integral of the sum. The integral is a constant times an integrand is a constant times the integral of the integrand itself. In other words-- sparing you the details, because they follow right from the definition. And that is assuming that f and g have Laplace transforms-- in other words that that improper integral from 0 to infinity, e to the minus st f of t dt, converges.

Then what's true is that l of f plus g is l of f plus l of g. And l of c times f is c times l of f. See-- the linearity properties. Now, how does that help us? And why does that give us a hint that there will be some usage of the Laplace transform in solving linear differential equations with constant coefficients? Well, perhaps the best way to see this is by means of an illustration.

Let's assume that we're given that y is a twice differentiable function of t and that y double prime plus 2a y prime plus by where a and b are constants is some function f of t. The idea is this. I simply take the Laplace transform of both sides. See?

By linearity, since the Laplace transform is a linear function, a linear operator, l of y double prime plus to 2ay prime plus by is simply l of y double prime plus 2a l of y prime plus b l of y. And the Laplace transform of f of t, assuming that f of t has a Laplace transform-- in particular, if f of t happens to be a function of exponential order, we've already seen that it will have a Laplace transform. All we're saying is, let's call a Laplace transform of f of t, as usual, f bar of s.

And now what we have is an equation involving the Laplace transform of y, the Laplace transform of y prime, the Laplace transform of y double prime. In terms of f bar of s, if somehow we could manipulate this to solve for l of y-- in other words, if we could find what the Laplace transform of y was in terms of s-- maybe we could then invert, so to speak, and find out what the function itself must have been. In other words, the idea of inverse functions comes up the same way here as it comes up every place.

Namely, starting with a function that maps x into y, one often says, if I know what y is, can I determine what x must have been? And that's why 1 to 1-ness is so important. In other words, if it turns out that the Laplace transform is 1 to 1, once I know the Laplace transform, I essentially know the function, because if two different functions can't have the same Laplace transform-- and by the way, as a finale for today's lecture, we will show that this is essentially the case that the Laplace transform is 1 to 1. And what I mean by essentially, it will be pointed out in the learning exercises.

But don't worry about that now. The idea is simply this. Suppose there was some way of expressing l of y prime and l of y double prime in terms of l of y. I could then solve the resulting equation for l of y in terms of s. Having a table of Laplace transforms, I could then locate what that function of s is the Laplace transform of and then invert this to find what my y must have been.

The problem is, how do I know that I can express l of y prime, l of y double prime, l of y triple prime? I'm only going up to the second derivative in terms of our usual convention of illustrating everything by second-order equations, even though the results hold the higher orders. The question though is, how do we know that we can express these Laplace transforms in terms of the Laplace transform of y itself?

And this brings up the second reason why that fact-- that e to the minus st has a multiplying factor-- is so important. You see, aside from the fact that, for most functions, e to the minus st makes sure that the resulting integral will converge, the other key property is that when you integrate or differentiate e to the minus st, because s is a parameter-- meaning it's being treated as a constant at a given time-- what this thing means is that the integral or the derivative of e to the minus st is simply a constant times e to the minus st. Well, let me show you how this is used.

Let's suppose, for example, that I wanted to find the Laplace transform into f prime of t. And I didn't know any tricks. And by the way, I hope that this is one of the fringe benefits that this course is teaching you-- that we do not have to be tricky in mathematics. All we have to know is the basic definitions and how to manipulate them. I do admit that it is a stroke of genius sometimes in inventing the basic definition that we're going to use.

For example, it's my own belief that it's a lot easier to use the Laplace transform than it was to have invented the concept in the first place. But once I've defined what the Laplace transform is, what happens here is very simple. Namely, by definition, what is the Laplace transform of f prime of t? I simply multiply f prime of t by e to the minus st and integrate that from 0 to infinity. And if that integral converges, then the Laplace transform exists.

