

MIT OpenCourseWare  
<http://ocw.mit.edu>

*Continuum Electromechanics*

For any use or distribution of this textbook, please cite as follows:

Melcher, James R. *Continuum Electromechanics*. Cambridge, MA: MIT Press, 1981. Copyright Massachusetts Institute of Technology. ISBN: 9780262131650. Also available online from MIT OpenCourseWare at <http://ocw.mit.edu> (accessed MM DD, YYYY) under Creative Commons license Attribution-NonCommercial-Share Alike.

For more information about citing these materials or our Terms of Use, visit:  
<http://ocw.mit.edu/terms>.



To Janet Damman Melcher

# **Continuum Electromechanics**

---

**James R. Melcher**

Copyright ©1981 by

The Massachusetts Institute of Technology

No part of this book may be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information storage and retrieval system, without permission in writing from the publisher.

Printed and bound in the United States of America

Library of Congress Cataloging in Publication Data

Melcher, James R.

Continuum electromechanics.

Includes index.

1. Electric engineering. 2. Electrodynamics.  
3. Continuum mechanics. 4. Electromagnetic fields.

I. Title.

TK145.M616 621.3 81-1578

ISBN 0-262-13165-X AACR2

# Preface

The three stages in which this text came into being give some insight as to how the material has matured. As "notes" written in the early 1960's, it was intended to serve as an introduction to the subject of electrohydrodynamics. Thus, it reflected the author's early research interests. During this period, the author had the privilege of collaborating with Herbert H. Woodson (now University of Texas) on the development of an undergraduate subject, "Fields, Forces and Motion". That effort resulted in the text Electromechanical Dynamics (Wiley, 1968). There has also been a strong influence from Hermann A. Haus, with whom the author has collaborated for a number of years in the development and teaching of an undergraduate electromagnetic field theory subject. Both Woodson, with his interests in rotating machinery and magnetohydrodynamics, and Haus, who then worked in areas ranging from electron beam engineering and plasmas to the electrodynamics of continuous media, stimulated the notion that there was a set of fundamental ideas that permeated many different "specialty areas". To be taught were widely applicable basic laws, approaches to modeling and mathematical techniques for disclosing what the models had to say.

The text took its second form in 1972-1973, when the objective was to achieve this broader and more enduring aspect of the material. Much of the writing was done while the author was on a Guggenheim Fellowship and a Fellow of Churchill College, Cambridge University, England. During that year, as a guest of George Batchelor's Department of Applied Mathematics and Theoretical Physics, and with the privilege of working with Sir Geoffrey Taylor, there was the opportunity to further broaden the perspective. Here, the influences were toward the disciplines of continuum mechanics.

Unfortunately, the manuscript resulting from this second writing was more in the nature of two books than one. More integration and culling of material was required if the self-imposed objective was to be achieved of helping to define a discipline rather than simply covering a number of interrelated topics.

The third version, this text, would probably not have come into being had it not been for the active encouragement of Aina Sils. Her editorial help and typewriter artistry provided teaching material that was immediately sufficiently attractive to serve as an incentive to commit nights and weekends to yet another rewrite.

As a close colleague who has been instrumental in establishing as an area the continuum electromechanics of biological systems, Alan J. Grodzinsky has been both a source of technical insight and an inspiration to complete the publication of material that for so many years had been referenced in theses as "notes."

Research carried out by still other colleagues at MIT will be seen to have influenced the scope and content. The Electric Power Systems Engineering Laboratory, directed by Gerald L. Wilson, is an example with its activities in superconducting machinery (James L. Kirtley, Jr.) and its model power system (Steven D. Umans). Others are the High Voltage Laboratory (John G. Trump and Chathan M. Cooke), the National Magnet Laboratory (Ronald R. Parker and Richard D. Thornton), the Research Laboratory of Electronics (Paul Penfield, Jr. and David H. Staelin), the Materials Processing Center (Merton C. Flemings), the Energy Laboratory (Janos M. Beer and Jean F. Louis), the Polymer Processing Program (Nam P. Suh), and the Laboratory for Insulation Research, (Arthur R. Von Hippel and William B. Westphal).

A great satisfaction and motivation has come from seeing the ideas promulgated here serve the needs of industry. The author's consulting activities, for more than 30 different companies, provided many useful examples. In the face of an increasing awareness of the importance of energy to our societal institutions and our way of life, it has been satisfying to see the concepts presented here applied not only to the development of new energy systems, but to the conflicting problem of environmental control as well.

Where possible, examples have intentionally been chosen that can be illustrated with generally available films. Referenced in Appendix C, these are in two series. The series from the National Committee on Fluid Mechanics Films was being developed at the Education Development Center while the author was active in making three films in the series from the National Committee on Electrical Engineering Films. Interaction with such individuals as Ascher H. Shapiro and J. A. Shercliff fostered an interest in using films to enliven and undergird classroom education.

While graduate students involved with the subject or carrying out their PhD theses, a number of people have made substantial contributions. Some of these are James F. Hoburg (Secs. 8.17 and 8.18), Jose Ignacio Perez Arriaga (Secs. 4.5 and 4.8), Peter W. Dietz (Sec. 5.17), Richard S. Withers (Secs. 5.8 and 5.9), Kent R. Davey (Sec. 8.5), and Richard M. Ehrlich (Sec. 5.9).

Problems at the ends of chapters were typed by Eleanor J. Nicholson. Figures were drawn by the author.

Solutions to the problems have been prepared in the form of a manual. Intended as an aid to those either presenting this material in the classroom or using it for self-study, this manual is available for the cost of reproduction from the author. Requests should be over the signature of either a member of a university faculty or the industrial equivalent.

James R. Melcher

Cambridge, Massachusetts  
January, 1981

# Contents

1. INTRODUCTION TO CONTINUUM ELECTROMECHANICS
  - 1.1 Background 1.1
  - 1.2 Applications 1.2
  - 1.3 Energy Conversion Processes 1.4
  - 1.4 Dynamical Processes and Characteristic Times 1.4
  - 1.5 Models and Approximations 1.4
  - 1.6 Transfer Relations and Continuum Dynamics of Linear Systems 1.6
2. ELECTRODYNAMIC LAWS, APPROXIMATIONS AND RELATIONS
  - 2.1 Definitions 2.1
  - 2.2 Differential Laws of Electrodynamics 2.1
  - 2.3 Quasistatic Laws and the Time-Rate Expansion 2.2
  - 2.4 Continuum Coordinates and the Convective Derivative 2.6
  - 2.5 Transformations between Inertial Frames 2.7
  - 2.6 Integral Theorems 2.9
  - 2.7 Quasistatic Integral Laws 2.10
  - 2.8 Polarization of Moving Media 2.11
  - 2.9 Magnetization of Moving Media 2.13
  - 2.10 Jump Conditions 2.14
    - Electroquasistatic Jump Conditions 2.14
    - Magnetoquasistatic Jump Conditions 2.18
    - Summary of Electroquasistatic and Magnetoquasistatic Jump Conditions 2.19
  - 2.11 Lumped Parameter Electroquasistatic Elements 2.19
  - 2.12 Lumped Parameter Magnetoquasistatic Elements 2.20
  - 2.13 Conservation of Electroquasistatic Energy 2.22
    - Thermodynamics 2.22
    - Power Flow 2.24
  - 2.14 Conservation of Magnetoquasistatic Energy 2.26
    - Thermodynamics 2.26
    - Power Flow 2.28
  - 2.15 Complex Amplitudes; Fourier Amplitudes and Fourier Transforms 2.29
    - Complex Amplitudes 2.29
    - Fourier Amplitudes and Transforms 2.30
    - Averages of Periodic Functions 2.31
  - 2.16 Flux-Potential Transfer Relations for Laplacian Fields 2.32
    - Electric Fields 2.32
    - Magnetic Fields 2.32
    - Planar Layer 2.32
    - Cylindrical Annulus 2.34
    - Spherical Shell 2.38
  - 2.17 Energy Conservation and Quasistatic Transfer Relations 2.40
  - 2.18 Solenoidal Fields, Vector Potential and Stream Function 2.42
  - 2.19 Vector Potential Transfer Relations for Certain Laplacian Fields 2.42
    - Cartesian Coordinates 2.45
    - Polar Coordinates 2.45
    - Axisymmetric Cylindrical Coordinates 2.45
  - 2.20 Methodology 2.46
    - PROBLEMS 2.47
3. ELECTROMAGNETIC FORCES, FORCE DENSITIES AND STRESS TENSORS
  - 3.1 Macroscopic versus Microscopic Forces 3.1
  - 3.2 The Lorentz Force Density 3.1
  - 3.3 Conduction 3.2
  - 3.4 Quasistatic Force Density 3.4
  - 3.5 Thermodynamics of Discrete Electromechanical Coupling 3.4
    - Electroquasistatic Coupling 3.4
    - Magnetoquasistatic Coupling 3.6
  - 3.6 Polarization and Magnetization Force Densities on Tenuous Dipoles 3.6

- 3.7 Electric Korteweg-Helmholtz Force Density 3.9
  - Incompressible Media 3.11
  - Incompressible and Electrically Linear 3.12
  - Electrically Linear with Polarization Dependent on Mass Density Alone 3.12
  - Relation to the Kelvin Force Density 3.12
- 3.8 Magnetic Korteweg-Helmholtz Force Density 3.13
  - Incompressible Media 3.15
  - Incompressible and Electrically Linear 3.15
  - Electrically Linear with Magnetization Dependent on Mass Density Alone 3.15
  - Relation to Kelvin Force Density 3.15
- 3.9 Stress Tensors 3.15
- 3.10 Electromechanical Stress Tensors 3.17
- 3.11 Surface Force Density 3.19
- 3.12 Observations 3.21
- PROBLEMS 3.23
- 4. ELECTROMECHANICAL KINEMATICS: ENERGY-CONVERSION MODELS AND PROCESSES
  - 4.1 Objectives 4.1
  - 4.2 Stress, Force, and Torque in Periodic Systems 4.1
  - 4.3 Classification of Devices and Interactions 4.2
    - Synchronous Interactions 4.4
    - D-C Interactions
    - Synchronous Interactions with Instantaneously Induced Sources 4.5
  - 4.4 Surface-Coupled Systems: A Permanent Polarization Synchronous Machine 4.8
    - Boundary Conditions 4.8
    - Bulk Relations 4.10
    - Torque as a Function of Voltage and Rotor Angle ( $v, \theta_r$ ) 4.10
    - Electrical Terminal Relations 4.11
  - 4.5 Constrained-Charge Transfer Relations 4.13
    - Particular Solutions (Cartesian Coordinates) 4.14
    - Cylindrical Annulus 4.15
    - Orthogonality of  $\Pi_i$ 's and Evaluation of Source Distributions 4.16
  - 4.6 Kinematics of Traveling-Wave Charged-Particle Devices 4.17
    - Single-Region Model 4.19
    - Two-Region Model 4.20
  - 4.7 Smooth Air-Gap Synchronous Machine Model 4.21
    - Boundary Conditions 4.23
    - Bulk Relations 4.23
    - Torque as a Function of Terminal Currents and Rotor Angle 4.23
    - Electrical Terminal Relations 4.25
    - Energy Conservation 4.25
  - 4.8 Constrained-Current Magnetoquasistatic Transfer Relations 4.26
  - 4.9 Exposed Winding Synchronous Machine Model 4.28
    - Boundary Conditions 4.30
    - Bulk Relations 4.30
    - Torque as a Function of Terminal Variables 4.30
    - Electrical Terminal Relations 4.31
  - 4.10 D-C Magnetic Machines 4.33
    - Mechanical Equations 4.36
    - Electrical Equations 4.37
    - The Energy Conversion Process 4.39
  - 4.11 Green's Function Representations 4.40
  - 4.12 Quasi-One-Dimensional Models and the Space-Rate Expansion 4.41
  - 4.13 Variable-Capacitance Machines 4.44
    - Synchronous Condition 4.46

- 4.14 Van de Graaff Machine 4.49
  - Quasi-One-Dimensional Fields 4.49
  - Quasistatics 4.51
  - Electrical Terminal Relations 4.52
  - Mechanical Terminal Relations 4.52
  - Analogy to the Magnetic Machine 4.52
  - The Energy Conversion Process 4.53
- 4.15 Overview of Electromechanical Energy Conversion Limitations 4.53
  - Synchronous Alternator 4.54
  - Superconducting Rotating Machine 4.54
  - Variable-Capacitance Machine 4.55
  - Electron-Beam Energy Converters 4.56
- PROBLEMS 4.57

## 5. CHARGE MIGRATION, CONVECTION AND RELAXATION

- 5.1 Introduction 5.1
- 5.2 Charge Conservation with Material Convection 5.2
- 5.3 Migration in Imposed Fields and Flows 5.5
  - Steady Migration with Convection 5.6
  - Quasistationary Migration with Convection 5.7
- 5.4 Ion Drag Anemometer 5.7
- 5.5 Impact Charging of Macroscopic Particles: The Whipple and Chalmers Model 5.9
  - Regimes (f) and (i) for Positive Ions; (d) and (g) for Negative Ions 5.14
  - Regimes (d) and (g) for Positive Ions; (f) and (i) for Negative Ions 5.14
  - Regimes (j) and (k) for Positive Ions; (b) and (c) for Negative Ions 5.15
  - Regime (l) for Positive Ions; (a) for Negative Ions 5.15
  - Regime (e), Positive Ions; Regime (h), Negative Ions 5.15
  - Regime (h) for Positive Ions; (e) for Negative Ions 5.15
  - Positive and Negative Particles Simultaneously 5.16
  - Drop Charging Transient 5.16
- 5.6 Unipolar Space Charge Dynamics: Self-Precipitation 5.17
  - General Properties 5.18
  - A Space-Charge Transient 5.19
  - Steady-State Space-Charge Precipitator 5.20
- 5.7 Collinear Unipolar Conduction and Convection: Steady D-C Interactions 5.22
  - The Generator Interaction 5.24
  - The Pump Interaction 5.24
- 5.8 Bipolar Migration with Space Charge 5.26
  - Positive and Negative Ions in a Gas 5.26
  - Aerosol Particles 5.27
  - Intrinsically Ionized Liquid 5.27
  - Partially Dissociated Salt in Solvent 5.27
  - Summary of Governing Laws 5.27
  - Characteristic Equations 5.28
  - One-Dimensional Characteristic Equations 5.28
  - Numerical Solution 5.29
  - Numerical Example 5.30
- 5.9 Conductivity and Net Charge Evolution with Generation and Recombination: Ohmic Limit 5.33
  - Maxwell's Capacitor 5.35
  - Numerical Example 5.36

