PROFESSOR: Ladies and gentlemen, welcome to this set of lectures on the finite element method. In these lectures I would like to give you an introduction to the linear analysis of solids and structures. You are probably well aware that the finite element method is now widely used for analysis of structural engineering problems. The method is used in civil, aeronautical, mechanical, ocean, mining, nuclear, biomechanical, and other engineering disciplines.

Since the first applications two decades ago of the finite element method we now see applications in linear, nonlinear, static, and dynamic analysis. However, in this set of lectures, I would like to discuss with you only the linear, static, and dynamic analysis of problems. The finite element method is used today in various computer programs. And its use is very significant.

My objective in this set of lectures is to introduce to you the finite element methods or some of the finite element methods that are used for linear analysis of solids and structures. And here we understand linear to mean that we're talking about infinitesimally small displacements and that we are using a linear elastic material law. In other words, Hooke's law applies.

We will consider, in this set of lectures, the formulation of the finite element equilibrium equations, the calculation of finite element matrices of the matrices that arise in the equilibrium equations. We will be talking about the methods for solution of the governing equations in static and dynamic analysis. And we will talk about actual computer implementations. I will emphasize modern and effective techniques and their practical usage.

The emphasis, in this set of lectures, is given to physical explanations of the methods, techniques that we are using rather than mathematical derivations. The techniques that we will be discussing are those employed largely in the computer programs SAP and ADINA. SAP stands for Structural Analysis Program and you might very well be aware that there is a series of such programs, SAP I to SAP VI now.

And ADINA stands for Automatic Dynamic Incremental Nonlinear Analysis. However, this program is also very effectively employed for linear analysis. The nonlinear analysis being then a next step in the usage of the program. In fact, the elements in ADINA, the numerical methods that are used in ADINA, I consider to be the most effective, the most modern state of the art techniques that are currently available.

These few lectures really represent a very brief and compact introduction to the field of finite element analysis. We will go very rapidly through some or the basic concepts, practical applications, and so on. We shall follow quite closely, however, certain sections in my book entitled *Finite Element Procedures in Engineering Analysis* to be published by Prentice Hall. And I will be referring in the study guide of this set of lectures extensively to this book to the specific sections that we're considering in the lectures in this book.

The finite element solution process can be described as given on this viewgraph. You can see here that we talk about a physical problem. We want to analyze an actual physical problem. And our first step, of course, is to establish a finite element model of that physical problem.

Then, in the next step, we solve that model. And then we have to interpret the results. Because the interpretation of the results depends very much on how we established the finite element model, what kind of model we used, and so on. And in establishing the finite element model, we have to be aware of what kinds of elements, techniques, and so on are available to us.

Well, therefore, I will be talking, in the set of lectures, about these three steps basically here for different kinds of physical problems. Once we have interpreted the results we might go back from down here to there to revise or refine our model and go through this process again until we feel that our model has been an adequate one for the solution of the physical problem of interest.

Let me give you or show you some models that have been used in actual structural analysis. You might have seen similar models in textbooks, in publications already. This, for example, is a model that was used for the analysis of a cooling tower. The basic process of the finite element method is that we are taking the continuous system, and we are idealizing it as an assemblage of elements. I'm drawing here a typical three-noded triangular shell element that was used in the analysis of this cooling tower.

We talk about very many elements in order to obtain an accurate response prediction. And, of course, that means that we will be dealing with a large set of equations to be solved. And there's a significant computer effort required. I will be addressing all of these questions in these lectures.

Here you see the finite element model of a dam. The earth below the dam was idealized as an assemblage all such elements here, triangular elements now. And the dam itself was also idealized as an assemblage of elements. We will be talking about how such assemblages are best created, what kinds of elements to select, what assumptions are in the selection of these elements, and then how do we solve the resulting finite element equilibrium equations.

Here you see the finite element analysis, or the mesh that was used in the finite element analysis of a tire. This wall is half of the tire, as you can see, and this was the finite element mesh used. Again, we have to judiciously choose the kinds of finite elements to be employed. And we will be talking about that in this set of lectures.

Here you see the finite element model employed in the analysis of a spherical cover of a laser vacuum target chamber. This is the finite element mesh used. Again, specific elements were employed here. And we will be talking about the characteristics of these elements in this set of lectures.

Here you see the model of the shell structure subjected to a pinching load. There's a load up here and a load down there. These are the triangular elements that were used in the idealization of that shell and the resulting bending moments and displacements along the line DC are plotted here that have been predicted by the finite element analysis.

