SOME BASIC CONCEPTS OF ENGINEERING ANALYSIS

LECTURE 1

46 MINUTES

LECTURE 1

Introduction to the course, objective of lectures

Some basic concepts of engineering analysis, discrete and continuous systems, problem types: steady-state, propagation and eigenvalue problems

Analysis of discrete systems: example analysis of a spring system

Basic solution requirements

Use and explanation of the modern direct stiffness method

Variational formulation

TEXTBOOK: Sections: 3.1 and 3.2.1, 3.2.2, 3.2.3, 3.2.4

Examples: 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.11, 3.12, 3.13, 3.14

INTRODUCTION TO LINEAR ANALYSIS OF SOLIDS AND STRUCTURES

- The finite element method is now widely used for analysis of structural engineering problems.
- In civil, aeronautical, mechanical, ocean, mining, nuclear, biomechanical,... engineering
- Since the first applications two decades ago,
  - we now see applications in linear, nonlinear, static and dynamic analysis.
  - various computer programs are available and in significant use

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My objective in this set of lectures is:

- to introduce to you finite element methods for the linear analysis of solids and structures.

  ["linear" meaning infinitesimally small displacements and linear elastic material properties (Hooke's law applies)]

- to consider
  - *the formulation of the finite element equilibrium equations*
  - the calculation of finite element matrices
  - methods for solution of the governing equations
  - *computer implementations*
- to discuss modern and effective techniques, and their practical usage.

REMARKS

- Emphasis is given to physical explanations rather than mathematical derivations