Let me assume, for the sake of argument now, that the Laplace transform of f of t exists. And by the way, again remember, I'm not really assuming anything when I'm dealing with linear differential equations with constant coefficients, because the functions that come up as possible solutions there are of exponential order. And we have already seen that, for functions of exponential order, the Laplace transform does indeed exist.

But anyway, the idea is this. By definition, this is the Laplace transform of f prime of t. I would like to be able to integrate this. Well, the idea is that the integral of f prime of t is certainly f of t. And if I-- well, to a constant. But that's not important. What I'm thinking of is integrating by parts.

Notice that if I let u equal e to the minus st and dv be f prime of t dt, then du is minus se to the minus st. v would just be f of t. Remembering the recipe for integration by parts-- remember, just by way of quick review-- integral u dv is equal to uv minus the integral v du. What we're saying here is that to evaluate this integral, we simply take, what? u times v-- e to the minus st f of t.

Evaluate that as t goes from 0 to infinity, minus the integral v du. du is already negative. s is a constant. So minus integral v du is just plus s integral 0 to infinity, e to the minus st f of t dt. And lo and behold, you see that's that beautiful property of e to the minus st. It's still in here. The integral is precisely what we mean by, what? l of f of t. In other words, the Laplace transform of f prime of t is this thing plus s times the Laplace transform of f of t.

But let me not be so informal. Let me not refer to this as "this thing." Let's see what "this thing" really is. Notice that, because the Laplace transform of f of t is assumed to exist, the fact that the integral of this from 0 to infinity is finite means, in particular, that ad infinity, the integrand must be 0. You see, otherwise, how could the area under the curve be finite if the curve didn't at least asymptotically approach the t axis?

So in other words, the assumption that f has a Laplace transform means that the upper limit gives me 0. The lower limit gives me, what? When I plug in t equals 0, this is e to the minus s 0, which is 1. This is f of 0. So the lower limit is f of 0.

And since I'm subtracting the lower limit, I have what? That the thing that I call "this thing" is, more precisely, minus f of 0. In other words, to find the Laplace transform of f prime of t, it's simply minus f of 0 plus s times the Laplace transform of f of t.

Well, look. If I happen to know what f of 0 is-- in other words, if I pick a particular member of the family f of t, given the curve y equals f of t-- I certainly know what f of 0 is. This says, what? The Laplace transform of f prime of t is some constant plus s times the Laplace transform of f of t itself. In other words, somehow or other, I have now managed to express the Laplace transform as f prime of t in terms of a polynomial involving s and the Laplace transform of f of t itself.

By the way, notice that generically here there is nothing sacred about f of 0, or f. Think of f as being any function and f prime as being its derivatives. In particular, I could allow f prime to play the role of f. See-- this says, what? The Laplace transform of the derivative is minus the function evaluated at 0 plus s times the Laplace transform of that function.

So if I now take my function to be f prime, its derivative is f double prime. The function is, itself, f prime. So all I really do is, what? Go into this recipe here. Every place I see a prime, replace it by a double prime. Every place I see no prime, put a prime in. And I have that the Laplace transform of f double prime of t is minus f prime of 0 plus s times the Laplace transform of f prime of t.

By the way, to make sure you see this, notice-- what would the Laplace transform of f triple prime be? The Laplace transform of f triple prime of t would be minus f double prime of 0 plus s times the Laplace transform of f double prime of t. So assuming that f prime also has a Laplace transform, we see what the Laplace transform of f double prime of t looks like in terms of f prime of t-- the Laplace transform of f prime of t.

We know what the Laplace transform of f prime of t looks like in terms of the Laplace transform of t. In other words, from here, I simply replace l of f prime of t by its value in this equation. And I wind up with that the Laplace transform of f double prime of t is nothing more than minus f prime of 0 minus s times f of 0 plus s squared l of f of t. And now, you see, I've solved the problem that I was discussing over here.