### DYNAMICS OF OHMIC CONDUCTORS

- 5.10 Charge Relaxation in Deforming Ohmic Conductors 5.38
  - Region of Uniform Properties 5.39
  - Initial Value Problem 5.39
  - Injection from a Boundary 5.40
- 5.11 Ohmic Conduction and Convection in Steady State: D-C Interactions 5.42
  - The Generator Interaction 5.43
  - The Pump Interaction 5.43

- 5.12 Transfer Relations and Boundary Conditions for Uniform Ohmic Layers 5.44
  - Transport Relations 5.44
  - Conservation of Charge Boundary Condition 5.44
- 5.13 Electroquasistatic Induction Motor and Tachometer 5.45
  - Induction Motor 5.46
  - Electroquasistatic Tachometer 5.46
- 5.14 An Electroquasistatic Induction Motor; Von Quincke's Rotor 5.49
- 5.15 Temporal Modes of Charge Relaxation 5.54
  - Temporal Transients Initiated from State of Spatial Periodicity 5.54
  - Transient Charge Relaxation on a Thin Sheet 5.55
  - Heterogeneous Systems of Uniform Conductors 5.56
- 5.16 Time Average of Total Forces and Torques in the Sinusoidal Steady State 5.60
  - Fourier Series Complex Amplitudes 5.60
  - Fourier Transform Complex Amplitudes 5.60
- 5.17 Spatial Modes and Transients in the Sinusoidal Steady State 5.61
  - Spatial Modes for a Moving Thin Sheet 5.62
  - Spatial Transient on Moving Thin Sheet 5.66
  - Time-Average Force 5.67
- PROBLEMS 5.71
- 6. MAGNETIC DIFFUSION AND INDUCTION INTERACTIONS
  - 6.1 Introduction 6.1
  - 6.2 Magnetic Diffusion in Moving Media 6.1
  - 6.3 Boundary Conditions for Thin Sheets and Shells 6.4
    - Translating Planar Sheet 6.5
  - 6.4 Magnetic Induction Motors and a Tachometer 6.6
    - Two-Phase Stator Currents 6.6
    - Fields 6.7
    - Time-Average Force 6.8
    - Balanced Two-Phase Fields and Time-Average Force 6.8
    - Electrical Terminal Relations 6.8
    - Balanced Two-Phase Equivalent Circuit 6.10
    - Single-Phase Machine 6.10
    - Tachometer 6.11
  - 6.5 Diffusion Transfer Relations for Materials in Uniform Translation or Rotation 6.11
    - Planar Layer in Translation 6.13
    - Rotating Cylinder 6.13
    - Axisymmetric Translating Cylinder 6.14
  - 6.6 Induction Motor with Deep Conductor: A Magnetic Diffusion Study 6.15
    - Time-Average Force 6.15
    - Thin-Sheet Limit 6.17
    - Conceptualization of Diffusing Fields 6.17
  - 6.7 Electrical Dissipation 6.19
  - 6.8 Skin-Effect Fields, Relations, Stress and Dissipation 6.20
    - Transfer Relations 6.21
    - Stress 6.21
    - Dissipation 6.22
  - 6.9 Magnetic Boundary Layers 6.22
    - Similarity Solution 6.24
    - Normal Flux Density 6.25
    - Force 6.26
  - 6.10 Temporal Modes of Magnetic Diffusion 6.26
    - Thin-Sheet Model 6.26
    - Modes in a Conductor of Finite Thickness 6.27
    - Orthogonality of Modes 6.29

- 6.11 Magnetization Hysteresis Coupling: Hysteresis Motors 6.30
  - PROBLEMS 6.35
- 7. LAWS, APPROXIMATIONS AND RELATIONS OF FLUID MECHANICS
  - 7.1 Introduction 7.1
  - 7.2 Conservation of Mass 7.1
    - Incompressibility 7.1
  - 7.3 Conservation of Momentum 7.2
  - 7.4 Equations of Motion for an Inviscid Fluid 7.2
  - 7.5 Eulerian Description of the Fluid Interface 7.3
  - 7.6 Surface Tension Surface Force Density 7.4
    - Energy Constitutive Law for a Clean Interface 7.4
    - Surface Energy Conservation 7.5
    - Surface Force Density Related to Interfacial Curvature 7.5
    - Surface Force Density Related to Interfacial Deformation 7.6
  - 7.7 Boundary and Jump Conditions 7.8
  - 7.8 Bernoulli's Equation and Irrotational Flow of Homogeneous Inviscid Fluids 7.9
    - A Capillary Static Equilibrium 7.10
  - 7.9 Pressure-Velocity Relations for Inviscid, Incompressible Fluid 7.11
    - Streaming Planar Layer 7.11
    - Streaming Cylindrical Annulus 7.13
    - Static Spherical Shell 7.13
  - 7.10 Weak Compressibility 7.13
  - 7.11 Acoustic Waves and Transfer Relations 7.13
    - Pressure-Velocity Relations for Planar Layer 7.14
    - Pressure-Velocity Relations for Cylindrical Annulus 7.15
  - 7.12 Acoustic Waves, Guides and Transmission Lines 7.15
    - Response to Transverse Drive 7.16
    - Spatial Eigenmodes 7.17
    - Acoustic Transmission Lines 7.17
  - 7.13 Experimental Motivation for Viscous Stress Dependence on Strain Rate 7.18
  - 7.14 Strain-Rate Tensor 7.20
    - Fluid Deformation Example 7.21
    - Strain Rate as a Tensor 7.21
  - 7.15 Stress-Strain-Rate Relations 7.21
    - Principal Axes 7.22
    - Strain-Rate Principal Axes the Same as for Stress 7.22
    - Principal Coordinate Relations 7.23
    - Isotropic Relations 7.23
  - 7.16 Viscous Force Density and the Navier-Stokes's Equation 7.24
  - 7.17 Kinetic Energy Storage, Power Flow and Viscous Dissipation 7.25
  - 7.18 Viscous Diffusion 7.26
    - Convection Diffusion of Vorticity 7.26
    - Perturbations from Static Equilibria 7.27
    - Low Reynolds Number Flows 7.27
  - 7.19 Perturbation Viscous Diffusion Transfer Relations 7.28
    - Layer of Arbitrary Thickness 7.29
    - Short Skin-Depth Limit 7.31
    - Infinite Half-Space of Fluid 7.31
  - 7.20 Low Reynolds Number Transfer Relations 7.32
    - Planar Layer 7.32
    - Axisymmetric Spherical Flows 7.33
  - 7.21 Stokes's Drag on a Rigid Sphere 7.36
  - 7.22 Lumped Parameter Thermodynamics of Highly Compressible Fluids 7.36
    - Mechanical Equations of State 7.37
    - Energy Equation of State for a Perfect Gas 7.37
    - Conservation of Internal Energy in CQS Systems 7.37

- 7.23 Internal Energy Conservation in a Highly Compressible Fluid 7.38
  - Power Conversion from Electromagnetic to Internal Form 7.39
  - Power Flow Between Mechanical and Internal Subsystems 7.39
  - Integral Internal Energy Law 7.39
  - Combined Internal and Mechanical Energy Laws 7.39
  - Entropy Flow 7.40
- 7.24 Overview 7.41
  - PROBLEMS 7.43
- 8. STATICS AND DYNAMICS OF SYSTEMS HAVING A STATIC EQUILIBRIUM
  - 8.1 Introduction 8.1
    - STATIC EQUILIBRIA
  - 8.2 Conditions for Static Equilibria 8.1
  - 8.3 Polarization and Magnetization Equilibria: Force Density and Stress Tensor Representations 8.4
    - Kelvin Polarization Force Density 8.4
    - Korteweg-Helmholtz Polarization Force Density 8.6
    - Korteweg-Helmholtz Magnetization Force Density 8.6
  - 8.4 Charge Conserving and Uniform Current Static Equilibria 8.8
    - Uniform Charged Layers 8.8
    - Uniform Current Density 8.10
  - 8.5 Potential and Flux Conserving Equilibria 8.11
    - Antiduals 8.12
    - Bulk Relations 8.13
    - Stress Equilibrium 8.13
    - Evaluation of Surface Deflection 8.13
    - Evaluation of Stress Distribution 8.14
    - HOMOGENEOUS BULK INTERACTIONS
  - 8.6 Flux Conserving Continua and Propagation of Magnetic Stress 8.16
    - Temporal Modes 8.19
    - Spatial Structure of Sinusoidal Steady-State Response 8.19
  - 8.7 Potential Conserving Continua and Electric Shear Stress Instability 8.20
    - Temporal Modes 8.23
  - 8.8 Magneto-Acoustic and Electro-Acoustic Waves 8.25
    - Magnetization Dilatational Waves 8.27
    - PIECEWISE HOMOGENEOUS SYSTEMS
  - 8.9 Gravity-Capillary Dynamics 8.28
    - Driven Response 8.29
    - Gravity-Capillary Waves 8.30
    - Temporal Eigenmodes and Rayleigh-Taylor Instability 8.30
    - Spatial Eigenmodes 8.31
  - 8.10 Self-Field Interfacial Instabilities 8.33
  - 8.11 Surface Waves with Imposed Gradients 8.38
    - Bulk Relations 8.38
    - Jump Conditions 8.39
    - Dispersion Equation 8.39
    - Temporal Modes 8.39
  - 8.12 Flux Conserving Dynamics of the Surface Coupled  $z=0$  Pinch 8.40
    - Equilibrium 8.42
    - Bulk Relations 8.42
    - Boundary and Jump Conditions 8.42
    - Dispersion Equation 8.43
  - 8.13 Potential Conserving Stability of a Charged Drop: Rayleigh's Limit 8.44
    - Bulk Relations 8.45
    - Boundary Conditions 8.45
    - Dispersion Relation and Rayleigh's Limit 8.45

8.14	Charge Conserving Dynamics of Stratified Aerosols 8.46
	Planar Layer 8.46
	Boundary Conditions 8.47
	Stability of Two Charge Layers 8.47
8.15	The z Pinch with Instantaneous Magnetic Diffusion 8.50
	Liquid Metal z Pinch 8.51
	Bulk Relations 8.51
	Boundary Conditions 8.52
	Rayleigh-Plateau Instability 8.53
	z Pinch Instability 8.54
8.16	Dynamic Shear Stress Surface Coupling 8.54
	Static Equilibrium 8.54
	Bulk Perturbations 8.55
	Jump Conditions 8.55
	Dispersion Equation 8.56
	SMOOTHLY INHOMOGENEOUS SYSTEMS AND THEIR INTERNAL MODES
8.17	Frozen Mass and Charge Density Transfer Relations 8.57
	Weak-Gradient Imposed Field Model 8.59
	Reciprocity and Energy Conservation 8.60
8.18	Internal Waves and Instabilities 8.62
	Configuration 8.62
	Normalization 8.62
	Driven Response 8.63
	Spatial Modes 8.66
	Temporal Modes 8.66
	PROBLEMS 8.69
9.	ELECTROMECHANICAL FLOWS
9.1	Introduction 9.1
9.2	Homogeneous Flows with Irrotational Force Densities 9.2
	Inviscid Flow 9.2
	Uniform Inviscid Flow 9.2
	Inviscid Pump or Generator with Arbitrary Geometry 9.4
	Viscous Flow 9.5
	FLOWS WITH IMPOSED SURFACE AND VOLUME FORCE DENSITIES
9.3	Fully Developed Flows Driven by Imposed Surface and Volume Force Densities 9.5
9.4	Surface-Coupled Fully Developed Flows 9.7
	Charge-Monolayer Driven Convection 9.7
	EQS Surface Coupled Systems 9.10
	MQS Systems Coupled by Magnetic Shearing Surface Force Densities 9.10
9.5.	Fully Developed Magnetic Induction Pumping 9.11
9.6	Temporal Flow Development with Imposed Surface and Volume Force Densities 9.13
	Turn-On Transient of Reentrant Flows 9.13
9.7	Viscous Diffusion Boundary Layers 9.16
	Linear Boundary Layer 9.17
	Stream-Function Form of Boundary Layer Equations 9.17
	Irrotational Force Density; Blasius Boundary Layer 9.18
	Stress-Constrained Boundary Layer 9.20
9.8	Cellular Creep Flow Induced by Nonuniform Fields 9.22
	Magnetic Skin-Effect Induced Convection 9.22
	Charge-Monolayer Induced Convection 9.24
	SELF-CONSISTENT IMPOSED FIELD
9.9	Magnetic Hartmann Type Approximation and Fully Developed Flows 9.25
	Approximation 9.25
	Fully Developed Flow 9.26