- Techniques discussed are those employed in the computer programs

  SAP and ADINA

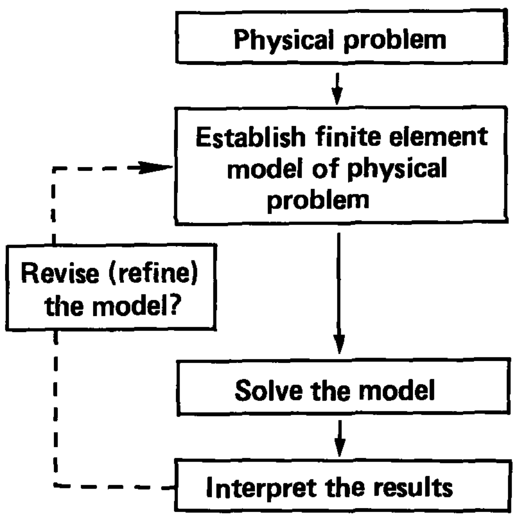
  SAP ≡ Structural Analysis Program

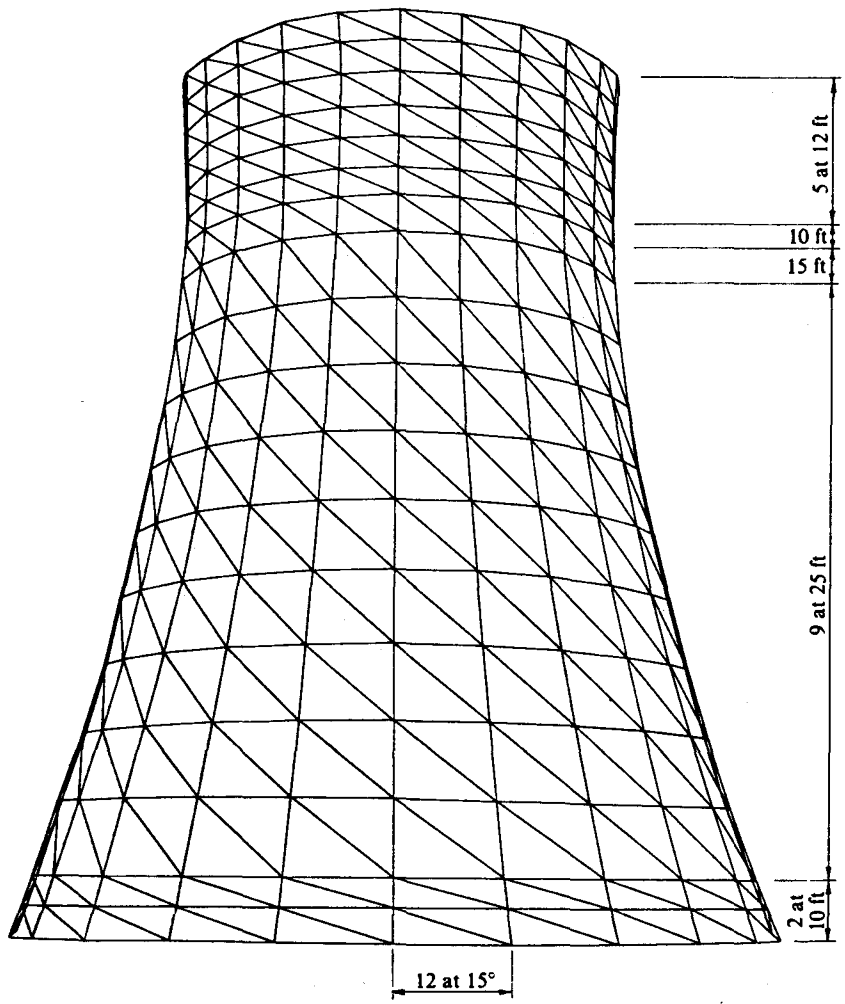
  ADINA ≡ Automatic Dynamic Incremental Nonlinear Analysis

- These few lectures represent a very brief and compact introduction to the field of finite element analysis

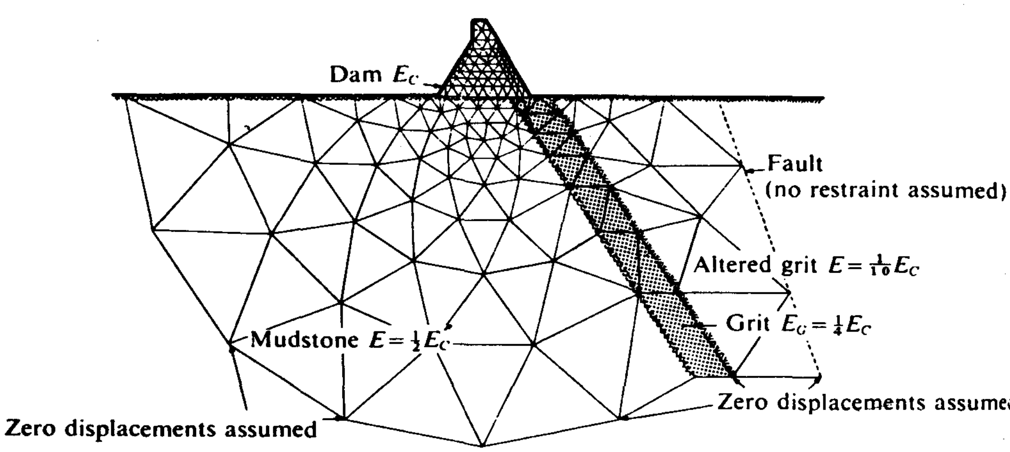
- We shall follow quite closely certain sections in the book
  Finite Element Procedures in Engineering Analysis,
  Prentice-Hall, Inc.
  (by K.J. Bathe).