Namely, once I know what y of 0 is-- by the way, don't confuse the f here with the f here. That's an unfortunate choice, which I've just noticed. The f that I'm referring to here names the function whose Laplace transform I'm trying to find.

Notice that the f prime and f double prime are served by y prime and y double prime here. What we're saying in terms of this notation is that I can express l of y double prime and l of y prime in terms of the Laplace transform of l of y plus powers of s, provided I know only what y of 0 is and what y prime of 0 is. And by the way, again, notice that in a physical problem, it is very natural to look at y and y prime when t equals 0 because, in many instances, t equals 0 represents the time at which we've started the measurement at our experiment. And this becomes a very natural interpretation for what you mean by the initial conditions-- namely, what's going on when t equals 0.

At any rate, we are now in a position to apply our results to an actual linear differential equation with constant coefficients given initial conditions as to what's happening at 0 in terms of the y-coordinate, and the y-prime coordinate, et cetera-- in other words, as an application to a linear differential equations with constant coefficients. Suppose we want to solve-- meaning, what? Find what function y is of t, if y double prime minus 4y prime plus 3y equals e to the 2t, given that my initial conditions-- namely, when t is 0, y is going to be 0, and y prime is going to be 1.

This could be any conditions I want over here. I just chose these to give me a rather simple algebraic example. And I'll pick more complicated things in the exercises. But the idea is this. We have already seen by linearity that, if I take the Laplace transform of both sides here, I have the l of y double prime minus 4l of y prime plus 3l of y is the Laplace transform of e to the 2t.

Now, we just saw, right over here, that the Laplace transform of y double prime is minus y prime evaluated at 0 minus sy of 0 plus s squared l of y. The Laplace transform of y prime was minus, you see, y of 0 plus sl y. So minus 4 times that is just 4 times y of 0 minus 4s l of y plus 3l of y.

And we saw earlier that the Laplace transform of e of the at is 1 over s minus a where s is any number greater than 2. In particular, this is that formula with a equal-- s is any number greater than a. In particular, this is that formula with a equal to 2, so that the Laplace transform of e to the 2t is 1 over s minus 2, provided s is greater than 2.

By the way, notice, up to this point, I have not used the initial conditions. I have not used the fact that y of 0 is 0 and that y prime of 0 is 1. Notice that, even without this, the l of y is here by itself. It's going to be multiplied by s squared minus 4s plus 3.

These terms can go over onto the other side of the equation. I will get a function as s alone. I then divide through by s squared minus 4s plus 3. That will give me l of y as a function of s. And if I can then find one function which has that function of s as its Laplace transform, I can conclude that that function is the one I'm looking for, provided only that l is a 1 to 1 operator-- that the Laplace transform is a 1 to 1 function.

At any rate, all I did in our problem was to simplify this by specifying that y of 0 was 0 and that y prime of 0 was 1. These two terms drop out. This is minus 1. It comes over onto the other side as plus 1. And so I wind up with this particular equation.

And by the way, notice that this simply says, what? s minus 2 plus 1 over s minus 2. In other words, this is s minus 1 over s minus 2. This factors into s minus 1 times s minus 3. So in other words, what I'm now faced with is that s minus 1 times s minus 3 times l of y is equal to s minus 1 over s minus 2.

I have to be careful. You see, I wanted to divide-- see, I want to be able to cancel out the s minus 1's here. I want to be able to divide through by s minus 3. I have to be careful of 0 denominators. And the safest way to take care of everything is that all I really care about is whether the Laplace transform exists for sufficiently large values of s.

To get rid of my headache, I'll simply assume that I'm not even looking at the Laplace transform until, shall we say, s is greater than 3. You see, once s is greater than 3, this factor can't be 0. This factor can't be 0. Consequently, I can solve for l of y and concluded that l of y is 1 over s minus 3 times s minus 2, provided that s is greater than 3.