Finally here you see the finite element idealization of a wind tunnel that was used for the dynamic analysis of this tunnel. You can see a large number of shell elements were employed in the idealization of the shelf of the tunnel. Then, of course, supports were provided here for that shell. And this was a very large system that was solved. And the eigenvalues of this system were calculated using the subspace iteration method that we would be also talking about in this set of lectures.

Well, with this short introduction then, I would like to go now and discuss with you some basic concepts of engineering analysis. There's a lot of work ahead in this set of lectures. So let me take off my jacket with your permission, and let us just go right on with the actual discussion of the theory of the finite element method.

The basic concepts that I address here, in this first lecture, is summarized basically here once more. We are talking about the idealization of a system. We are talking about the formulation of the equilibrium equations, then the solution of the equations, and then, as I mentioned earlier already, the interpretation of the results. These are really the four steps that have to be performed in the analysis of an engineering system or of a physical system that we want to analyze.

Now when we talk about systems, we are really talking about discrete and continuous systems where, however, in reality, we recognize that all systems are really continuous. However, if the system consists of a set of springs, dashpots, beam elements, then we might refer to this continuous system as a discrete system because we can see already, it is obvious, so to say, how to idealize a system into a set of elements, discrete elements. In that case, the response is described by variables at a finite number of points. And this means that we have to set up a set of algebraic questions to solve that system.

So here I'm talking about elementary systems ofl springs, dashpots, discrete beam elements, and so on. In the analysis of a continuous system the response is described really by variables at an infinite number of points. And, in this case, we really come up with a differential equation, obviously a set of differential equations, that we have to solve.

The analysis of a complex continuous system requires a dissolution of the differential equations using numerical procedures. And this solution via numerical procedures-- and, of course, in this set of lectures we will be talking about the finite element method numerical procedures-- really reduces a continuous system to a discrete form. The powerful mechanism that we talk about here is the finite element method implemented on a digital computer.

The problem types that I will be talking about are steady-state problems, or static analysis, propagation problems, dynamic analysis, and eigenvalue problems. And these three types of problems, of course, arise for discrete and continuous systems.

Now let us talk first about the analysis of discrete systems in this first lecture. Because many of the characteristics that we are using in the analysis of discrete systems, discrete meaning, springs, dashpots, et cetera, we can directly see the discrete elements of the system. The steps involved in the analysis of such discrete systems are very similar to the analysis of complex finite element systems. The steps involved are the system idealization into elements. And that idealization is somewhat obvious because we have the discrete elements already. The evaluation of the element equilibrium requirements, the element assemblage, and the solution of the response.

Notice when we later on talk about the analysis of continuous systems instead of discrete systems, then the system idealization into finite elements here is not an obvious step and needs much attention. But these three steps here are the same in the finite element analysis of a continuous system. And I would like to now discuss all of these steps here just to show you, basically, some of the basic concepts that we're using in the finite element analysis.

Let us look at this discrete system here as an example. And let us display the basic ideas in the analysis of this discrete system. Here we have a set of rigid carts, three rigid carts, vertical carts that are supported on rollers down here. This means that each of these carts can just roll horizontally. The carts are connected via springs, k2, k3, k4, k5. And the first cart here is connected via k1 to a rigid support that does not move.

The displacement all of this cart here is u1. The load applied is R1. Notice that u1 is the displacement of each of these springs since this cart is rigid.

The displacement of this cart here is u2. And R2 is the load applied. The displacement of this cart is u3. And R3 is the load applied.

We now want to analyze this system when R1, R2, looking at it, we can directly see the elements of the system k1 to k5, and we can see directly, of course, how these elements are interconnected. The steps that we will be talking about in the analysis of this discrete system are really very similar to the steps that we're using in the finite element analysis of continuous systems.

What we will be doing is that we look at the equilibrium requirements for each spring as a first step. Then we look at the interconnection requirements between these springs that, in other words, the force on these springs here at this cart, and that spring, must be balanced by R1. And then, of course, we have a compatibility requirement that u1 is a displacement of each of these springs here.

So we are talking about the constitutive relations, the equilibrium requirements, and the compatibility requirements. These are, of course, the three requirements that we also have to satisfy in the analysis of a continuous system using, later on, finite element methods.