Finite Element Solution Process

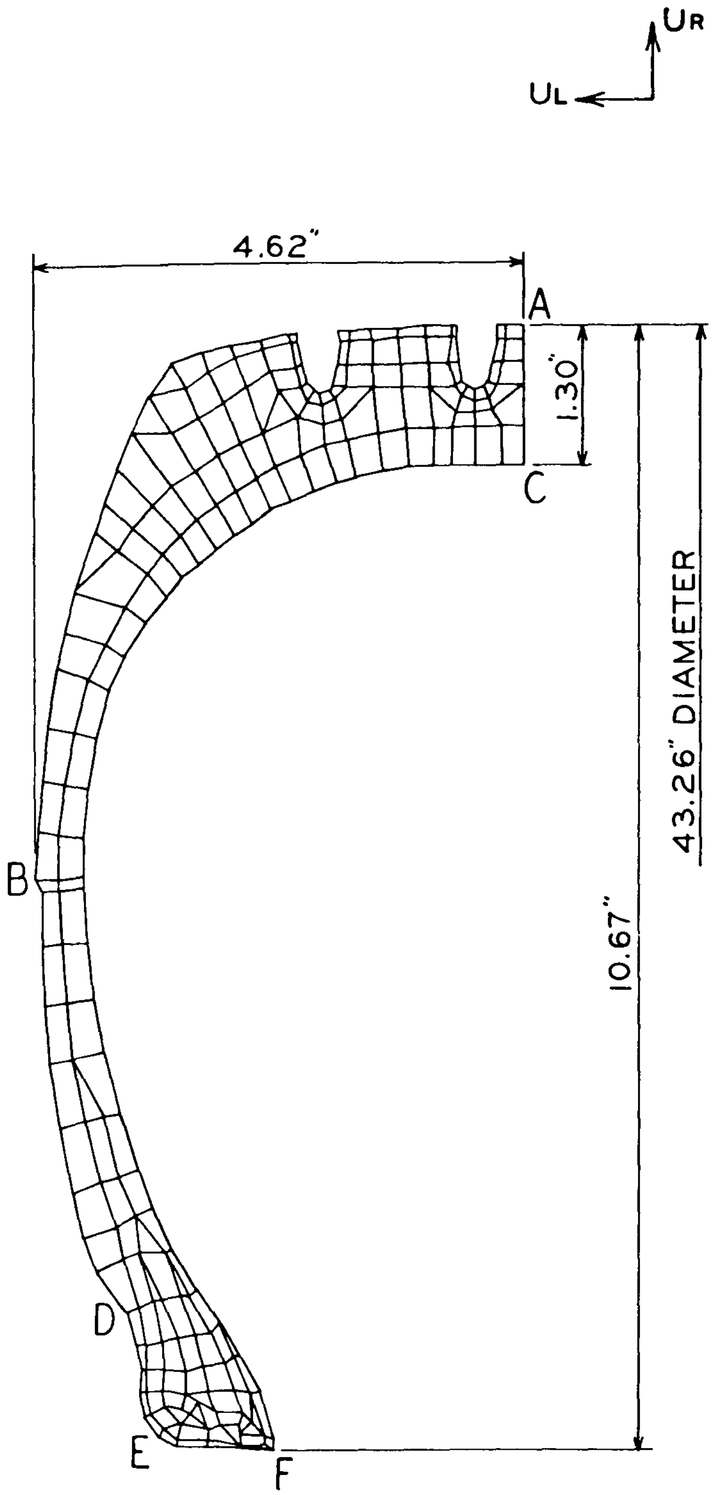




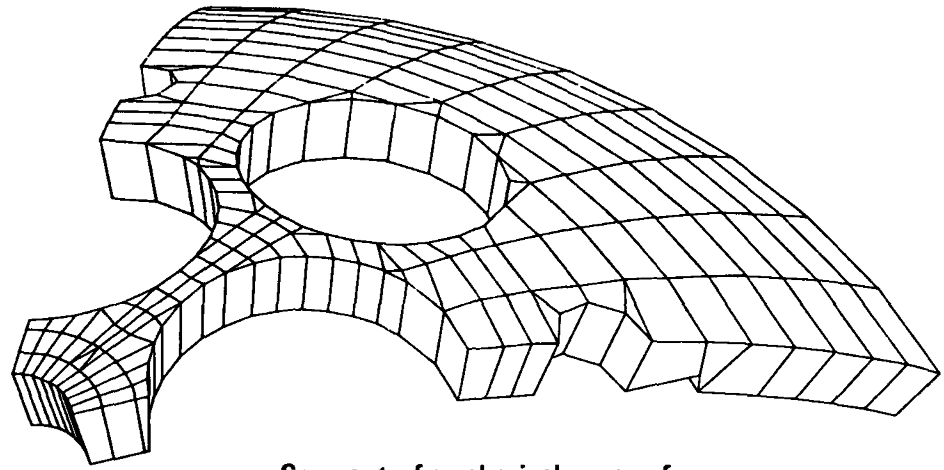
Analysis of cooling tower.