9.10	Flow Development in the Magnetic Hartmann Approximation	9.28
9.11	Electrohydrodynamic Imposed Field Approximation	9.32
9.12	Electrohydrodynamic "Hartmann" Flow	9.33
9.13	Quasi-One-Dimensional Free Surface Models	9.35
	Longitudinal Force Equation	9.36
	Mass Conservation	9.36
	Gravity Flow with Electric Surface Stress	9.37
9.14	Conservative Transitions in Piecewise Homogeneous Flows	9.37
	GAS DYNAMIC FLOWS AND ENERGY CONVERTERS	
9.15	Quasi-One-Dimensional Compressible Flow Model	9.41
9.16	Isentropic Flow Through Nozzles and Diffusers	9.42
9.17	A Magnetohydrodynamic Energy Converter	9.45
	MHD Model	9.46
	Constant Velocity Conversion	9.47
9.18	An Electrogasdynamic Energy Converter	9.48
	The EGD Model	9.49
	Electrically Insulated Walls	9.51
	Zero Mobility Limit with Insulating Wall	9.51
	Constant Velocity Conversion	9.52
9.19	Thermal-Electromechanical Energy Conversion Systems	9.53
	PROBLEMS	9.57
10.	ELECTROMECHANICS WITH THERMAL AND MOLECULAR DIFFUSION	
10.1	Introduction	10.1
10.2	Laws, Relations and Parameters of Convective Diffusion	10.1
	Thermal Diffusion	10.1
	Molecular Diffusion of Neutral Particles	10.2
	Convection of Properties in the Face of Diffusion	10.3
	THERMAL DIFFUSION	
10.3	Thermal Transfer Relations and an Imposed Dissipation Response	10.5
	Electrical Dissipation Density	10.5
	Steady Response	10.6
	Traveling-Wave Response	10.6
10.4	Thermally Induced Pumping and Electrical Augmentation of Heat Transfer	10.8
	Electric Relations	10.8
	Mechanical Relations	10.8
	Thermal Relations	10.9
10.5	Rotor Model for Natural Convection in a Magnetic Field	10.10
	Heat Balance for a Thin Rotating Shell	10.11
	Magnetic Torque	10.12
	Buoyancy Torque	10.12
	Viscous Torque	10.12
	Torque Equation	10.12
	Dimensionless Numbers and Characteristic Times	10.13
	Onset and Steady Convection	10.14
10.6	Hydromagnetic Bénard Type Instability	10.15
	MOLECULAR DIFFUSION	
10.7	Unipolar-Ion Diffusion Charging of Macroscopic Particles	10.19
10.8	Charge Double Layer	10.21
10.9	Electrokinetic Shear Flow Model	10.23
	Zeta Potential Boundary Slip Condition	10.24
	Electro-Osmosis	10.24
	Electrical Relations; Streaming Potential	10.25
10.10	Particle Electrophoresis and Sedimentation Potential	10.25
	Electric Field Distribution	10.26
	Fluid Flow and Stress Balance	10.27

10.11	Electrocapillarity	10.27
10.12	Motion of a Liquid Drop Driven by Internal Currents	10.32
	Charge Conservation	10.33
	Stress Balance	10.34
	PROBLEMS	10.37
11.	STREAMING INTERACTIONS	
11.1	Introduction	11.1
	BALLISTIC CONTINUA	
11.2	Charged Particles in Vacuum; Electron Beams	11.1
	Equations of Motion	11.1
	Energy Equation	11.2
	Theorems of Kelvin and Busch	11.2
11.3	Magnetron Electron Flow	11.3
11.4	Paraxial Ray Equation: Magnetic and Electric Lenses	11.6
	Paraxial Ray Equation	11.6
	Magnetic Lens	11.8
	Electric Lens	11.8
11.5	Plasma Electrons and Electron Beams	11.10
	Transfer Relations	11.10
	Space-Charge Dynamics	11.10
	Temporal Modes	11.11
	Spatial Modes	11.12
	DYNAMICS IN SPACE AND TIME	
11.6	Method of Characteristics	11.13
	First Characteristic Equations	11.13
	Second Characteristic Lines	11.14
	Systems of First Order Equations	11.14
11.7	Nonlinear Acoustic Dynamics: Shock Formation	11.16
	Initial Value Problem	11.16
	The Response to Initial Conditions	11.17
	Simple Waves	11.18
	Limitation of the Linearized Model	11.20
11.8	Nonlinear Magneto-Acoustic Dynamics	11.21
	Equations of Motion	11.21
	Characteristic Equations	11.21
	Initial Value Response	11.22
11.9	Nonlinear Electron Beam Dynamics	11.23
11.10	Causality and Boundary Conditions: Streaming Hyperbolic Systems	11.27
	Quasi-One-Dimensional Single Stream Models	11.28
	Single Stream Characteristics	11.29
	Single Stream Initial Value Problem	11.30
	Quasi-One-Dimensional Two-Stream Models	11.32
	Two-Stream Characteristics	11.33
	Two-Stream Initial Value Problem	11.34
	Causality and Boundary Conditions	11.35
	LINEAR DYNAMICS IN TERMS OF COMPLEX WAVES	
11.11	Second-Order Complex Waves	11.37
	Second Order Long-Wave Models	11.37
	Spatial Modes	11.39
	Driven Response of Bounded System	11.40
	Instability of Bounded System	11.41
	Driven Response of Unbounded System	11.45

- 11.12 Distinguishing Amplifying from Evanescent Modes 11.46
  - Laplace and Fourier Transform Representation in Time and Space 11.47
  - Laplace Transform on Time as the Sum of Spatial Modes: Causality 11.48
  - Asymptotic Response in the Sinusoidal Steady State 11.50
  - Criterion Based on Mapping Complex  $k$  as Function of Complex  $\omega$  11.53
- 11.13 Distinguishing Absolute from Convective Instabilities 11.54
  - Criterion Based on Mapping Complex  $k$  as Function of Complex  $\omega$  11.54
  - Second Order Complex Waves 11.55
- 11.14 Kelvin-Helmholtz Types of Instability 11.56
- 11.15 Two-Stream Field-Coupled Interactions 11.15
- 11.16 Longitudinal Boundary Conditions and Absolute Instability 11.16
- 11.17 Resistive-Wall Electron Beam Amplification 11.68

PROBLEMS 11.71

APPENDIX A. Differential Operators in Cartesian, Cylindrical, and Spherical Coordinates

APPENDIX B. Vector and Operator Identities

APPENDIX C. Films

INDEX

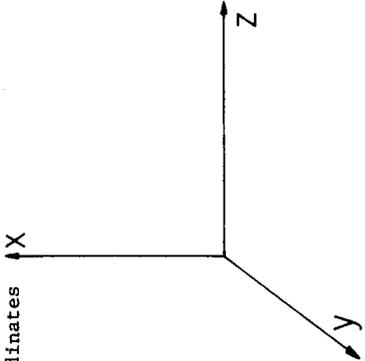
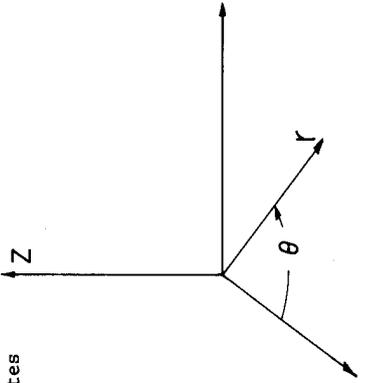
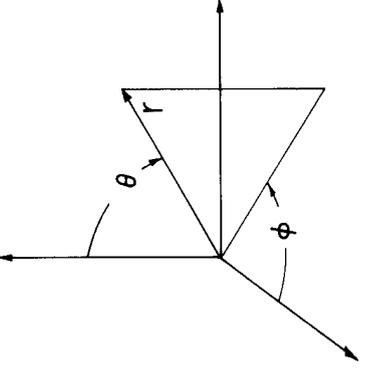


# Appendix A

---

## Differential Operators in Cartesian, Cylindrical and Spherical Coordinates

APPENDIX A. Differential Operators in Cartesian, Cylindrical and Spherical Coordinates

Operator	Cartesian coordinates 	Cylindrical coordinates 	Spherical coordinates 
$(\nabla \cdot \vec{A})$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$
$\nabla \phi$	$\frac{\partial \phi}{\partial x} \vec{i}_x + \frac{\partial \phi}{\partial y} \vec{i}_y + \frac{\partial \phi}{\partial z} \vec{i}_z$	$\frac{\partial \phi}{\partial r} \vec{i}_r + \frac{1}{r} \frac{\partial \phi}{\partial \theta} \vec{i}_\theta + \frac{\partial \phi}{\partial z} \vec{i}_z$	$\frac{\partial \phi}{\partial r} \vec{i}_r + \frac{1}{r} \frac{\partial \phi}{\partial \theta} \vec{i}_\theta + \frac{1}{r \sin \theta} \frac{\partial \phi}{\partial \phi} \vec{i}_\phi$
$(\nabla^2 \phi)$	$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial \phi}{\partial r}) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial \phi}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial \phi}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \phi}{\partial \phi^2}$
$(\nabla \times \vec{A})$	$\left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \vec{i}_x + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \vec{i}_y + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \vec{i}_z$	$\left( \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} - \frac{\partial A_r}{\partial z} \right) \vec{i}_r + \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \vec{i}_\theta + \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right) \vec{i}_z$	$\left( \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{1}{r \sin \theta} \frac{\partial A_\theta}{\partial \phi} \right) \vec{i}_r + \left( \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) \right) \vec{i}_\theta + \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right) \vec{i}_\phi$

$\nabla^2 \vec{A}$	$\left[ \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \right] \hat{i}_x$ $+ \left[ \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \right] \hat{i}_y$ $+ \left[ \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \right] \hat{i}_z$	$\left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_r) \right) + \frac{1}{r^2} \frac{\partial^2 A_r}{\partial \theta^2} - \frac{2}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial^2 A_r}{\partial z^2} + \frac{\partial^2 A_r}{\partial z^2} \right] \hat{i}_r$ $+ \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 A_\theta}{\partial \theta^2} + \frac{2}{r} \frac{\partial A_r}{\partial \theta} + \frac{\partial^2 A_\theta}{\partial z^2} + \frac{\partial^2 A_\theta}{\partial z^2} \right] \hat{i}_\theta$ $+ \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \theta^2} + \frac{\partial^2 A_z}{\partial z^2} \right] \hat{i}_z$	$\left[ \nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r} \frac{\partial A_\theta}{\partial \theta} - \frac{2A_\theta \cot \theta}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \right] \hat{i}_r$ $+ \left[ \nabla^2 A_\theta - \frac{2A_\theta}{r^2} - \frac{2}{r} \frac{\partial A_r}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\phi}{\partial \phi} \right] \hat{i}_\theta$ $+ \left[ \nabla^2 A_\phi - \frac{A_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial A_r}{\partial \theta} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\theta}{\partial \theta} \right] \hat{i}_\phi$
$\vec{\nabla} \cdot \vec{A}$	$\left( C_x \frac{\partial A_x}{\partial x} + C_y \frac{\partial A_x}{\partial y} + C_z \frac{\partial A_x}{\partial z} \right) \hat{i}_x$ $+ \left( C_x \frac{\partial A_y}{\partial x} + C_y \frac{\partial A_y}{\partial y} + C_z \frac{\partial A_y}{\partial z} \right) \hat{i}_y$ $+ \left( C_x \frac{\partial A_z}{\partial x} + C_y \frac{\partial A_z}{\partial y} + C_z \frac{\partial A_z}{\partial z} \right) \hat{i}_z$	$\left( C_r \frac{\partial A_r}{\partial r} + C_\theta \frac{\partial A_r}{\partial \theta} + C_\phi \frac{\partial A_r}{\partial \phi} \right) \hat{i}_r$ $+ \left( C_r \frac{\partial A_\theta}{\partial r} + C_\theta \frac{\partial A_\theta}{\partial \theta} + C_z \frac{\partial A_\theta}{\partial z} + C_\phi \frac{\partial A_\theta}{\partial \phi} \right) \hat{i}_\theta$ $+ \left( C_r \frac{\partial A_z}{\partial r} + C_\theta \frac{\partial A_z}{\partial \theta} + C_z \frac{\partial A_z}{\partial z} \right) \hat{i}_z$	$\left( C_r \frac{\partial A_r}{\partial r} + \frac{C_\theta}{r} \frac{\partial A_r}{\sin \theta} + \frac{C_\phi}{r} \frac{\partial A_r}{\sin \theta} - \frac{C_\theta A_\theta}{r} - \frac{C_\phi A_\phi}{r} \right) \hat{i}_r$ $+ \left( C_r \frac{\partial A_\theta}{\partial r} + \frac{C_\theta}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{C_\phi}{r} \frac{\partial A_\theta}{\sin \theta} + \frac{C_\theta A_r}{r} - \frac{C_\phi A_\theta \cot \theta}{r} \right) \hat{i}_\theta$ $+ \left( C_r \frac{\partial A_z}{\partial r} + \frac{C_\theta}{r} \frac{\partial A_z}{\partial \theta} + \frac{C_\phi}{r} \frac{\partial A_z}{\sin \theta} + \frac{C_\theta A_r}{r} + \frac{C_\phi A_\theta \cot \theta}{r} \right) \hat{i}_\phi$
$\vec{\nabla} \cdot (\vec{T} \cdot \vec{A})$	$\left( T_{xx} \frac{\partial A_x}{\partial x} + T_{yy} \frac{\partial A_x}{\partial y} + T_{zz} \frac{\partial A_x}{\partial z} \right) \hat{i}_x$ $+ \left( T_{xy} \frac{\partial A_x}{\partial y} + T_{yx} \frac{\partial A_x}{\partial x} + T_{zz} \frac{\partial A_x}{\partial z} \right) \hat{i}_y$ $+ \left( T_{xz} \frac{\partial A_x}{\partial z} + T_{yz} \frac{\partial A_x}{\partial y} + T_{zz} \frac{\partial A_x}{\partial z} \right) \hat{i}_z$	$T_{rr} \left( \frac{\partial A_r}{\partial r} \right) + T_{\theta\theta} \left( \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{A_\theta}{r} \right) + T_{\phi\phi} \left( \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} + \frac{A_\phi}{r} + \frac{A_\theta \cot \theta}{r} \right)$ $+ T_{r\theta} \left( r \frac{\partial}{\partial r} \left( \frac{A_\theta}{r} \right) + \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right) + T_{\theta z} \left( \frac{1}{r} \frac{\partial A_z}{\partial \theta} + \frac{\partial A_\theta}{\partial z} \right)$ $+ T_{rz} \left( \frac{\partial A_z}{\partial r} + \frac{\partial A_r}{\partial z} \right)$	$T_{rr} \left( \frac{\partial A_r}{\partial r} \right) + T_{\theta\theta} \left( \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{A_\theta}{r} \right) + T_{\phi\phi} \left( \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} + \frac{A_\phi}{r} + \frac{A_\theta \cot \theta}{r} \right)$ $+ T_{r\theta} \left( r \frac{\partial}{\partial r} \left( \frac{A_\theta}{r} \right) + \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right) + T_{r\phi} \left( \frac{\partial A_\phi}{\partial r} + \frac{1}{r} \frac{\partial A_r}{\sin \theta} + \frac{1}{r} \frac{\partial A_\theta}{\partial \phi} - \frac{A_\theta}{r} \right)$ $+ T_{\theta\phi} \left( \frac{1}{r} \frac{\partial A_\phi}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\theta}{\partial \phi} - \frac{\cot \theta}{r} A_\phi \right)$
$\vec{\nabla} \cdot \vec{T}$	$\left( \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \right) \hat{i}_x$ $+ \left( \frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \right) \hat{i}_y$ $+ \left( \frac{\partial T_{zx}}{\partial x} + \frac{\partial T_{zy}}{\partial y} + \frac{\partial T_{zz}}{\partial z} \right) \hat{i}_z$	$\left( \frac{1}{r} \frac{\partial}{\partial r} (r T_{rr}) + \frac{1}{r} \frac{\partial}{\partial \theta} T_{r\theta} - \frac{1}{r} T_{\theta\theta} + \frac{\partial T_{rz}}{\partial z} \right) \hat{i}_r$ $+ \left( \frac{1}{r} \frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{\partial T_{r\theta}}{\partial r} + \frac{2}{r} T_{r\theta} + \frac{\partial T_{\theta z}}{\partial z} \right) \hat{i}_\theta$ $+ \left( \frac{1}{r} \frac{\partial}{\partial r} (r T_{rz}) + \frac{1}{r} \frac{\partial T_{z\theta}}{\partial \theta} + \frac{1}{r} \frac{\partial T_{zz}}{\partial z} + \frac{\partial T_{z\theta}}{\partial z} \right) \hat{i}_z$	$\left( \frac{1}{r} \frac{\partial}{\partial r} (r^2 T_{rr}) + \frac{1}{r} \frac{\partial}{\partial \theta} (T_{r\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial T_{\theta\theta}}{\partial \theta} - \frac{T_{\theta\theta}}{r} + \frac{\partial T_{r\phi}}{\partial r} - \frac{\cot \theta}{r} T_{\phi\phi} \right) \hat{i}_r$ $+ \left( \frac{1}{r} \frac{\partial}{\partial r} (r^2 T_{r\theta}) + \frac{1}{r} \frac{\partial}{\partial \theta} (T_{\theta\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial T_{r\theta}}{\partial \theta} + \frac{1}{r} \frac{\partial T_{r\phi}}{\partial r} - \frac{\cot \theta}{r} T_{\phi\phi} \right) \hat{i}_\theta$ $+ \left( \frac{1}{r} \frac{\partial}{\partial r} (r^2 T_{r\phi}) + \frac{1}{r} \frac{\partial T_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial T_{r\theta}}{\partial \theta} + \frac{1}{r} \frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{2 \cot \theta}{r} T_{\theta\phi} \right) \hat{i}_\phi$