Notice that these springs here are our finite elements, if you want to think of it that way, a very simple set of elements. In a more complex analysis, these springs here would be plane stress elements, plane strain elements, three dimension elements, shell elements. And we will be talking about how we derive the characteristics of these elements.

And we will, however, interconnect these elements, these more complex elements, later in exactly the same way as we connect these simple elements. So the connections between the elements are established in the same way, and the solution of the equilibrium equations is also performed in the same way. But in this simple analysis, we are given directly the spring stiffnesses. And one other important point is that the spring stiffnesses here are exact stiffnesses.

In a finite element analysis of a continuous system, we have a choice on what kind of interpolations we can use for an element. We have a choice on what assumptions we want to lay down for an element. And then using different assumptions we are coming up with different stiffnesses of the element domain that we will be talking about.

And we will also find that the equilibrium in that element domain is not satisfied. It will only be satisfied in the limit as the elements become smaller, and smaller, and smaller. Whereas in the analysis of this discrete system, the equilibrium in each spring is always satisfied. So this is a very simple finite element analysis if you want to think of it that way.

The elements here than are k1. And notice that the equilibrium requirement for this element says simply that k1u1 is equal to the force applied to this node. It's a force, the external force, applied to this node. The equilibrium requirement of this element, k2, is written down here in matrix form. k2 is the physical stiffness of the spring. And F1, F2 are the forces applied at these two ends.

Notice, please, that the superscript here refers to the element number, superscript 1 here for element 1, superscript 2 here for element 2. And notice that we would find that F1(2) is minus F2(2) given u1 and u2. Of course, that means the element is in equilibrium.

Notice also, if you look at this matrix closer, that if u1 is greater than u2, then we would find that, in other words, u1 greater than u2 means that the spring is in compression. We would find that F1(2) is positive by simply multiplying this out here. And F2(2) is negative, which corresponds to the physical situation that we actually have. If u1 is greater than u2, this force here is positive, and that force is negative because the spring is compressed.

Well, similarly, we can write down the equilibrium requirement for the spring 3. And I've written down the matrix here. The only difference to the equilibrium requirements for spring 2 are that we're using now k3 here. And, of course, the superscript now is 3.

We can then proceed to write down the equilibrium equation for spring 4, which is the same form as before, now k4 here and the 4 superscript denoting element 4. And, finally for k5, we have k5 here and superscripts 5 here to denote element 5.

Now we should also point out one other important point. Namely, if we look at this cart systems here, notice that this k1 spring is connected to u1. It's connected to u1. k4 is connected to u1 and u3. So if we look at the equilibrium requirements here, you will notice that I have F1 here for k1 because this is u1 here. That is the global displacement u1. And looking now at k4, a more complicated case which is connected to u1 and u3, I have for that spring the u1 and u3 denoted here. And we have F1 and F3 here, F1 and F3. So these are the forces that go directly into the degrees of freedom 1 and 3 respectively, and similarly for the other springs.

Now if we want to assemble the global equilibrium equations for this structure with the unknowns u1, u2, u3, the loads R1, R2, and R3 are known, then we have to use now the equilibrium requirement at these degrees of freedom u1, u2, and u3, or rather at the cart 1, 2, and 3. And that equilibrium requirement then means that the sum of the forces acting onto the individual springs 1, 2, 3, and 4 at degree of freedom 1 must be equal to R1.

Now let us look once at this first equation back here again. Notice u1 couples into this spring 1, spring 2, spring 3, and spring 4. And that coupling is seen right here in spring 1, 2, 3, and 4. And summing all these forces that are acting individually onto the springs, the sum of these forces must be equal to the external load. That is the interconnection requirement between the springs.

The equilibrium requirements within the springs are expressed by these individual matrices here that we looked at already. These are the equilibrium requirements for the individual springs. Now I'm talking about the equilibrium requirement at the carts, the interconnection requirements between the springs.

Similarly, we can sum the forces that have to be equal to R2 and sum the forces that have to be equal to R3. And these three equations then set up in matrix form by substituting for F1(1), F1(2), and so on from the equilibrium requirements of the springs, we directly obtain this set off equations, KU equals R. Notice that K now is a 3 by 3 matrix. U is a 3 by 1 vector. R is a 3 by 1 vector. I denote matrices and vectors by bars under the symbols. As you can see here there are bars under these symbols.