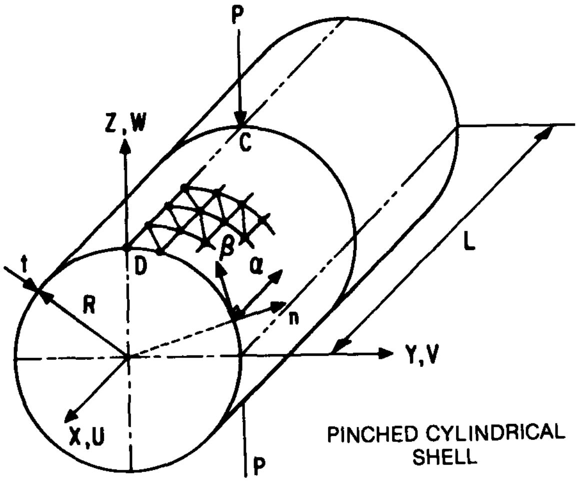
Analysis of dam.



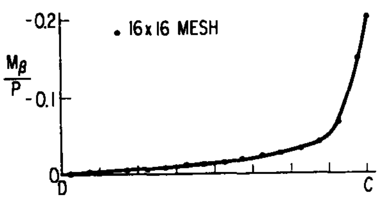
Finite element mesh for tire inflation analysis.



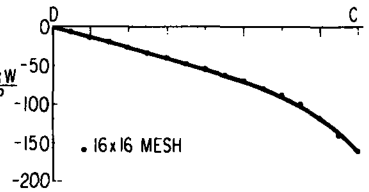
Segment of a spherical cover of a laser vacuum target chamber.



