NICHOLAS BURNAND: Hello and welcome to this short video on Mohr's circle. The goal of this video is to show you how Mohr's circle works, how they could be useful for you, and to make some interesting observations as well.

So first of all, Mohr's circle is a graphical way to find the principle normal stresses and the maximal shear stress of a given stress state, which could be the one right here, where you have some normal forces acting on sigma x, y, and z, and also some different shear stresses-- the tau's you have here. Your stress state could also be shown using a tensor-- stress tensor-- which is shown right here by this symmetric 3 by 3 matrix. It has to be symmetric because we don't want to have any movement acting on our stress element to prevent any rotation. And also of course, we're using an isotropic material.

So I'm going to start with the 2D Mohr's circle-- the simplest one. So now obviously, my stress element is a square. We have sigma x and sigma y as normal forces acting on it. And the shear stress tau x,y.

So now, let's move to the Mohr's circle itself and I'll explain to you how to draw it. So first of all, I have to put my two stresses on my graph right here. I have normal axis right here and a shear axis right here. So I'm going to take sigma x, which is [INAUDIBLE] here, and tau x,y, which is going to give me my first point right here. And then sigma y and tau y,x-- which is the opposite of tau x,y-- and this is going to give me my second point right here.

Now, I'm going to say that this is the diameter of my Mohr's circle. And this allows me to draw the outermost circle right here, as you can see. Now, I see that this Mohr's circle crosses the normal axis on two points-- the two endpoints that you see right here.

And B, these are my principal normal stresses acting on my stress element. And I can actually also get on the Mohr's circle, the angle how much I have to turn my stress element to get this principal normal stresses. And it's important to remember that the angle I see on the Mohr's circle-- so this angle here-- is actually twice the real angle I have to turn my stress element by, which is shown right here.

And then I have a blue point here, which is my maximal shear stress. So its maximal stresses, as you can see, is the radius of my Mohr's circle. And again, I have an angle of maximal shear stress, which is displayed right here. And again, the angle of my Mohr's circle is twice the real angle.

Now, some interesting observations we can do right here is that even if I don't apply any shear stress to my stress element, I will still have some maximal shear stress. And as you can see, the angle of the maximal shear stress is, well, 90 degree on my Mohr's circle, but in reality, 45 degrees. And as you probably already know, if I'm doing for example a tensile stress, then my maximum shear stress is going to be on a 45 degrees plane.

Now, the only way to avoid having any shear stresses in my stress element is actually to have sigma x being equal to sigma y. And that's the only way I can avoid any shear stresses. Now, the other way to get my principal stresses would be to get the eigenvalues of my two by two stress tensor. And that's what we're going to see later for the 3D case.

So in 3D now, what I have to do is get this principal normal stresses by, as I've told you, find the eigenvalues of my stress tensor, which is right here. That's the random one I've chosen. It's symmetric, as I've explained to you in the beginning. And finding the eigenvalues gives me my actual principal normal stresses sigma 1, sigma 2, and sigma 3.

So now, let's move to the 3D Mohr's circles. So it's the same principle, but now I have three principal stresses instead of two. So I also have three Mohr's circle instead of one because I have one Mohr's circle between sigma 1 and sigma 2, another Mohr's circle between sigma 2 and sigma 3, and then bigger one between sigma 1 and sigma 3.

So now, the orange zone right here-- same at the bottom-- is the allowed pairs of stresses sigma n-- the principal normal force-- and tau, which is my shear force. And the only [INAUDIBLE] pairs of normal and shear stresses are in this two orange zones because it has to be inside the bigger circle and outside of the two smaller circles.

Now, the observation we can do here is that first of all, the maximal shear stress only depends on the difference between the smallest and the largest principal stresses. So as you can see, if I move my sigma 2, which is the middle one, well, my maximal shear stress doesn't change at all.

And the other observation we can make is that if I equal sigma 3 to 0, as it is right now, and this would be the same actually-- or almost the same if we look in 2D, as the example I've shown you before. But actually, the fact that I'm now in 3D and that even if I don't apply a force on the z-axis-- any normal force-- well, this still has an influence on the maximal shear stress because in 2D, my maximal shear stress would lie right here at the top of the first gray Mohr's circle. But now, because I am in 3D, my maximal shear stress is actually a bit higher.

And now, actually another way to get this principal normal stresses using Mohr's circles-- so instead of as I've done it here, get the eigenvalues of my stress tensor, I can actually use projections. So I have my 3D stress tensor here. And what I'm going to do is project it all along my x,y, and z-axis. So that's for example, projection on the z-axis. And what I get is a 2D stress tensor.

So doing this, what I get again of course is my 2D Mohr's circle, as we've seen it in the beginning of this video. And that's for example, the projection along the x-axis. And I again, get my principal normal stresses, two endpoints, and my maximal shear stress blue point right here.

And what's more interesting is to see for example, the angle of principal stresses, which is minus 19 right here and to half the angle between the red line here and the normal axis. And doing three different projections along the x,y, and z-axis, i get three different angles. So here, that's another one and that's the third one.

And now if I actually apply this angle, so if I actually rotate my stress element by minus 19.9539 degrees around the x-axis and do the same around the y-axis and the same again around the z-axis, then I actually get my new set of coordinates where I get my principal normal stresses acting on my stress element. So I would get again, these three values up here. Just by applying three rotation, I've found by projecting my stress state on my three different axis.

So that's it. I hope you enjoyed the video and actually learned something on Mohr's circles. Thanks for watching.