Well, if we look at these equilibrium equations, we notice that our U vector, this vector U here contains u1, u2, and u3 as the unknowns. Notice this T here, this superscript T means transpose. The actual vector U actually looks this way u1, u2, u3. It lists the displacements vertically downwards. But it is easier to write it this way by transposing as a vector. So UT, capital T there, means transpose.

Similarly for R we have R1, R2, and R3 as the components. And the K matrix that we have obtained by substituting into these equations from the element equilibrium requirements, the K matrix is this one here. Now let us look a little closer at how do we construct this K matrix.

Well, we note that the total K matrix can be constructed by summing all of the individual element matrices from 1 to 5. And these individual element matrices are, for two extremes, written down here. K1 is a 3 by 3 matrix now. Not anymore the 1 by 1 or 2 by 2. It's a 3 by 3 matrix with just k1 in the 1,1 position. All the other elements are 0.

K2 is this matrix. So what I have done then is I have taken the 2 by 2 matrix which appeared in the element equilibrium requirement and has blown this matrix up filling zeroes for the third degree of freedom. Similarly we would obtain K3 and so on. The zeroes always appear in those rows and columns into which the element does not couple, in other words, into those degrees of freedom that the element does not couple.

For example, k1, this element 1 here, couples only into the degree of freedom 1. So, therefore, we have the second and third rows be 0. Element 2 couples only into degree of freedom 1 and 2. Therefore, the third degree of freedom contains all zeroes, and so on.

This assemblage process is called the direct stiffness method, an extremely important concept that is very well implemented in a computer program. It represents the basis of the implementation of the finite element method in almost every code that is currently in use. The direct stiffness method has also a very nice physical explanation. And this is what I really want to talk to you about now for the next five minutes.

The steady-state analysis, of course, then is completed. The steady-state analysis of this system, of course, is completed by solving this system of equations here, equations a. Once we know U we can go back to the elements and calculate the forces in the elements themselves by going to the element equilibrium requirements.

Well let us look then at what we are doing when we perform this process here by summing, in other words, the K element, the stiffnesses of the elements into a global stiffness matrix. And let us look at what we're doing physically. Because that really, of course, is the direct stiffness method that we are using here. And it is, I think, very nice if you can clearly see what is happening in that method.

Well, the basic process is the following. Here I have drawn the carts without any strings. Of course, our degrees of freedom are here, u1, u2, and u3. And the loads are R1, R2, R3. I don't need to put them in again. This system here corresponds to a K matrix with zeroes everywhere. Blanks in these positions here denote zeroes. So this is a system that we're starting off with in this direct stiffness method, a system without any elements, a matrix without any elements also.

The process, then, is the following. We are using this cart system. And we're adding one spring on. That is the first edition. That is spring k1.

Mathematically this means that we're going through the following process. We are taking our K matrix with blanks everywhere, and we're adding into it this one element, k1. Now this is a K matrix, this stiffness matrix governing-- and this is very important-- governing this system. Once again, this is a K matrix governing this system. Of course, this is not a stable system yet because there are no connections between these carts here.

Well, with a second edition we're adding in the second spring. And that means we are putting this spring there. That is k2. Well in our matrix formulation then, what that means in our direct stiffness method is that we are going from this system over on this K matrix to that K matrix.

We are adding this second spring stiffness into the K matrix. So this is the stiffness matrix that governs the equilibrium of this physical system. Notice that this spring here, the second spring, couples into u1 and u2. And therefore we have added these blue elements corresponding to the second spring into degrees of freedom 1 and 2.

Next in the direct stiffness method we're adding the next spring element, and that is spring element number 3. Again, it couples into u1 and u2. And the stiffness matrix that we are now talking about is the following. We're going from this stiffness matrix to that stiffness matrix here, adding the green k3 in there.

Now next we go from this system to add into the system the spring 4, spring 4 now. Please notice that this is now a stable system. It is a stable system because if I want to put u3 over here, then I have to do work on this spring. So this is now a stable system.

In our mathematical formulation, or in our direct stiffness method rather, what this then corresponds to is that we are going from this system here, or this stiffness matrix, to that stiffness matrix. Notice we have added a k4 into here. And that, in fact, allows us now to solve at this level. We could solve the equations KU equals R. Of course, this now is a stiffness matrix corresponding to this system.

We have not quite yet reached the system that we want to analyze. But we reach it by adding the final spring in k5. k5 now is here. And that corresponds in our direct stiffness method to adding this spring in there. These elements here, k5.