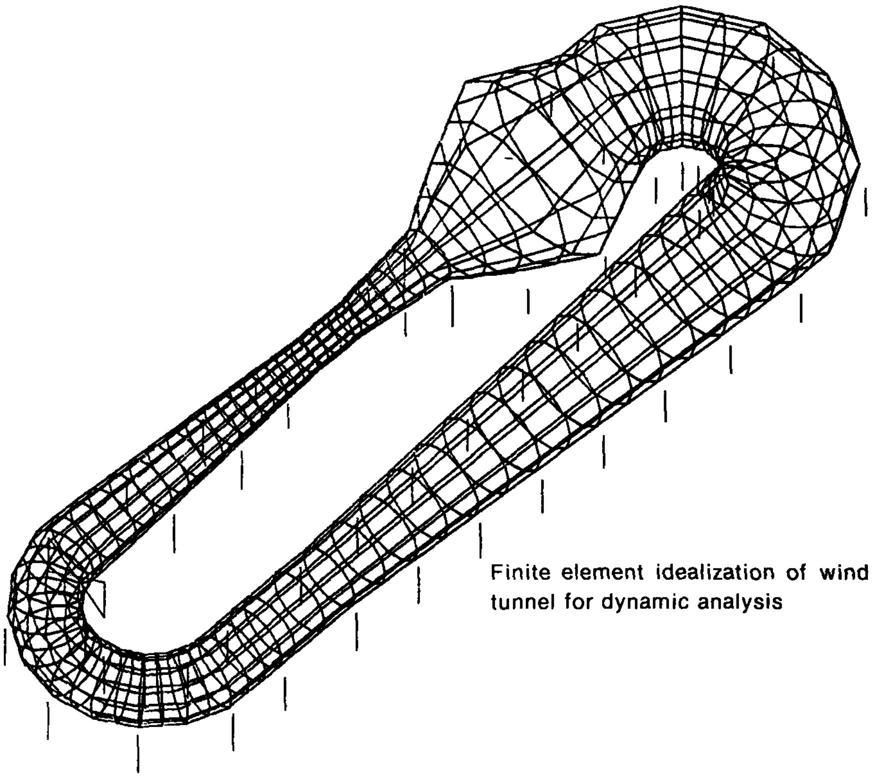
PINCHED CYLINDRICAL SHELL



BENDING MOMENT DISTRIBUTION ALONG DC OF PINCHED CYLINDRICAL SHELL



DISPLACEMENT DISTRIBUTION ALONG DC OF PINCHED CYLINDRICAL SHELL



Finite element idealization of wind tunnel for dynamic analysis

SOME BASIC CONCEPTS OF ENGINEERING ANALYSIS

The analysis of an engineering system requires:

- **idealization of system**
- **formulation of equilibrium equations**
- **solution of equations**
- **interpretation of results**

SYSTEMS

DISCRETE	CONTINUOUS
response is described by variables at a finite number of points	response is described by variables at an infinite number of points
set of algebraic equations	set of differential equations

PROBLEM TYPES ARE

- STEADY-STATE (statics)
- PROPAGATION (dynamics)
- EIGENVALUE

For discrete and continuous systems

Analysis of complex continuous system requires solution of differential equations using numerical procedures

reduction of continuous system to discrete form

powerful mechanism:

the finite element methods, implemented on digital computers

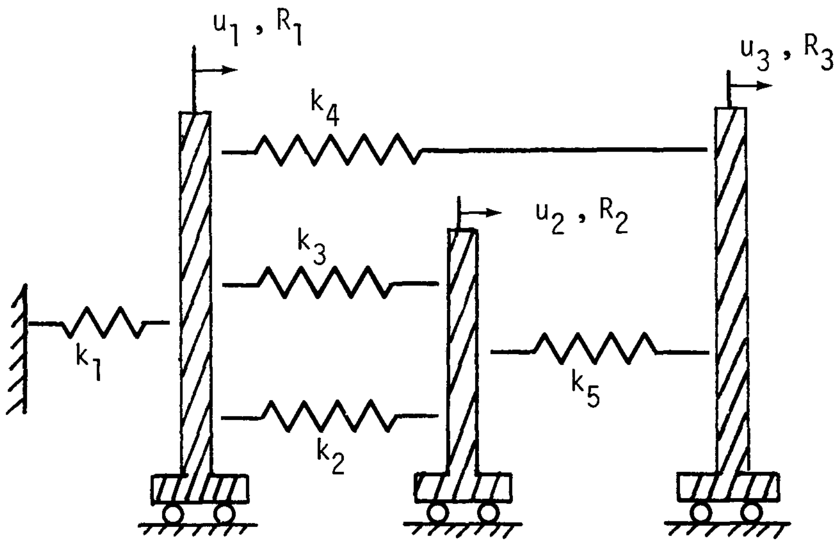
ANALYSIS OF DISCRETE SYSTEMS

Steps involved:

- system idealization into elements
- evaluation of element equilibrium requirements
- element assemblage
- solution of response

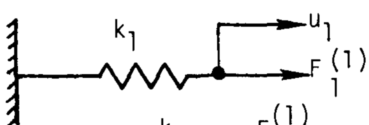
Example:

steady-state analysis of system of rigid carts interconnected by springs

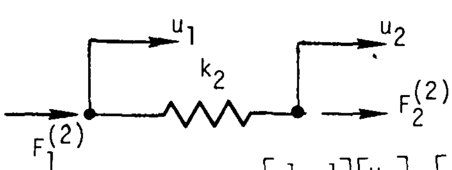


Physical layout

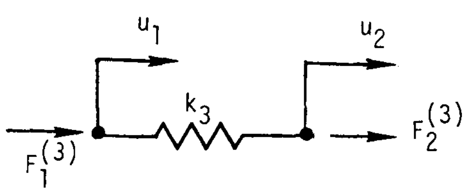
ELEMENTS



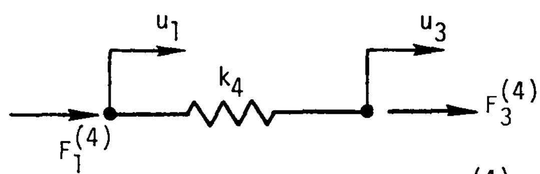
$$k_1 u_1 = F_1^{(1)}$$



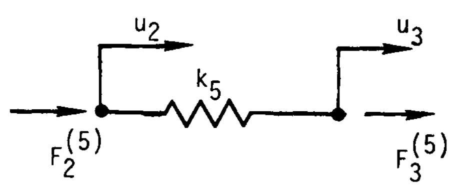
$$k_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1^{(2)} \\ F_2^{(2)} \end{bmatrix}$$



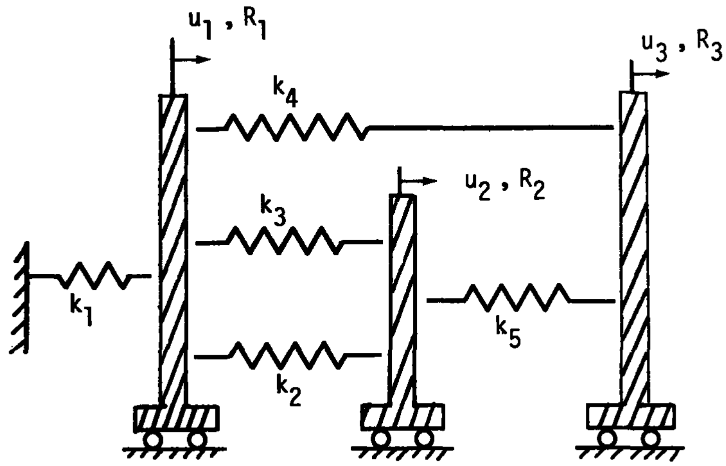
$$k_3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1^{(3)} \\ F_2^{(3)} \end{bmatrix}$$



$$k_4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1^{(4)} \\ F_3^{(4)} \end{bmatrix}$$



$$k_5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_2^{(5)} \\ F_3^{(5)} \end{bmatrix}$$



Element interconnection requirements :

$$F_1^{(1)} + F_1^{(2)} + F_1^{(3)} + F_1^{(4)} = R_1$$

$$F_2^{(2)} + F_2^{(3)} + F_2^{(5)} = R_2$$

$$F_3^{(4)} + F_3^{(5)} = R_3$$

These equations can be written in the form

$$\underline{K}\ \underline{U} = \underline{R}$$

Equilibrium equations

$$\underline{K}\ \underline{U} = \underline{R} \qquad \text{(a)}$$

$$\underline{U}^T = [u_1 \quad u_2 \quad u_3] \ ;$$

$$\underline{R}^T = [R_1 \quad R_2 \quad R_3]$$

$$\underline{K} = \begin{bmatrix} k_1 + k_2 + k_3 + k_4 & -k_2 - k_3 & -k_4 \\ -k_2 - k_3 & k_2 + k_3 + k_5 & -k_5 \\ -k_4 & -k_5 & k_4 + k_5 \end{bmatrix}$$

and we note that

$$\underline{K} = \sum_{i=1}^{5} \underline{K}^{(i)}$$

where

$$\underline{K}^{(1)} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

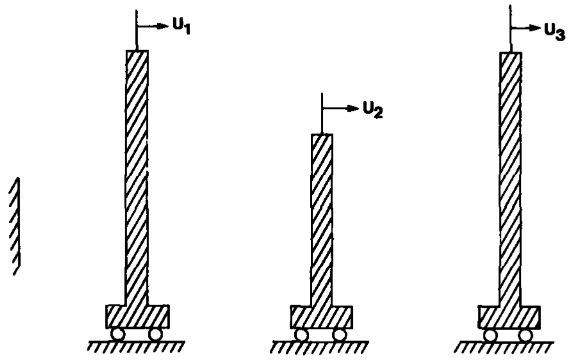
$$\underline{K}^{(2)} = \begin{bmatrix} k_2 & -k_2 & 0 \\ -k_2 & k_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

etc...