For

# Appendix B

---

## Vector and Operator Identities

$$\vec{A} \times \vec{B} \cdot \vec{C} = \vec{A} \cdot \vec{B} \times \vec{C} \quad (1)$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \quad (2)$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi \quad (3)$$

$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B} \quad (4)$$

$$\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B} \quad (5)$$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi \quad (6)$$

$$\nabla \cdot (\psi\vec{A}) = \vec{A} \cdot \nabla\psi + \psi\nabla \cdot \vec{A} \quad (7)$$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B} \quad (8)$$

$$\nabla \cdot \nabla\phi = \nabla^2\phi \quad (9)$$

$$\nabla \cdot \nabla \times \vec{A} = 0 \quad (10)$$

$$\nabla \times \nabla\phi = 0 \quad (11)$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A} \quad (12)$$

$$(\nabla \times \vec{A}) \times \vec{A} = (\vec{A} \cdot \nabla)\vec{A} - 1/2 \nabla(\vec{A} \cdot \vec{A}) \quad (13)$$

$$\nabla(\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) \quad (14)$$

$$\nabla \times (\phi\vec{A}) = \nabla\phi \times \vec{A} + \phi\nabla \times \vec{A} \quad (15)$$

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} \quad (16)$$

# Appendix C

---

## Films

Developed for educational purposes with the support of the National Science Foundation at the Education Development Center, films cited fall in one of two series.

Produced by the National Committee for Fluid Mechanics Films and distributed by Encyclopedia Britannica Educational Corp., 425 N. Michigan Ave., Chicago, Illinois (60611) are:

- (1) Channel Flow of a Compressible Fluid
- (2) Current-induced Instability of a Mercury Jet
- (3) Eulerian and Lagrangian Descriptions in Fluid Mechanics
- (4) Flow Instabilities
- (5) Fundamentals of Boundary Layers
- (6) Low-Reynolds Number Flows
- (7) Magnetohydrodynamics
- (8) Pressure Fields and Fluid Acceleration
- (9) Surface Tension and Fluid Mechanics
- (10) Waves in Fluids

Produced by the National Committee for Electrical Engineering Films and distributed by Education Development Center, 39 Chapel Street, Newton, Mass. 02160 are:

- (11) Complex Waves I and Complex Waves II
- (12) Electric Fields and Moving Media

---

## Index

Absolute from convective instability, distinguishing, 11.32  
 Absolute instability, 11.32  
 Absolute instability, boundary conditions and, 11.41  
 Accelerator, linear particle, 4.18  
 Accelerator operation, d-c machine, 4.39  
 Acoustic dynamics, nonlinear, 11.16  
 Acoustic guides, 7.15  
 Acoustic impedance, numerical values of, 7.14  
 Acoustic surface waves, 7.48  
 Acoustic transmission lines, 7.15, 7.17  
 Acoustic velocity, ideal gas, 7.40  
 Acoustic velocity, numerical values of, 7.14  
 Acoustic wave spatial modes, 7.17  
 Acoustic wave transit time, 1.5  
 Acoustic wave velocity, 7.13  
 Acoustic waves, 7.13, 7.15  
 Acoustic waves in solids, 7.48  
 Aerosol, dynamics of stratified charged, 8.46  
 Aerosol, impact charging of, 8.50  
 Aerosol, space-charge static equilibrium of, 8.8  
 Agglomeration, self-, 5.27  
 Alfvén waves, 8.16  
 Alfvén waves, demonstration of, 8.20  
 Alfvén waves, effect of finite conductivity and viscosity on, 8.16  
 Ampère's law, differential form of, 2.2  
 Amplifying from evanescent modes, distinguishing, 11.46  
 Amplifying wave, 11.32, 11.41, 11.44  
 Amplitude parameter expansion (see also linearization), 1.4  
 Anemometer, ion drag, 5.7  
 Anisotropic conductor, charge relaxation in deformable, 8.20  
 Antiduals, 8.12  
 Applications, 1.2  
 Atomization, 8.37  
 Average of periodic functions, 2.31  
 Averaging theorem, complex amplitude, 2.31  
 Averaging theorem, Fourier series, 2.31  
 Axisymmetric spherical creep flow, 7.33  
  
 Bénard instability, hydromagnetic, 10.15  
 Bernoulli's equation, 7.9  
 Bernoulli's equation, streamline form of, 7.9  
 Bessel functions, 2.36  
 Bessel functions modified, 2.36  
 Biharmonic equation, 7.32  
 Biological locomotion, 10.32  
 Bipolar migration, 5.26  
 Bipolar migration, laws for, 5.27  
 Blasius boundary layer, 9.18  
 Boltzmann constant, 5.3  
 Boundary condition, thin conducting permeable sheet, 6.35  
 Boundary condition, thin conductor, 6.4  
 Boundary condition, thin permeable sheet, 6.35  
 Boundary conditions, causality and, 11.35, 11.40, 11.45  
 Boundary conditions, electromagnetic, 2.14, 2.19  
 Boundary conditions, fluid mechanics, 7.8  
 Boundary conditions, magnetoquasistatic, 2.18, 2.19  
 Boundary layer, Blasius, 9.18  
 Boundary layer, irrotational force density and viscous, 9.18  
 Boundary layer, linear viscous, 9.17  
 Boundary layer, magnetic, 6.22  
 Boundary layer, stress-constrained, 9.20  
 Boundary layer, viscous diffusion, 9.16  
 Boundary layer equations, 9.16  
 Boundary layer equations, integration of, 9.18  
 Boundary layer equations, streamfunction form of, 9.17  
 Brake, d-c electric ohmic, 5.43  
 Brake, Hartmann magnetic, 9.28  
 Brake, unipolar electric, 5.22  
 Brake operation, d-c machine, 4.39  
 Breakdown strength of gases, 4.55  
 Bulk electromechanical interactions, homogeneous, 8.16  
 Buoyancy, 10.12  
 Busch's theorem for electron beam, 11.2  
  
 Capacitance, 2.1, 2.20  
 Capillary instability of cylindrical jet, 8.53, 8.73  
 Capillary ripples, 7.4  
 Capillary rise, demonstration of, 7.6  
 Capillary static equilibrium, 7.10  
 Cauchy integral theorem, 5.66  
 Causality, characteristics and boundary conditions, 11.27  
 Causality and boundary conditions, two-stream systems, 11.35  
 Cellular convection, magnetic field-induced liquid metal, 9.23  
 Channel flow, compressible fluid, 9.41  
 Channel flow of a compressible fluid, film Reference 1, Appendix C, 9.45  
 Channel flows, 9.35  
 Characteristic equations, 11.13, 11.16, 11.21, 11.23  
 Characteristic equations, charge relaxation in terms of, 5.39  
 Characteristic equations, determinantal conditions, 11.14  
 Characteristic equations, first and second, 11.14  
 Characteristic equations, first order systems, 5.5, 5.7, 5.17, 5.26, 5.33, 5.38  
 Characteristic equations, method of undetermined coefficients, 11.14

Characteristic time, capillary, 9.1  
 Characteristic time, charge relaxation, 2.3, 8.23  
 Characteristic time, electroinertial, 8.49, 9.32  
 Characteristic time, electroviscous, 8.23, 9.8  
 Characteristic time, gravity, 9.1  
 Characteristic time, Hartmann flow establishment, 9.31  
 Characteristic time, magnetic diffusion, 6.3, 8.11, 9.25  
 Characteristic time, magneto-inertial, 8.17, 9.25  
 Characteristic time, migration, 10.19  
 Characteristic time, molecular diffusion, 10.3, 10.19  
 Characteristic time, reciprocal cyclotron frequency, 11.4  
 Characteristic time, reciprocal plasma frequency, 11.10, 11.23  
 Characteristic time, thermal diffusion, 10.2, 10.13  
 Characteristic time, thermal relaxation, 10.13  
 Characteristic time, viscous diffusion, 7.27, 7.32, 7.42, 8.17, 8.23, 9.25  
 Characteristic time, viscous relaxation, 7.42  
 Characteristic times, 1.4, 1.5  
 Characteristic times, ambipolar diffusion and, 10.4  
 Characteristics, boundary condition and, 5.6  
 Charge, 2.1  
 Charge, conservation of, 2.2, 5.2  
 Charge, lumped parameter variable of, 2.20  
 Charge conservation boundary conditions, 5.44, 5.45, 5.50  
 Charge conserving continua, 8.8, 8.57  
 Charge conserving continua, transfer relations for, 8.46  
 Charge conserving interfacial dynamics, 8.54  
 Charge convection, 5.1  
 Charge convection, instability and, 8.49  
 Charge density, free, 2.1  
 Charge density, magnetic, 2.13  
 Charge density, polarization, 2.12  
 Charge diffusion, heterogeneous ohmic, 5.56  
 Charge generation, 5.2  
 Charge migration, 5.1  
 Charge monolayer driven convection, 9.7  
 Charge monolayer induced cellular convection, 9.24  
 Charge recombination, 5.2  
 Charge relaxation, 5.1  
 Charge relaxation, deforming ohmic conductor, 5.38  
 Charge relaxation, heterogeneous system of uniform conductors, 5.56  
 Charge relaxation temporal modes, 5.54  
 Charge relaxation time, 1.5, 2.3, 5.34  
 Charge relaxation time, imposed field flow approximation, 9.32  
 Charge relaxation time, particle, 5.76  
 Charge relaxation transmission line, 4.51  
 Charged aerosol equilibrium, stability of, 8.9  
 Charged drop dynamics, 8.44  
 Charged particle beam, kinematics of, 4.17  
 Charged particle migration, 5.5  
 Charged particles in vacuum, 11.1  
 Charging diagram, negative ion impact, 5.13  
 Charging diagram, positive ion impact, 5.12  
 Charging of macroscopic particle, ion diffusion, 10.19  
 Choking, compressible channel flow, 9.44  
 Chu formulation, 2.1  
 Classification, energy converter, 4.3  
 Coefficient of heat transfer, surface, 10.11  
 Coenergy, electroquasistatic, 2.24  
 Coenergy, magnetoquasistatic, 2.28  
 Coenergy density, electroquasistatic, 2.24  
 Coenergy density, magnetoquasistatic, 2.28  
 Coenergy function, electrocapillary, 10.29  
 Collision frequency, 3.2  
 Combustor, energy conversion cycle, 9.53  
 Complex amplitudes, definition of, 2.29  
 Complex amplitudes, polyphase currents represented by, 4.21  
 Complex waves, 11.37  
 Complex waves, second order, 11.37  
 Complex waves I, film Reference 11, Appendix C, 11.42  
 Complex waves II, film Reference 11, Appendix C, 8.32, 11.43, 11.44  
 Compressibility constitutive law, weak, 7.13  
 Compressible channel flow, conservative transition in, 9.44  
 Compressible channel flow, electrogasdynamic, 9.48, 9.62  
 Compressible flow, channel, 9.41  
 Compressible flow, free surface flow analogous to, 9.43, 9.60  
 Compressible flow, velocity-area diagram, 9.44, 9.45  
 Compressible fluid, weakly, 7.13  
 Compressible quasistatic, 7.42  
 Compressible quasistatic limit (CQS), 7.16  
 Compressional waves, 8.25  
 Compressional waves in solids, 7.48  
 Compressional waves, magnetization, 8.27  
 Conduction, electrical, 3.2  
 Conduction machines, 4.33  
 Conductivity, evolution of bipolar, 5.33  
 Conductivity, numerical values of electrical, 6.3  
 Conductivity, surface electrical, 5.44, 5.45, 6.4  
 Conductivity, temperature dependent electrical, 10.3, 10.8  
 Confinement, plasma, 8.40  
 Conservation of energy, internal and kinetic energy combined, 7.39  
 Conservation of energy, surface, 7.5  
 Conservation of free charge, boundary condition for, 2.17, 2.19  
 Conservation of internal energy, highly compressible fluid, 7.38