Notice that this spring now here couples into degrees of freedom 4 and five 5, and that's why it appears in quadrant column 4 and 5 here. And this spring. I should've have said, this spring here couples into column 2 and 3, column 2 and 3 meaning u2 and u3. And here we see, of course, that the spring indeed goes into degree of freedom u2 and u3, into degree of freedom u2 and u3.

So this then is to final system that we want to analyze, and this is the final stiffness matrix that we had to obtain. Notice, once again, this matrix has been obtained by taking the sum over all the element stiffness matrices. We are summing from i equals 1 to 5.

And this mathematical process, once again, which we call the direct stiffness method has a physical analog. You can understand it physically in the way I've shown here. Namely you're starting off with a blank K matrix, no elements in it at all, and you simply add one element after the other into that K matrix filling up the K matrix that way. And the additions are carried out-- this is important-- by taking the element matrices and adding them into the appropriate columns and rows of the K matrix.

For example, this element here couples into degree of freedom 1 and 3. And if we go once more back to the process that we have been carrying out here, notice our k4 here corresponds to degree of freedom 1 and degree of freedom 3, the first row and column and third row and column. That's where these elements appear. So there's a neat physical explanation for the direct stiffness method which I wanted to discuss with you here.

Now as another approach, instead of using the direct formulation of the equations KU equals R, the equilibrium equations of the system, we can also use a variational approach. We will be talking about that variational approach in the second lecture. And I would like to discuss it, or introduce it to you, now very briefly for the analysis of this discrete system that we just looked at.

The basic process here is that we are constructing a functional pi which is equal to u minus w where u is the strain energy of the system and w is the total potential of the loads. The equilibrium equations that we just looked at, KU equals R in other words, are obtained by invoking that del-pi shall be 0, the stationality condition on pi. And this means that del-pi, del-ui, shall be 0 for all ui. This then gives three equations, And these three questions are obtained as follows.

If we use u, the strain energy of the system is given right here, 1/2 U transpose KU. If you were to multiply this out substituting for U and for K with the values that I've given to you, you would find that this indeed is the strain energy in the system. The potential of the total loads is given by U transposed R. Notice please that there is no 1/2 here in front. Simply U transpose R is the potential of the loads. Now if we invoke this condition that del-pi, del-ui shall be 0, we directly obtain KU equals R.

Now there's one important point. To obtain u and w, u and w here, we again, can add up the contributions from all the elements using the direct stiffness method. In other words, this K here can be constructed as we have shown by summing over the elements, by summing the contributions over all of the elements. And since this is true, we can also write this total u as being the sum of the ui's, if you want to, the strain energies of all of the individual elements.

So here too we could use the direct stiffness method. Of course, in actuality, in actual practical analysis, we never form this u, we never form that w when we want to calculate KU equals R. This is simply a theoretical concept that I wanted to introduce to you, a theoretical concept that we will be using later on in the construction of KU equals R. We never really calculate these measures if we only want to calculate KU equals R.

It might be of interest to us to calculate this in order to find out how much strain energy is put into individual elements in finite element analysis. But this is only done if you want to evaluate error bounds on the finite element solution and so on. If we only want the calculate KU equals R and obtain the use in other words, to be able to predict the displacements and the stresses in the elements, then we would not calculate these two quantities.

Now this then were the essence of the analysis of a steady-state problem for discrete systems. I pointed out already that if we have an extra finite element system there, of course, many additional concepts that we have to talk about, a selection of elements, the kinds of interpolations to be used, and, of course, we then have to also talk about how do we solve these equations, and so on. We will address these questions in the later lectures.

However, another class of problems that we will be talking about are propagation problems. The main characteristics of propagation problems are that the response changes with time. Therefore, we need to include the d'Alembert forces. Now basically what we are saying then is that we're looking at static equilibrium as a function of time but also taking into account the d'Alembert forces. And that, together then, makes it a dynamic problem.

Of course, if the displacement varies very slow, in other words, the load varies very slow, then the inertia forces can be neglected, and we would simply have this set of equations where R of t is a function of time and U of t would be a function of time. However, when R of t acts rapidly or suddenly inertia conditions are applied to the system, then the inertia forces can be very important. We have to include their effect. And then we have a true propagation problem, a truly dynamic problem that has to be solved.