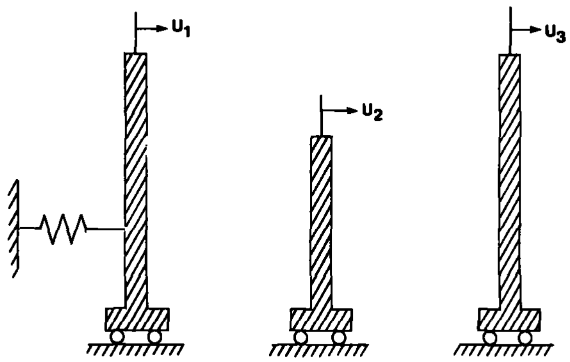
This assemblage process is called the <u>direct stiffness method</u>

The steady-state analysis is completed by solving the equations in (a)

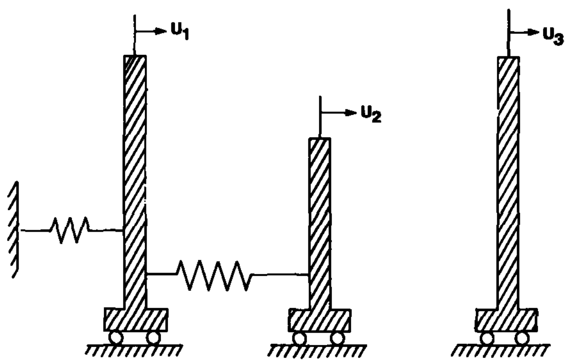
$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$



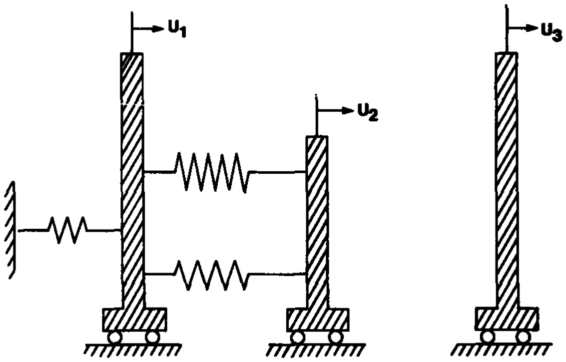
$$\underline{K} = \begin{bmatrix} K_1 & & \\ & & \\ & & \end{bmatrix}$$



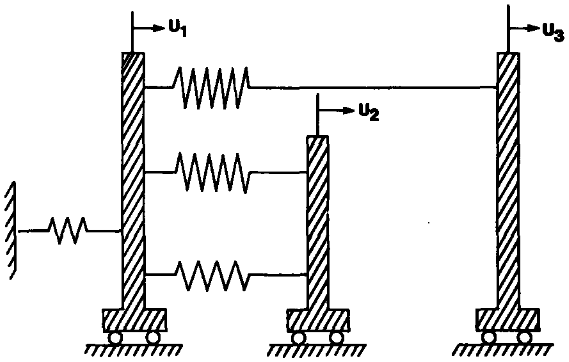
$$\underline{K} = \begin{bmatrix} K_1 + K_2 & -K_2 & \\ -K_2 & K_2 & \\ & & \end{bmatrix}$$