Now I resort to that same method of partial fractions that we used when we integrated back in part 1 using partial fractions. I simply try to find constants a and b, such that I can write this as a over s minus 3 plus b over s minus 2. In other words, break this down into simpler parts. Sparing you the details, it follows very simply in this case that this expression is simply 1 over s minus 3 minus 1 over s minus 2. In other words, that if I put this over a common denominator, I get 1 over s minus 3 times s minus 2.

Now here's where the kicker comes in. Remember, we have already seen that 1 over s minus a is the Laplace transform of e to at, provided s is greater than a. In particular, the Laplace transform of e to the 3t would be 1 over s minus 3, provided s was greater than 3. Similarly, the Laplace transform into e to the 2t would be 1 over s minus 2, provided s was greater than 2.

By the way, notice that both of these conditions are obeyed as soon as s is greater than 3. Obviously, if s is greater than 3, it must be greater than 2. So I'm sure that both of these results are true once s is greater than 3. You see the same 3 I have up here.

Now, by linearity, I know that the Laplace transform of a difference is the difference of the Laplace transforms. You see, by equals added to equals, this tells me that l of e to the 3t minus l of e to the 2t is 1 over s minus 3 minus 1 over s minus 2. But by linearity, l of e to the 3t minus l of e to the 2t is l of the quantity e to the 3t minus e to the 2t.

In particular then, do I know one function whose Laplace transform is 1 over s minus 3 minus 1 over s minus 2? And the answer is, yes. We've just constructed it-- namely, the function y equals e to the 3t minus e to the 2t. In other words what we do know is that, in any event, whether y is the given function or not, whatever l of y is, it's l of e to the 3t minus e to the 2t. And therefore, we could conclude that y must equal e to the 3t minus e to the 2t, provided that l is 1 to 1-- provided that there was only one function that can have a given Laplace transform.

You see, you cannot automatically conclude that, because these two things are equal-- that the whole expression's equal-- that the inputs must be equal. For example, you cannot conclude that x equals pi over 6, simply because you know that sine x equals sine pi over 6. You see, the sine function is not 1 to 1.

Well, at any rate, I think you can sense intuitively that-- because we already know that, for linear differential equations with constant coefficients, there are no singular solutions and that, once you've found one solution, subject to given initial conditions, you've found them all. I think you can guess that, at least for functions of exponential order, this should be a true result. And the fact that it is true is known in the literature under the name of Lerch's theorem-- "Lerch," not L-U-R-CH, but L-E-R-CH. It's known under the name of Lerch's theorem. And I have taken the liberty of stating it in a less controversial form, saving the generalization for the learning exercises.

But essentially, all Lerch's theorem says is this. If f and g are continuous and of exponential order, then if the Laplace transform of f equals the Laplace transform of g, for all sufficiently large values of s-- in other words, if beyond a certain s, f bar of s equals g bar of s, in other words, f and g have the same Laplace transform-- then f and g must be identical. And there is a slight modification of this that comes up if you leave out the word "continuous" here. Lerch's theorem is stated only in terms of f and g being piecewise continuous. In other words, there could be jump discontinuities in the curves and things like this, at which case this would hold, except possibly at the points at which you had jump discontinuities.

I prefer not to get into this at this particular time, but rather to wrap up our lecture at this particular point, and to simply say that we have done-- what now? We have studied linear differential equations. We have used this last lecture to introduce the concept of the Laplace transform, which has secondary value in terms of solving differential equations, but which has other applications that go far beyond the scope of this particular course. And what I'm hoping is that this rather introductory lecture, coupled with well-chosen exercises, will give you enough insight to the Laplace transform so that you will be able to handle it not only in terms of solving linear differential equations with constant coefficients but in other contexts where they might occur in the particular line of work and research that you're doing.

At any rate, this completes our work on block seven. And in our next lecture, what we shall do is start the final block of our course and revisit the concept of vector spaces from, hopefully, a more productive point of view than what we've had before. But more about that next time. Until next time, goodbye.

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