For our example, the M matrix here would be this 3 by 3 matrix where m1 is simply the mass of the cart 1, m2 is the mass of the cart 2, m3 is the mass of the cart 3. Of course these masses would have to be given. And notice that we would evaluate them by basically saying that this total mass here can be evaluated by taking the mass per unit volume times the volume. And that would be the mass that we're talking about when we accelerate that cart into this direction.

So these masses here are very simply evaluated. When we talk later on about actually finite elements, we will be talking about similar mass matrices where we simply take the total volume of an element and lump that volume of the element to its nodes. We will also talk about consistent mass matrices where this mass matrix is a little bit more complicated. In other words, some of these off-diagonal elements are not 0.

Finally, we will also talk about eigenvalue problems. In the solution of eigenvalue problems, we will be talking about generalized eigenvalue problems, in particular, which are Av equals lambda Bv, which can be written down in this form where A and B are symmetric matrices of order n, v is a vector of order n, and lambda is a scalar.

As an example, for example here in dynamic analysis, what we will see there is K phi equals omega squared M phi where K is the stiffness matrix that I talked about already. This which would be for the cart system here simply as 3 by 3, this 3 by 3 stiffness matrix that I introduced to you. And it's a mass matrix that we just had here on this viewgraph. That is the mass matrix. And phi is the vector.

If we find a solution, in other words, if this equation is satisfied, we put an i on there and satisfy for phi i and omega i squared. Omega i squared will be a frequency. I will be discussing it just now a little more. And then we're talking about an eigenpair. But notice that is a typical problem that we will be discussing which arises, in other words, in dynamic analysis.

Notice also that what we're really saying here is that the right-hand side is a load vector. And if we know v, if we know lambda, then we know the load vector. What we would calculate then is the same v that we have substituted here. In other words, if we consider this to be a set of loads where v is now known, lambda is known, then we could evaluate R. In solving Av equals R, we would get back our v that we substituted into here. And that is the main characteristic of an eigenvalue problem.

Well, they arise in dynamic and buckling analysis, and let us look at one example where we actually obtain this eigenvalue problem. And the example is simply the system of rigid carts that we considered already earlier. We obtain the eigenvalue problem by looking at the equilibrium equations when no loads are applied. And we call these the free vibration conditions, free because there are no loads applied, free of loads.

If we let U be equal to phi times sine omega t minus tau where the time dependency now in the response is in this function here, in the sine function only, and if we take the second derivative of U, meaning that we get a cosine and then a minus sign in here, and, of course, this omega twice outside, so we have a sign change here. We have a minus omega squared M phi sine omega t minus tau for this part here. And for this part KU we obtain K phi sine omega t minus tau by simply substituting from here into there. And, of course, the sum of these two must be equal to 0.

Now this equation must hold for any time, t. So we can simply cancel out this part and that part. And the resulting set of equations that we are obtaining then are given on the last viewgraph, namely those equations being K phi equals omega squared M phi.

So that is the generalized eigenvalue problem which we obtain in dynamic analysis. We will be later on talking about how we solve this generalized eigenvalue problem for the eigenvalues and eigenvectors.

In the case of of the 3 by 3 system that we are considering here, in other words, the analysis of the cart system, we only have three solutions, omega 1 phi 1, omega2 phi2, omega3 phi3. And we call each of the solutions an eigenpair. So there are three eigenpairs that satisfy this particular equation.

Notice that this is, in other words, the equation that I talked about here earlier. And the eigenpairs, phi i, omega i squared are the solutions to this equation. We are really interested in omega i because that is the frequency in radians per second, and the eigenvalue, however, being omega squared.

In general when we have an n by n system-- and I have already written down here the n by n, let me put it bigger once more here-- and we have a general n by n system, in other words, and not being equal to 3 just as we have in our cart system, then we have n solutions. And, however, we will find that in finite element analysis we do not necessarily need to calculate all n solutions.

In fact, when we consider large eigensystems where n is equal to 1,000 or even more, then certainly we do not want to calculate all eigenvalues. It would be exorbitantly expensive, much too expensive to calculate all of the eigenvalues and eigenvectors. We don't need to have them all in analysis.

And, therefore, we will talk about eigenvalue solution methods that only calculate the eigenvalues and eigenvectors that we are actually interested in. We also, of course, have to, before we actually get to that topic which is the topic of the last lecture, we will talk about how we actually construct these K matrices, how we calculate them, construct them for different finite element systems.

Well, this then does complete what I wanted to say in this lecture. Thank you very much for your attention.