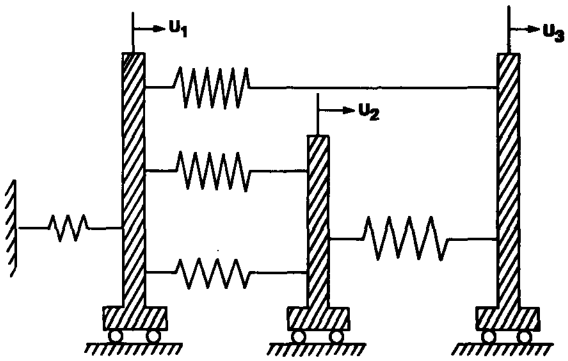
$$\underline{K} = \begin{bmatrix} K_1 + K_2 + K_3 & -K_2 - K_3 & \\ -K_2 - K_3 & K_2 + K_3 & \\ & & \end{bmatrix}$$



$$\underline{K} = \begin{bmatrix} K_1 + K_2 + K_3 + K_4 & -K_2 - K_3 & -K_4 \\ -K_2 - K_3 & K_2 + K_3 & \\ -K_4 & & K_4 \end{bmatrix}$$



$$\underline{K} = \begin{bmatrix} K_1 + K_2 + K_3 + K_4 & -K_2 - K_3 & -K_4 \\ -K_2 - K_3 & K_2 + K_3 + K_5 & -K_5 \\ -K_4 & -K_5 & K_4 + K_5 \end{bmatrix}$$



$$\underline{K} = \sum_{i=1}^{5} \underline{K}^{(i)}$$

In this example we used the direct approach; alternatively we could have used a variational approach.

In the variational approach we operate on an extremum formulation:

$$\Pi = \mathcal{U} - \mathcal{W}$$

$\mathcal{U}$ = strain energy of system

$\mathcal{W}$ = total potential of the loads

Equilibrium equations are obtained from

$$\frac{\partial \Pi}{\partial u_i} = 0 \qquad \text{(b)}$$

In the above analysis we have

$$\mathcal{U} = \tfrac{1}{2} \underline{U}^T \underline{K} \underline{U}$$

$$\mathcal{W} = \underline{U}^T \underline{R}$$

Invoking (b) we obtain

$$\underline{K} \underline{U} = \underline{R}$$

Note: to obtain $\mathcal{U}$ and $\mathcal{W}$ we again add the contributions from all elements

PROPAGATION PROBLEMS

main characteristic: the response changes with time ⇛ need to include the d'Alembert forces:

$$\underline{K}\,\underline{U}(t) = \underline{R}(t) - \underline{M}\,\ddot{\underline{U}}(t)$$

For the example:

$$\underline{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

EIGENVALUE PROBLEMS

we are concerned with the generalized eigenvalue problem (EVP)

$$\underline{A}\,\underline{v} = \lambda\,\underline{B}\,\underline{v}$$

$\underline{A}$, $\underline{B}$ **are symmetric matrices of order n**

$\underline{v}$ **is a vector of order n**

λ **is a scalar**

EVPs arise in dynamic and buckling analysis

Example: system of rigid carts

$$\underline{M}\,\ddot{\underline{U}} + \underline{K}\,\underline{U} = \underline{0}$$

Let

$$\underline{U} = \underline{\phi} \sin \omega(t-\tau)$$

Then we obtain

$$-\omega^2 \underline{M}\,\underline{\phi} \sin \omega(t-\tau)$$

$$+ \underline{K}\,\underline{\phi} \sin \omega(t-\tau) = \underline{0}$$

Hence we obtain the equation

$$\underline{K}\,\underline{\phi} = \omega^2 \underline{M}\,\underline{\phi}$$

There are 3 solutions

$$\left.\begin{array}{l} \omega_1 , \underline{\phi}_1 \\ \omega_2 , \underline{\phi}_2 \\ \omega_3 , \underline{\phi}_3 \end{array}\right\} \text{eigenpairs}$$

In general we have n solutions

MIT OpenCourseWare
http://ocw.mit.edu

Resource: Finite Element Procedures for Solids and Structures
Klaus-Jürgen Bathe

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