Conservation of internal energy,  
   lumped parameter, 7.37  
 Conservation of kinetic energy,  
   fluid, 7.25  
 Conservation of mass, fluid, 7.1  
 Conservation of momentum, fluid, 7.2  
 Conservative transition, compressible  
   flow, 9.44  
 Conservative transitions, piecewise  
   homogeneous flow, 9.37  
 Constitutive laws, polarization, 3.9  
 Continua, current conserving, 8.70, 8.71  
 Convection, natural, 10.10  
 Convection, rotor model for  
   thermal, 10.10  
 Convective derivative, 2.6, 2.7  
 Convective derivative, fluid  
   acceleration and, 7.2  
 Convective from absolute instabilities,  
   distinguishing, 11.54  
 Convective instability, 11.32  
 Convective magnetic diffusion, 6.2  
 Convective magnetic diffusion, skin-  
   effect approximation to, 6.21  
 Coordinates, Eulerian, 2.6  
 Coordinates, Lagrangian, 2.6  
 Corona discharge, electrostatic  
   precipitator, 5.9  
 Couette flow, generalized, 9.6  
 Couette mixer, electrohydro-  
   dynamic, 8.24  
 CQS (see compressible quasi-  
   static)  
 Creep flow, 7.27  
 Creep flow, cellular electro-  
   mechanical, 9.22  
 Critical angle, particle  
   charging, 5.13  
 Critical charge (also saturation  
   charge), 5.10  
 Critical depth of gravity flows, 9.40  
 Critical flow, free surface, 9.39  
 Critical lines (or points), charge  
   trajectory, 5.8, 5.17  
 Current, 2.1  
 Current, electroquasistatic  
   lumped parameter, 2.20  
 Current, Hall, 3.3  
 Current-charge relation, lumped  
   parameter, 2.20  
 Current conserving continua, 8.70, 8.71  
 Current density, free, 2.1  
 Current-induced instability of a  
   mercury jet, film Reference 2,  
   Appendix C, 8.51  
 Current tube, 2.26  
 Curvature, radius of, 7.5  
 Cyclotron frequency, 11.4  
 Cylindrical shell, boundary condition  
   for rotating conducting, 6.5  
 Cylindrical shell, boundary condition  
   for translating conducting, 6.5  
  
 D-c interactions, conditions for, 4.5  
 D-c interactions, magnetohydro-  
   dynamic, 9.45  
 D-c interactions, ohmic electric, 5.42  
 D-c machine, energy conservation  
   in, 4.39  
 D-c machine, magnetic, 4.33  
 D-c machine, model for, 4.33  
 D-c machine, unipolar electric, 5.22  
 D-c machine, winding for, 4.35  
 D-c machines, classification of, 4.2  
 D-c pump, unipolar electric, 5.22  
 Debye length, 10.22  
 Diffuser, compressible flow, 9.44  
 Diffusion, ambipolar, 10.3  
 Diffusion, charged particle, 5.2  
 Diffusion, magnetic traveling wave, 6.18  
 Diffusion, migration relative to, 5.3  
 Diffusion, transient magnetic, 6.25  
 Diffusion, unipolar ion, 10.19  
 Diffusion charging of macroscopic  
   particle, 10.20  
 Diffusion coefficient, mobility related  
   to, 5.3  
 Diffusion coefficient, numerical values  
   of molecular, 10.4  
 Diffusion equation, magnetic, 6.2  
 Diffusion time, numerical values of  
   magnetic, 6.4  
 Diffusion time, viscous, 7.27  
 Diffusion wave, phase of, 6.18  
 Dimensionless numbers, 1.5  
 Dipole, force on, 3.7  
 Direction cosine, 3.16  
 Dissipation, calculation of elec-  
   trical, 6.20  
 Dissipation, relation of magnetic  
   stress to, 6.22  
 Dissipation density, electrical, 9.39  
 Dissipation density, electrical to  
   thermal, 10.5  
 Dissipation density, fluid, 7.25  
 Dissociation, formation of charge  
   by, 5.27  
 Dorn effect, 10.27  
 Double layer, 2.15, 10.21  
 Double layer, boundary conditions  
   for, 2.16, 2.19  
 Double layer, ideally polarized, 10.11,  
   10.32  
 Double layer charge conservation,  
   ideally polarized, 10.33  
 Double layer incremental capacitance, 10.30  
 Double layer surface force density, 3.20  
   10.31  
 Drag, rigid sphere viscous, 7.36  
 Drag, Stokes's, 7.36  
 Drag-cup tachometer, 6.11  
 Drop charging, ion impact, 5.10  
 Drop formation, 8.53  
 Drop formation, electric field and, 8.73  
 Dynamical processes, characteristic  
   times and, 1.4  
 Dynamics in space and time, 11.13  
  
 EGD (see electrogasdynamic)  
 EHD (see electrohydrodynamic)  
 Eigenfrequencies, temporal mode Alfvén  
   wave, 8.20  
 Eigenfrequencies, integral condition  
   on, 8.66  
 Einstein relation, 5.3  
 Elastic modulus, numerical values of, 7.46  
 Elastic properties, numerical values  
   of, 7.46

Electric displacement, definition of, 2.1, 2.2  
 Electric field intensity, 2.1  
 Electric field intensity, breakdown, 4.55  
 Electric field intensity, limits on, 4.53  
 Electric fields and moving media, film Reference 12, Appendix C, 5.53, 8.5, 9.9  
 Electric Hartmann number, 1.5, 5.76, 9.9  
 Electric Prandtl number, 5.76  
 Electric Reynolds number, 1.5, 9.8  
 Electric Reynolds number, unipolar, 5.20, 5.23  
 Electrical energy subsystem, 8.60  
 Electrical dissipation, magnetoquasistatic ohmic, 6.19  
 Electrical dissipation density, 7.39  
 Electrification, thunderstorm, 5.10  
 Electro-acoustic compressional waves, 8.25  
 Electro-acoustic velocity, 8.26  
 Electrocapillarity, 10.27  
 Electrocapillary mobility, 10.40  
 Electrocapillary motions of drop, 10.32  
 Electrodynamics laws, differential, 2.1  
 Electrogasdynamic compressible energy converter, 9.48  
 Electrohydrodynamic imposed field approximation, 9.32  
 Electro-inertial time, 1.5, 8.23, 8.49  
 Electrokinetics, 10.23, 10.25  
 Electromagnetic flight, 6.23  
 Electromagnetic wave transit time, 1.5, 2.3  
 Electromagnetic waves, quasistatics and, 1.1  
 Electromagnetics, branches of, 1.1  
 Electromechanical coupling, thermal or molecular subsystem and, 10.1  
 Electromechanical energy conversion configurations, 4.3  
 Electromechanical kinematics, 1.1, 2.1, 4.1  
 Electron beam, equation of motion for, 11.1  
 Electron beam, magnetic confinement of, 4.17  
 Electron beam, resistive wall amplification on, 11.68  
 Electron beam, transfer relations for, 11.10  
 Electron beam, traveling-wave interaction with, 11.78  
 Electron beam devices, 4.17  
 Electron beam dynamics, nonlinear, 11.23  
 Electron beam energy converters, 4.56  
 Electro-osmosis, 10.23, 10.24  
 Electrophoresis, 10.25  
 Electroquasistatic jump conditions, 2.14  
 Electroquasistatics, 2.3  
 Electrostatic precipitator, 5.9  
 Electroviscous time, 1.5, 5.76, 8.23, 9.8  
 Energy, conservation of, 2.22  
 Energy, electroquasistatic, 2.22  
 Energy, gas internal, 7.36  
 Energy, magnetoquasistatic, 2.28  
 Energy and coenergy, relation between, 2.24  
 Energy conservation, electrocapillary surface, 10.29  
 Energy conservation, electroquasistatic subsystem, 2.25  
 Energy conservation, lumped parameter electroquasistatic, 2.24  
 Energy conservation, magnetoquasistatic, 2.26  
 Energy conservation, magnetoquasistatic subsystem, 2.28  
 Energy conversion, d-c machine, 4.39  
 Energy conversion limitations, 4.53  
 Energy conversion limitations, electro-gasdynamic, 9.53  
 Energy conversion limitations, magneto-hydrodynamic, 9.48  
 Energy conversion processes, 1.3  
 Energy converters, classification of, 1.4  
 Energy density, electroquasistatic, 2.24  
 Energy density, magnetoquasistatic, 2.28  
 Energy equation, electron beam, 11.2  
 Energy function, permanent polarization, 4.12  
 Energy state equation, fluid, 7.37  
 Enthalpy, electrical power generated and change in, 9.48, 9.55  
 Enthalpy, specific, 7.38  
 Entropy, 7.37  
 Entropy, MHD generator increase in, 9.48  
 Entropy, specific, 7.38  
 Entropy flow, 7.40  
 EQS (see electroquasistatic)  
 Equation of state, mechanical fluid, 7.37  
 Equations of elasticity, 7.46, 7.47  
 Equations of motion for inviscid fluid, 7.2  
 Equivalent circuit, charge relaxation diffusion line, 5.56  
 Equivalent circuit, magnetic induction machine, 6.10  
 Error function, solution to diffusion equation, 6.24  
 Eulerian and Lagrangian coordinates, 2.48  
 Eulerian and Lagrangian descriptions in fluid mechanics, film Reference 3, Appendix C, 2.7  
 Eulerian coordinates, 2.6  
 Evanescent from amplifying modes, distinguishing, 11.46  
 Evanescent wave, 11.37, 11.41, 11.42  
 Exchange modes, MHD, 8.40  
 Exchange of stabilities, smoothly inhomogeneous temporal modes, 8.66  
 Expansion, space-rate, 4.41

Faraday's integral law, ohmic deformable media and, 6.2  
 Faraday's law, Chu formulation, 2.14  
 Faraday's law, differential form of, 2.1  
 Ferrofluid, 8.35  
 Ferrofluid, demonstration illustrating, 8.7  
 Ferrofluid interfacial instability, 8.37  
 Flow, low magnetic Reynolds number, 9.2, 9.10, 9.14, 9.17, 9.25  
 Flow development, Hartmann magnetic, 9.28  
 Flow development, temporal, 9.13  
 Flow instabilities, film Reference 4, Appendix C, 10.16  
 Flows, electromechanical, 9.1  
 Flows, fully developed, 9.5, 9.6, 9.7, 9.11, 9.26, 9.28, 9.33  
 Flows, imposed surface and volume force density, 9.5  
 Fluid interface, Eulerian description of, 7.3  
 Fluid mechanics, laws of, 7.1  
 Fluid mechanics boundary and jump conditions, 7.8  
 Fluid power flow, 7.25  
 Flux, lumped parameter variable of, 2.21  
 Flux conservation, surface of fixed identity and, 6.2  
 Flux conservation in conducting fluid, 8.16  
 Flux conserving continua, compressible, 8.25  
 Flux conserving continua, homogeneous bulk, 8.16  
 Flux conserving continua, static equilibrium of, 8.11  
 Flux conserving continua, z-theta pinch, 8.40  
 Flux-potential transfer relations, 2.16  
 Force, 2.1  
 Force, Lorentz, 3.1  
 Force, time-average, 5.60  
 Force, time-average in induction machine, 6.8, 6.16  
 Force coenergy relations, electroquasistatic, 3.5  
 Force coenergy relations, magnetoquasistatic, 3.6  
 Force densities, electromagnetic, 3.1  
 Force densities, tenuous dipole, 3.6  
 Force density, 2.1  
 Force density, electroquasistatic density dependent, 3.12  
 Force density, electroquasistatic free-charge, 3.4  
 Force density, flows with irrotational, 9.2  
 Force density, gradient of "pressure" as, 7.9  
 Force density, gravitational, 7.9  
 Force density, incompressible electrically linear electroquasistatic, 3.12, 3.18  
 Force density, incompressible electrically linear magnetoquasistatic, 3.15, 3.18  
 Force density, incompressible electroquasistatic, 3.11, 3.18  
 Force density, Kelvin magnetization, 3.8, 3.18  
 Force density, Kelvin polarization, 3.7, 3.18  
 Force density, Korteweg-Helmholtz electroquasistatic, 3.9, 3.11, 3.18  
 Force density, Korteweg-Helmholtz, magnetoquasistatic, 3.13, 3.18  
 Force density, Lorentz, 3.1  
 Force density, macroscopic vs. microscopic, 3.1  
 Force density, magnetic irrotational, 9.2  
 Force density, magnetoquasistatic density dependent, 3.15, 3.18  
 Force density, magnetoquasistatic free current, 3.4  
 Force density, quasistatic, 3.4  
 Force density, rotational, 9.3  
 Force density, viscous, 7.24  
 Force-energy relations, electroquasistatic, 3.5  
 Force-energy relations, magnetoquasistatic, 3.6  
 Force equation, fluid, 7.2  
 Force equation, isotropic perfectly elastic solid, 7.47  
 Force equation for viscous fluid, 7.24  
 Forces, electromagnetic, 3.1  
 Forces, lumped parameter, 3.4  
 Fourier amplitudes, definition of, 2.30  
 Fourier amplitudes, synchronous machine application of, 4.9  
 Fourier averaging theorem, synchronous machine application of, 4.10  
 Fourier series, 2.30  
 Fourier series complex amplitudes, time average of, 5.60  
 Fourier transform complex amplitudes, time average of, 5.60  
 Fourier transforms, definition of, 2.30  
 Frozen charge model, 8.57  
 Fully developed flow, mass conservation in, 9.7  
 Fully developed flows, surface coupled, 9.7  
 Fundamentals of boundary layers, film Reference 5, Appendix C, 9.19  
 Fusion experiments, 8.40  
 Galilean transformation, 2.7  
 Gas constant, 7.37  
 Gas constant, specific heats and, 7.38  
 Gasdynamic energy converters, 9.41  
 Gasdynamic flows, 9.41  
 Gauss' integral theorem, 2.9  
 Gauss' law, differential form of, 2.2  
 Gauss' law, multiple charge species and, 5.3

Gauss' theorem, tensor form, 3.15  
 Generalized Leibnitz rule, 2.10  
 Generating systems, open cycle, 9.53  
 Generation, bipolar, 5.33  
 Generation, charge, 5.2  
 Generator, d-c electric ohmic, 5.43  
 Generator, Hartmann magnetic, 9.28  
 Generator, inviscid magnetohydrodynamic, 9.4  
 Generator, magnetohydrodynamic gas-dynamic, 9.46  
 Generator, unipolar electric, 5.22  
 Generator, variable capacitance, 4.45  
 Generator operation, d-c machine, 4.39  
 Glass thickness control, 8.2, 8.10  
 Gradient field stabilization, 8.38  
 Gradient integral theorem, 2.9  
 Gravitational force density, 7.9  
 Gravitational subsystem, 8.60  
 Gravity-capillary dynamics, 8.28  
 Gravity-capillary modes of charge monolayer, 8.56  
 Gravity-capillary spatial modes, 8.31  
 Gravity-capillary wave, phase velocity, 8.30  
 Gravity-capillary waves, dispersion equation for, 8.29  
 Gravity flow, 9.35  
 Green's function field representations, 4.40  
  
 Hall current, 3.3  
 Hankel functions, 2.36  
 Hartmann channel flow, 9.26  
 Hartmann flow, electrohydrodynamic, 9.33  
 Hartmann layer, 9.29, 9.59  
 Hartmann number, Alfvén waves and magnetic, 8.18  
 Hartmann number, electric, 1.5, 9.9, 9.34  
 Hartmann number, ideally polarized double layer, 10.35  
 Hartmann number, magnetic, 1.5, 9.27  
 Hartmann profile, magnetic, 9.27  
 Hartmann type flows, magnetic, 9.25  
 Hartmann velocity profile, electric, 9.34  
 Head diagram, free surface flow, 9.39  
 Heat conduction and convection, imposed dissipation, 10.5  
 Heat transfer, 10.1  
 Helmholtz equation, 4.16  
 Hyperbolic systems, streaming, 11.27  
 Hysteresis loop, 6.30  
 Hysteresis motor, 6.30  
  
 Ideal gas equations of state, 7.37  
 Impact charging of macroscopic particles, 5.9  
 Impedance, acoustic, 7.18  
 Imposed vs. self-fields, 8.38  
 Imposed gradient surface interactions, 8.38  
 Incompressibility, fluid, 7.1  
 Incompressible elastic solid, 7.48  
 Incompressible inertialess solid, 7.49  
  
 Incompressible quasistatic, 7.42  
 Independent variables, electroquasistatic, 3.5  
 Inductance, 2.1, 2.21  
 Inductance matrix, 2.21, 4.21, 4.32  
 Induction heating, thermal response to magnetic, 10.5  
 Induction interactions, magnetic, 6.1  
 Induction machine, balanced two-phase, 6.8  
 Induction machine, end effect in magnetic linear, 6.36, 6.37  
 Induction machine, single-phase magnetic, 6.10  
 Induction motor, deep conductor, 6.15  
 Induction motor, electroquasistatic, 5.46  
 Induction motor, magnetic, 6.6  
 Induction pump, liquid metal magnetic, 9.11  
 Inertial quasistatic laws, 7.42  
 Inertial reference frame, 2.7  
 Inhomogeneity, mass density, 7.1  
 Initial value problem, method of characteristics and, 11.16  
 Initial value problem, single stream, 11.30  
 Ink jet printing, 8.44  
 Instability, absolute, 11.32, 11.37, 11.41  
 Instability, bulk electrohydrodynamic, 8.24  
 Instability, convective (also, amplifying wave), 11.32, 11.37, 11.41, 11.46, 11.54  
 Instability, critical conditions for, 8.36  
 Instability, electrohydrodynamic equipotential surface, 8.37  
 Instability, heavy fluid on top of light, 8.30  
 Instability, incipience of, 8.36  
 Instability, internal, 8.62  
 Instability, Kelvin-Helmholtz type, 11.56  
 Instability, nonlinear stages of surface, 8.31, 8.32, 8.37  
 Instability, Rayleigh-Plateau, 8.53  
 Instability, self-field interfacial, 8.33  
 Instability, two-stream, 11.34  
 Instability, z-pinch at low magnetic Reynolds number, 8.54  
 Instability of glycerin interface stressed by electric field, 8.37  
 Integral law, mass conservation, 7.1  
 Integral law, momentum conservation, 7.2  
 Integral laws, electroquasistatic, 2.10  
 Integral laws, magnetoquasistatic, 2.10  
 Integral theorem, Gauss', 2.9  
 Integral theorem, generalized Leibnitz, 2.9  
 Integral theorem, gradient, 2.9  
 Integral theorem, Stokes's, 2.9, 3.26  
 Integral theorem, Stokes's type, 3.25  
 Interface, fluid, 7.3  
 Interface, fluid velocity at an, 7.4  
 Interfacial coupling, charge monolayer, 8.54  
 Internal energy, specific, 7.37  
 Internal energy differential law 7.39

Internal energy integral law, 7.39  
 Internal energy of gas, 7.36  
 Internal energy storage, weakly compressible fluid, 7.26  
 Internal instabilities, 8.62  
 Internal waves, 8.62  
 Inviscid fluid, equations of motion for, 7.2  
 Inviscid irrotational flow, 9.2  
 Ion diffusion time, 5.34  
 Ion drag anemometer, 5.7  
 Ion drag brake, 5.22  
 Ion drag generator, 5.22  
 Ion drag pump, 5.22, 9.33  
 Ion mobility in gases, 5.4  
 Ion mobility in liquids, 5.4  
 Ionization, liquid, 5.27  
 IQS (see inertial or incompressible quasistatic)  
 Irrotational flow, fluid, 7.9  
 Irrotational force densities, homogeneous flows and, 9.2  
 Isentropic flow through nozzles and diffusers, 9.42  
 Isotropy relations, stress-strain-rate, 7.23

Jump conditions, electro-magnetic, 2.14, 2.19  
 Jump conditions, fluid mechanics, 7.8  
 Jump conditions, magnetoquasistatic, 2.18, 2.19

Kelvin and Korteweg-Helmholtz force densities compared, 8.4  
 Kelvin force density, static equilibrium in terms of the, 8.4  
 Kelvin-Helmholtz type instability, 11.56  
 Kelvin magnetization force density interaction between dipoles, 3.15  
 Kelvin polarization force density, 3.7, 3.12, 3.18  
 Kelvin polarization force density, interaction between dipoles and, 3.12, 3.18  
 Kelvin theorem, Busch's theorem and, 11.2  
 Kinematics, electromechanical, 1.1, 2.1, 4.1  
 Kinematics, mechanical, 2.1  
 Kinetic energy storage, fluid, 7.25  
 Kinetic energy subsystem, 8.60  
 Klystron, 11.24  
 Korteweg-Helmholtz and Kelvin force densities compared, 8.4  
 Korteweg-Helmholtz electroquasistatic force density, 3.9, 3.11, 3.18  
 Korteweg-Helmholtz magnetoquasistatic force density, 3.13, 3.18  
 Kronecker delta, 3.17

Lagrangian and Eulerian coordinates, 2.48  
 Lagrangian coordinates, 2.6  
 Lagrangian coordinates, electron motion in, 11.1  
 Langevin recombination coefficient, 5.26  
 Laplace-Fourier transform representation, 11.47  
 Laplace's equation, irrotational flow and, 7.10  
 Laplace's equation, numerical solution of, 8.14  
 Laval nozzle, 9.44  
 Legendre functions, associated, 2.40  
 Legendre polynomials, 2.40, 7.34  
 Leibnitz rule, generalized line integral, 2.49  
 Leibnitz rule, generalized surface integral, 2.49  
 Lens, electric electron, 11.6, 11.8  
 Lens, magnetic electron, 11.6, 11.8  
 Levitation, magnetic, 6.24, 8.2  
 Levitation force, relation of dissipation to, 6.22  
 Linearized model, limitations of acoustic wave, 11.20  
 Linearized models, 4.42  
 Lippmann equation, electrocapillary, 10.30  
 Liquid metal, motions in uniform magnetic field, 8.16  
 Liquid metal, static equilibrium of, 8.11  
 Liquid metal levitation, 8.2  
 Liquid metal z-pinch, 8.51  
 Longitudinal coordinate, 1.6, 4.53, 9.35  
 Long-wave free surface models, 9.35  
 Long-wave model, 4.42  
 Long-wave model, field coupled membrane, 11.37  
 Long-wave models, 1.4  
 Lorentz force, 3.1  
 Lorentz force density, 3.1  
 Low Reynolds number cellular convection, 9.22  
 Low Reynolds number flow, 7.27  
 Low Reynolds number flow, velocity-stress functions in, 7.35  
 Low Reynolds number flow in spherical coordinates, 7.33  
 Low Reynolds number flows, film Reference 6, Appendix C, 7.32  
 Lumped parameter forces, 3.4  
 Lumped parameters, electroquasistatic, 2.19  
 Lumped parameters, magnetoquasistatic, 2.20

Mach number, 1.5, 9.1  
 Mach number, channel flow, 9.43  
 Mach number, streaming membrane, 11.29  
 Magneplane, 6.24  
 Magnetic diffusion, 6.1  
 Magnetic diffusion, boundary layer and, 6.22

Magnetic diffusion, conducting sheet and, 6.4, 6.6  
 Magnetic diffusion, deep conductor, 6.15  
 Magnetic diffusion, instantaneous, 8.50  
 Magnetic diffusion, laws of, 6.1  
 Magnetic diffusion, temporal modes of, 6.28  
 Magnetic diffusion, thick conductor translation and rotation, 6.12  
 Magnetic diffusion, vector potential and, 6.13  
 Magnetic diffusion equation, normalized, 6.3  
 Magnetic diffusion time, 1.5, 2.3, 6.3, 9.25  
 Magnetic diffusion time, Alfvén waves and the, 8.17  
 Magnetic diffusion transfer relations, 6.12  
 Magnetic drag, boundary layer and, 6.23  
 Magnetic field intensity, 2.1  
 Magnetic flight, 6.24  
 Magnetic flux, 2.1  
 Magnetic flux density, 2.1  
 Magnetic flux density, definition of, 2.1, 2.2  
 Magnetic flux density, limits on, 4.53  
 Magnetic Hartmann number, 1.5  
 Magnetic lift, boundary layer and, 6.23  
 Magnetic propulsion, rail, 6.24  
 Magnetic Reynolds number, 1.5, 6.3, 9.12  
 Magnetic Reynolds number, free surface flow with low, 9.38  
 Magnetic Reynolds number, MHD flow and, 9.48  
 Magnetic saturation, magnetization force density with, 8.6  
 Magnetic-viscous Prandtl number, 1.5  
 Magnetization continua, compressible, 8.27  
 Magnetization continua, instability of bulk, 8.27  
 Magnetization density, 2.1, 2.13  
 Magnetization dilatational waves, 8.27  
 Magnetization force density, static equilibrium with, 8.6  
 Magnetization hysteresis motor, 6.30  
 Magnetization of moving media, 2.13  
 Magnetization surface instability, 8.33  
 Magnetization surface interaction dispersion equation, 8.35  
 Magneto-acoustic velocity, 8.26  
 Magneto-acoustic waves, linear, 8.25  
 Magneto-acoustic waves, non-linear, 11.21  
 Magnetohydrodynamic compressible flow, 9.45, 9.61  
 Magnetohydrodynamic energy conversion, 9.41, 9.45  
 Magnetohydrodynamic flow development, 9.28  
 Magnetohydrodynamic induction pump, 9.11  
 Magnetohydrodynamics, film Reference 7, Appendix C, 8.20, 9.3  
 Magneto-inertial time, Alfvén wave and the, 8.18  
 Magnetoquasistatic jump conditions, 2.18, 2.19  
 Magnetoquasistatics, 2.3  
 Magneto-viscous time, 1.5  
 Magnetron flow, 11.3  
 Mass conservation, fluid, 7.1  
 Mass conservation, free surface quasi-one-dimensional, 9.36  
 Mass conservation, incompressible, 7.2  
 Mass conservation jump condition, 7.8  
 Mass density, numerical values of, 7.14  
 Mass density, numerical values of fluid, 7.19  
 Mass density, solids, 7.46  
 Mass density, surface, 10.13  
 Maxwell's capacitor, bipolar model for, 5.35  
 Maxwell's equations, Chu formulation of, 2.1  
 Mechanical kinematics, 2.1  
 Mercury drop electrocapillary migration, 10.27  
 Mercury-electrolyte double layer, 10.28  
 Method of characteristics, 11.13  
 Method of characteristics, first order systems, 5.5, 5.7, 5.17, 5.26, 5.33, 5.38  
 Method of characteristics, higher order systems, 11.21  
 MHD (see magnetohydrodynamic)  
 Microwave generator, magnetron, 11.3, 11.5  
 Migration, bipolar, 5.26  
 Migration, diffusion relative to, 5.3  
 Migration, electrochemically induced, 10.32  
 Migration, imposed field and flow, 5.5, 5.7, 5.9  
 Migration time, 5.34  
 Migration time, imposed field flow approximation, 9.32  
 Migration time, particle, 1.5  
 Migration with convection, 5.6  
 Migration with convection, quasi-stationary, 5.7  
 Mixing, electrically induced, 8.24  
 Mobility, diffusion coefficient relative to, 5.3  
 Mobility, electrocapillary, 10.40  
 Mobility, macroscopic particle, 5.3, 5.4  
 Mobility in gases, ion, 5.4  
 Mobility in liquids, ion, 5.4  
 Models, charge conserving, 8.1  
 Models, electromechanical, 8.1  
 Models, flux conserving, 8.1, 8.2  
 Models, instantaneous charge relaxation, 8.1, 8.2  
 Models, instantaneous magnetic diffusion, 8.1  
 Molecular diffusion, 10.1, 10.19

Molecular diffusion, neutral particle, 10.2  
Molecular diffusion time, 1.5, 10.3, 10.19  
Molecular Peclet number, 1.5  
Molecular-viscous Prandtl number, 1.5  
Momentum conservation, fluid, 7.2  
Momentum conservation jump condition, 7.9  
Motor operation, d-c machine, 4.39  
Moving media, Ohm's law and, 6.1  
MQS (see magnetoquasistatic)

Navier-Stokes equation, 7.24  
Normal vector, surface deformation related to, 7.7  
Normal vector, surface geometry and, 7.3  
Normalization, convention for equations, 2.3  
Normalization, convention for symbols, 2.2  
Nozzle, compressible flow, 9.44  
Numerical integration by method of characteristics, 5.30, 5.36, 11.30, 11.32  
Numerical solution, superposition integral approach to, 8.14

Ohmic conduction transfer relations, 5.44  
Ohmic conduction with convection, 5.42  
Ohmic conductor, constitutive law for moving, 5.38  
Ohmic conductor, dynamics of, 5.38  
Ohmic limit, bipolar, 5.33  
Ohmic model, hierarchy of characteristic times for, 5.35  
Ohm's law, moving conductor, 3.3  
Ohm's law, moving media and, 6.1  
Orthogonal modes, representation of source distributions, 4.16  
Orthogonality, Helmholtz equation and, 4.16  
Orthogonality, magnetic temporal mode, 6.29  
Orthogonality, principal axes, 7.22  
Overview, text, 2.1  
Overview of energy conversion processes, 4.53

Paint spraying, electrostatic, 8.44  
Paraxial ray equation for electron beam, 11.6  
Particle charging, impact, 5.9  
Particle charging, ion diffusion, 10.19  
Particular solution, Poisson's equation, 4.13, 4.14  
Particular solution, vector Poisson's equation, 4.26  
Peclet number, molecular, 1.5, 10.3  
Peclet number, thermal, 1.5, 10.2, 10.9  
Periodic systems, stress, force and torque in, 4.1  
Permanent polarization motor, 6.30  
Permeability, free space, 2.1  
Permittivity, free space, 2.1  
Phase-plane, electron beam pictured in, 11.27  
Phase velocity, 2.30  
Piecewise homogeneous systems, 8.28  
Pinch, instantaneous magnetic diffusion, 8.50  
Pinch, low magnetic Reynolds number, 8.50  
Pinch, MHD z-theta, 8.40  
Pinch, sheet, 8.72  
Planar sheet conductor, boundary condition for translating conducting, 6.5  
Plasma, cold, 11.10  
Plasma column, stability of, 8.40  
Plasma frequency, 11.10, 11.23  
Plasma stability, z-theta pinch, 8.40  
Poiseuille flow, generalized, 9.6  
Poisson's equation, Green's function for, 4.40  
Poisson's equation, particular solutions of, 4.13, 4.14  
Poisson's equation, scalar, 4.13  
Poisson's equation, transfer relations for vector, 4.26  
Poisson's equation, vector, 2.45  
Poisson's ratio, numerical values of, 7.46  
Polarization, moving media, 2.11  
Polarization charge, conservation of, 2.12  
Polarization charge density, 2.12  
Polarization current density, 2.12, 2.13  
Polarization density, 2.1, 2.12  
Polarization force density, illustration of, 8.5  
Polarization stabilization of Rayleigh-Taylor instability, 8.31  
Polarization surface instability, 8.33  
Polarization surface interaction dispersion equation, 8.35  
Pollution control, 8.44  
Potential, velocity, 7.10  
Potential conserving continua, charged drop, 8.44  
Potential conserving continua, compressible, 8.25  
Potential conserving continua, homogeneous anisotropic bulk, 8.20  
Potential conserving continua, static equilibrium of, 8.11  
Potential well, magnetron, 11.4  
Power conversion, electromagnetic-to-internal, 7.39  
Power dissipation, vector potential magnetic field intensity evaluation of, 6.20  
Power flow, electroquasistatic, 2.24  
Power flow, magnetoquasistatic, 2.28  
Power flow density, magnetoquasistatic, 2.29  
Power flux density, electroquasistatic, 2.25  
Prandtl number, 10.13  
Prandtl number, magnetic-viscous, 1.5  
Prandtl number, molecular-viscous, 1.5  
Prandtl number, numerical values of molecular, 10.4

Prandtl number, thermal magnetic, 10.13  
 Prandtl number, thermal viscous, 1.5  
 Precipitator, electrostatic, 5.9  
 Precipitator, space-charge, 5.20  
 Pressure, force density and, 7.3  
 Pressure, irrotational flow and, 7.10  
 Pressure, stress tensor and, 7.3  
 Pressure fields and fluid acceleration,  
   film Reference 8, Appendix C, 7.10  
 Pressure in inviscid fluid, 7.3  
 Pressure-velocity transfer rela-  
   tions for inviscid fluid, 7.11  
 Principal axes of tensor, 7.22  
 Principal coordinate relations,  
   stress-strain-rate, 7.23  
 Principal mode, acoustic wave-  
   guide, 7.18  
 Principle of virtual power, 3.21  
 Pump, d-c electric ohmic, 5.43  
 Pump, Hartmann electric, 9.33  
 Pump, Hartmann magnetic, 9.28  
 Pump, inviscid magnetohydro-  
   dynamic, 9.4  
 Pump, ion drag, 9.33  
 Pump, traveling-wave surface MHD  
   and EHD, 9.10  
 Pumping, electroquasistatic  
   backward, 5.51  
 Pumping, electroquasistatic  
   thermally induced, 10.8  
  
 Quasi-one-dimensional model,  
   boundary layer as a, 9.16  
 Quasi-one-dimensional model,  
   compressible flow, 9.41  
 Quasi-one-dimensional model,  
   electrodynamic generator,  
   9.48, 9.62  
 Quasi-one-dimensional model,  
   electrokinetic, 10.23  
 Quasi-one-dimensional model,  
   free surface, 9.35, 9.37, 9.60  
 Quasi-one-dimensional models, 4.41  
 Quasi-one-dimensional models,  
   streaming, 11.28, 11.32  
 Quasistatic, compressible, 7.42  
 Quasistatic integral laws, 2.10  
 Quasistatic laws, electromag-  
   netic, 2.2  
 Quasistatic laws, electromagnetic  
   differential, 2.5  
 Quasistatic limit, fluid compres-  
   sible (CQS), 7.16  
 Quasistatics, electromagnetic waves  
   and, 2.3, 2.47  
 Quasistatics, fluid mechanics, 7.41  
 Quasistatics, inertial, 7.42  
 Quasistatics, instantaneous charge  
   relaxation, 4.51  
 Quasistatics, time-rate expansion  
   and, 2.2  
  
 Radii of curvature, double layer  
   surface force density and, 3.20  
 Radii of curvature, interfacial, 7.5  
 Rayleigh number, 10.13, 10.17  
 Rayleigh number, magnetic, 10.13, 10.17  
 Rayleigh-Plateau instability, 8.53  
  
 Rayleigh-Taylor instability, 8.30  
 Rayleigh-Taylor instability, polarization  
   stabilization of, 8.38  
 Rayleigh-Taylor instability in smoothly in-  
   homogeneous systems, 8.57  
 Rayleigh waves, 7.48  
 Rayleigh's limit of charge on a drop, 8.44  
 Reciprocity, rotating machine model  
   and, 4.12  
 Reciprocity and energy conservation in  
   smoothly inhomogeneous systems, 8.60  
 Reciprocity conditions, inductance matrix  
   and the, 4.26  
 Reciprocity relations, lumped parameter  
   electroquasistatic, 3.5  
 Reciprocity relations, lumped parameter  
   magnetoquasistatic, 3.6  
 Recombination, bipolar, 5.26, 5.33  
 Recombination, charge, 5.2  
 Red Sea, Moses' parting of, 8.1  
 Reentrant flows, turn-on transient, 9.13  
 Reflection coefficient, acoustic, 7.18  
 Residue theorem, Cauchy, 5.66  
 Resistive wall electron beam amplifica-  
   tion, 11.68  
 Reynolds number, 1.5, 7.27  
 Reynolds number, boundary layer and, 9.16  
 Rotating incompressible inviscid  
   fluid, 7.45  
 Rotational flow, magnetohydrodynamic, 9.3  
 Rotor model, MHD thermal convection,  
   10.10, 10.37  
 Rotor model, natural convection, 10.10,  
   10.37  
 Rotor model, single-phase induction  
   machine, 6.10, 6.37  
 Rotor model, two-phase induction  
   machine, 6.8  
 Rotor model, Von Quincke's, 5.49, 5.75  
  
 Salient pole machines, 4.3, 4.5  
 Salient pole machines, force from  
   stress tensor in, 4.6  
 Salt in solvent, 5.27  
 Saturation charge (see also critical  
   charge), 5.10  
 Scrubbers, charged drop, 8.44  
 Seal, magnetic fluid, 8.2  
 Sedimentation potential, 10.25  
 Self-precipitation, 5.17  
 Shear modulus, numerical values of, 7.46  
 Shear stress, electric bulk instability  
   and, 8.20  
 Shear stress, propagation of mag-  
   netic, 8.16  
 Shear waves in solids, 7.48  
 Sheets, boundary conditions for thin  
   conducting, 6.4  
 Shell, heat balance in rotating, 10.11  
 Shells, boundary conditions for thin  
   conducting, 6.4  
 Shock, compressible gas-dynamic, 9.45  
 Shock formation, 11.18  
 Similarity parameter, 6.24  
 Similarity solution, boundary  
   layer, 9.18, 9.20  
 Similarity solution, linear dif-  
   fusion, 6.24

Simple waves, method of characteristics and, 11.18

Skin depth, magnetic, 6.3

Skin depth, molecular, 10.3

Skin depth, moving frame of reference, 6.16

Skin depth, numerical values of magnetic, 6.4

Skin depth, thermal, 10.2, 10.3

Skin depth, viscous, 7.28, 9.16

Skin effect, 2.48

Skin effect, magnetic levitation and, 8.3

Skin effect, moving conductor, 6.16

Skin effect, transfer relations for magnetic, 6.21

Skin-effect induced cellular convection, 9.22

Skin-effect model, magnetic, 6.20

Skin-effect model, stress in magnetic, 6.20, 6.25

Smoothly inhomogeneous systems, 8.57

Solenoidal fields, representation of, 2.42

Space-average theorem, force and torque from, 4.2, 4.4, 4.5, 4.7, 4.10, 4.19, 4.24, 4.30, 4.31, 4.36, 4.47

Space-charge dynamics, particles in vacuum, 11.10

Space-charge dynamics, smoothly inhomogeneous fluid, 5.17, 8.59

Space-charge dynamics, unipolar migration, 5.17

Space-rate expansion, 1.4, 4.41

Space-rate expansion, boundary layer and, 9.16

Space-rate expansion, free surface flow, 9.60

Spatial modes, 5.61

Spatial modes, acoustic wave, 7.12

Spatial modes, electron beam and cold plasma, 11.11

Spatial modes, Fourier transform and, 5.66

Spatial modes, gravity-capillary, 8.31

Spatial modes, internal charge conserving, 8.66

Spatial modes, internal mass conserving, 8.66

Spatial modes, moving charged thin sheet, 5.62

Spatial modes, numerical solution for, 5.65

Spatial transients, sinusoidal steady state, 5.61

Specific entropy, ideal gas, 7.40

Specific heat, constant pressure, 7.38

Specific heat, constant volume, 7.37

Specific heat, numerical values, 10.2

Specific heats, ratio of, 7.40

Specific volume, 7.40

Specific volume of fluid, 7.37

Spherical shell, boundary condition for rotating conducting, 6.5

Stability, synchronous machine, 4.3

State equation, isentropic, 7.40

State space integration, energy function, 3.5, 4.12

Static equilibria, capillary, 7.10

Static equilibria, charge conserving, 8.8

Static equilibria, conditions for, 8.1

Static equilibria, constant potential, 8.2

Static equilibria, examples of, 8.2, 8.4, 8.6, 8.8, 8.10, 8.11

Static equilibria, flux conserving, 8.2

Static equilibria, force density conditions for, 8.2

Static equilibria, magnetization, 8.2, 8.4, 8.6

Static equilibria, numerical solution of, 8.14

Static equilibria, polarization, 8.2, 8.4, 8.31

Static equilibria, stability of, 8.9

Static equilibria, surface force density conditions for, 8.3

Static equilibria, uniform current, 8.8

Static equilibria, viscous fluid perturbations from, 7.27

Stokes's drag, 7.36

Stokes's integral theorem, 2.9

Strain-displacement relations, 7.47

Strain rate, normal, 7.20

Strain rate, shear, 7.20

Strain rate, viscous stress and 7.18

Strain-rate principal axes same as for stress, 7.22

Strain rate proved a tensor, 7.32

Strain-rate tensor, 7.20

Strain-stress relations, 7.47

Stratified media, smoothly, 8.57

Stream functions, 2.42

Stream functions, convective migration, 5.6

Stream functions, polar, 2.43, 5.6

Stream functions, spherical, 2.43, 5.6

Streaming interactions, 11.1

Streaming potential, 10.25

Streaming systems, single-stream prototype models, 11.28

Streaming systems, two-stream prototype models, 11.32

Streamlines unaltered by irrotational force density, 9.5

Stress-energy conversion relations, 4.53

Stress-strain-rate relation for isotropic fluid, 7.24

Stress-strain-rate relations, physical motivation for, 7.19

Stress-strain relations, 7.47

Stress-strain relations, general linear, 7.21, 7.41

Stress tensor, components defined, 3.16

Stress tensor, divergence of, 3.15

Stress tensor, force density related to, 3.15

Stress tensor, force found from, 4.1

Stress tensor, force in salient pole machine found from, 4.48

Stress tensor, physical significance of, 3.16

Stress tensor, torque found from, 4.1

Stress tensors, 3.15

- Stress tensors, electromagnetic, 3.1
- Stress tensors, electromechanical, 3.17
- Stress tensors, summary of electromagnetic, 3.18
- Subcritical free surface flow, 9.39
- Subsystems, electrical, kinetic and gravitational, 8.60
- Superconducting machine, power output of, 4.54
- Supercritical free surface flow, 9.39
- Supercritical waves, 11.40
- Superposition integral field solution, 4.40
- Surface acoustic waves, 7.48
- Surface charge density, free, 2.1, 2.15
- Surface charge density, polarization, 2.15
- Surface coupling, shear-stress, 8.54
- Surface current density, 2.1
- Surface dilatational modes of charge monolayer, 8.56
- Surface double layer density, 2.16
- Surface energy conservation, 7.5
- Surface force density, 3.19
- Surface force density, double layer, 10.28
- Surface force density, interfacial curvature and, 7.5
- Surface force density, surface tension, 7.4
- Surface force density, Young and Laplace, 7.5
- Surface heat transfer coefficient, 10.11
- Surface shaping, magnetic, 8.11
- Surface tension, 7.4
- Surface tension, clean interface and, 7.4
- Surface tension, energy constitutive law for, 7.4
- Surface tension, nonlinear static equilibrium with, 8.13
- Surface tension, numerical values of, 7.4
- Surface tension in fluid mechanics, film Reference 9, Appendix C, 7.6
- Surface tension surface force density, deformation related to, 7.6, 7.7
- Surface tension, voltage dependence of Hg-KNO<sub>3</sub> interface, 10.30
- Surface torque density, 3.17
- Synchronous alternator, power output of 4.54
- Synchronous interactions, conditions for, 4.4
- Synchronous machine, classification of, 4.2
- Synchronous machine, exposed winding, 4.28
- Synchronous machine, hysteresis, 6.30
- Synchronous machine, model for, 4.28
- Synchronous machine, model for smooth air-gap, 4.21
- Synchronous machine, permanent magnet, 4.3
- Synchronous machine, permanent polarization, 4.8
- Synchronous machine, salient pole, 4.3
- Synchronous machine, superconducting, 4.28
- Synchronous machine, variable capacitance, 4.42, 4.44
- Tachometer, drag-cup, 6.11
- Tachometer, electroquasi-static, 5.45
- Tachometer, magnetic induction, 6.6, 6.36
- Taylor pump, 9.9
- Taylor wavelength, 8.30
- Temporal flow development, imposed surface and volume forces and, 9.13
- Temporal mode orthogonality, 6.29
- Temporal modes, charge relaxation, 5.54
- Temporal modes, conducting fluid in uniform field, 8.19
- Temporal modes, constant potential continua, 8.23
- Temporal modes, eigenvalues of, 6.28
- Temporal modes, electron beam and cold plasma, 11.11
- Temporal modes, field-gradient coupled interfacial, 8.39
- Temporal modes, gravity-capillary, 8.30
- Temporal modes, hydromagnetic Bénard, 10.18
- Temporal modes, internal charge conserving, 8.66
- Temporal modes, magnetic diffusion thick conductor, 6.27
- Temporal modes, magnetic diffusion thin sheet, 6.26
- Temporal transient, Hartmann flow established by, 9.30
- Temporal transient, stress constrained flow, 9.14
- Temporal transient, velocity constrained flow, 9.14
- Tensor, strain-rate, 7.20
- Tensor, transformation of, 3.17
- Tensor integral theorem of Gauss, 3.15
- Terminal relations, electric d-c machine, 4.52
- Terminal relations, electrical rotating machine, 4.11, 4.21, 4.31
- Terminal relations, Van de Graaff machine, 4.52
- Thermal conductivity, definition of, 10.1
- Thermal conductivity, numerical values of, 10.2
- Thermal convection in a magnetic field, 10.10, 10.15
- Thermal diffusion, 10.1, 10.5
- Thermal diffusion time, 1.5, 10.2
- Thermal-electromechanical energy conversion, 9.53
- Thermal energy conversion cycle, 9.53, 9.63

Thermal diffusivity, definition of, 10.1  
 Thermal energy conversion efficiency, 9.55  
 Thermal expansion, coefficient of, 10.15  
 Thermal expansion, numerical values of coefficient of, 10.16  
 Thermal generation time, 5.34  
 Thermal Peclet number, 1.5  
 Thermal-viscous Prandtl number, 1.5  
 Thermal voltage, 5.3, 10.22  
 Thermodynamics, electroquasistatic, 2.22  
 Thermodynamics, equilibrium, 7.38  
 Thermodynamics, lumped parameter, of highly compressible fluid, 7.36  
 Thermodynamics, magnetoquasistatic subsystem, 2.26  
 Thermonuclear experiments, 8.40  
 Theta pinch, 8.43  
 Three-phase machine, 4.21  
 Thin sheet, charge relaxation on, 5.45, 5.55  
 Thin-sheet limit, magnetic diffusion in the, 6.17  
 Thunderstorm electrification, 5.10  
 Time-average force, 5.60  
 Time-average force, spatial transient, 5.67  
 Time-average torque, 5.60  
 Time-rate expansion, 1.4, 2.2  
 Time-rate parameter, 2.4  
 Time-rate parameter, acoustic, 7.42  
 Torque, surface torque density and, 3.17  
 Torque, time-average, 5.60  
 Torque-speed characteristic, hysteresis motor, 6.33  
 Torque-speed characteristic, induction motor, 6.9, 6.10, 6.16  
 Traction, 3.16  
 Transfer relations, 1.6, 2.46  
 Transfer relations, anisotropic ohmic conductor, 5.74  
 Transfer relations, cold plasma, 11.10  
 Transfer relations, conducting fluid in magnetic field, 8.18  
 Transfer relations, constrained charge, 4.13  
 Transfer relations, constrained current, 4.26  
 Transfer relations, electromagnetic planar, 2.52  
 Transfer relations, electron beam, 11.10  
 Transfer relations, electroquasistatic, 2.16  
 Transfer relations, electroquasistatic inhomogeneous dielectric planar, 2.53  
 Transfer relations, flux-potential, 2.16  
 Transfer relations, flux-potential cylindrical, 2.35  
 Transfer relations, flux-potential planar layer, 2.33  
 Transfer relations, flux-potential spherical, 2.38  
 Transfer relations, half-space of viscous fluid, 7.31  
 Transfer relations, implications of energy conservation for quasistatic, 2.40  
 Transfer relations, imposed force density fluid, 7.49  
 Transfer relations, incompressible elastic solid, 7.48  
 Transfer relations, incompressible inertialess solid, 7.49  
 Transfer relations, infinite half-space elastic material, 7.48  
 Transfer relations, inviscid fluid pressure-velocity, 7.11, 7.12  
 Transfer relations, Laplacian fields, 2.32  
 Transfer relations, low magnetic Reynolds number, 8.52  
 Transfer relations, low Reynolds number flow, 7.32, 7.33, 7.36  
 Transfer relations, magnetic diffusion, 6.12  
 Transfer relations, magnetoquasistatic, 2.16  
 Transfer relations, method of denoting variables, 2.46  
 Transfer relations, ohmic conduction, 5.44  
 Transfer relations, rotating incompressible inviscid fluid, 7.44  
 Transfer relations, smoothly inhomogeneous system, 8.57  
 Transfer relations, thermal diffusion, 10.5  
 Transfer relations, thermal diffusion with source, 10.6  
 Transfer relations, uniformly charged fluid layer, 8.46  
 Transfer relations, vector potential Laplacian field, 2.42  
 Transfer relations, viscous diffusion, 7.28  
 Transfer relations, viscous layer of arbitrary thickness, 7.30  
 Transfer relations, volume source, 4.13  
 Transfer relations, weak compressibility, 7.13  
 Transformation, Galilean, 2.7  
 Transformations, electroquasistatic, 2.9  
 Transformations, magnetoquasistatic, 2.9  
 Transformations between frames of reference, 2.7  
 Transverse coordinate, 1.6, 4.53, 9.35  
 Traveling space-charge wave, kinematics of, 4.17  
 Traveling-wave amplifier, 4.18  
 Traveling-wave induced convection, 5.51, 5.53  
 Traveling-wave-induced convection, surface, 9.10  
 Turn-on transient, electron beam in gap, 11.26

Turn-on transient, reentrant flow, 9.13  
 Two-phase surface currents, 6.6  
 Unipolar space-charge dynamics, 5.17  
 Units, electromagnetic, 2.1  
 Van de Graaff generator, 4.49  
 Van de Graaff machine, d-c machines and 4.53  
 Van de Graaff machine, energy conversion in, 4.53  
 Variable capacitance machine, 4.42, 4.44  
 Variable capacitance machine, output power of, 4.55  
 Vector, transformation of, 3.16  
 Vector potential, 2.42  
 Vector potential, magnetic diffusion and, 6.12, 6.13  
 Vector potential, velocity, 7.26  
 Velocity potential, 7.10  
 Virtual power, 3.21  
 Virtual work, 3.21  
 Viscometer, 7.18  
 Viscosity, absolute, 7.19  
 Viscosity, kinematic, 7.19  
 Viscosity, numerical values of, 7.19  
 Viscosity, unit conversion for, 7.19  
 Viscous diffusion, 7.26  
 Viscous diffusion, Alfvén waves and, 8.18  
 Viscous diffusion, boundary layer, 9.16  
 Viscous diffusion time, 1.5, 7.27, 7.42, 9.25, 9.32  
 Viscous diffusion time, Alfvén waves and, 8.17  
 Viscous diffusion transfer relations, 7.28  
 Viscous dissipation density, 7.25  
 Viscous force density, 7.24  
 Viscous relaxation time, 1.5, 7.42  
 Viscous skin depth, 7.28  
 Viscous skin depth, numerical values of, 7.29  
 Viscous stress, strain rate and, 7.18  
 Voltage, 2.1  
 Voltage, lumped parameter variable of, 2.20  
 Voltage-flux relation, lumped parameter, 2.22  
 Von Quincke's rotor, 5.49  
 Von Quincke's rotor, equations of motion for, 5.75  
 Vorticity, convective diffusion of, 7.26  
 Vorticity, fluid, 7.9  
 Vorticity, generation of, 7.26  
 Vorticity, surface of fixed identity conservation of, 7.10

Wall-less pipes, 9.35, 9.38  
 Wavelength, Taylor, 8.30  
 Waves, acoustic, 7.13  
 Waves, Alfvén, 9.8  
 Waves, amplifying, 11.31, 11.37, 11.41, 11.42  
 Waves, capillary-gravity surface, 8.28  
 Waves, charge conserving, 8.46  
 Waves, charge monolayer surface, 8.54, 8.75  
 Waves, complex, 11.37  
 Waves, current conserving, 8.71  
 Waves, elastic isotropic solid, 7.48  
 Waves, electro-acoustic, 8.25  
 Waves, evanescent, 11.31, 11.37, 11.41, 11.42  
 Waves, field-coupled surface, 8.33  
 Waves, imposed gradient polarization surface, 8.38  
 Waves, internal, 8.62  
 Waves, internal charge and mass conserving, 8.62  
 Waves, internal magnetization, 8.77  
 Waves, magnetic diffusion, 6.17  
 Waves, magneto-acoustic, 8.25, 11.21  
 Waves, magnetization dilatational, 8.27  
 Waves, prototype single-stream, 11.27  
 Waves, Rayleigh surface, 7.48  
 Waves, shock, 9.45, 11.19  
 Waves, space-charge gravity, 8.62  
 Waves, supercritical, 11.40  
 Waves, surface Alfvén, 8.40, 8.72  
 Waves, thermal diffusion, 10.5  
 Waves, viscous diffusion, 7.26, 8.16, 9.13  
 Waves in fluids, film Reference 10, Appendix C, 11.16  
 Weak-gradient imposed field model, 8.59, 8.64  
 Wetting, surface tension and liquid-solid, 7.6  
 Whipple and Chalmers model for particle charging, 5.9  
 Windings, two-phase, 6.6  
 Young and Laplace surface force density, 7.5  
 Zero-gravity liquid orientation, 8.2  
 Zeta potential, 10.22, 10.24  
 z pinch, MHD, 8.42  
 z-theta pinch, feedback stabilization of, 8.44