

## COMPUTATION OF THE DISCRETE FOURIER TRANSFORM - PART 1

## Solutions 18.1

The flow-graph of Fig. 9.3 of the text is based on the decomposition of  $X(k)$  in the form of equation 9.14 of text. For  $N=16$ , the corresponding flow-graph expressing  $X(k)$  as a combination of two eight-point DFT's is shown in Figure S18.1-1 below.

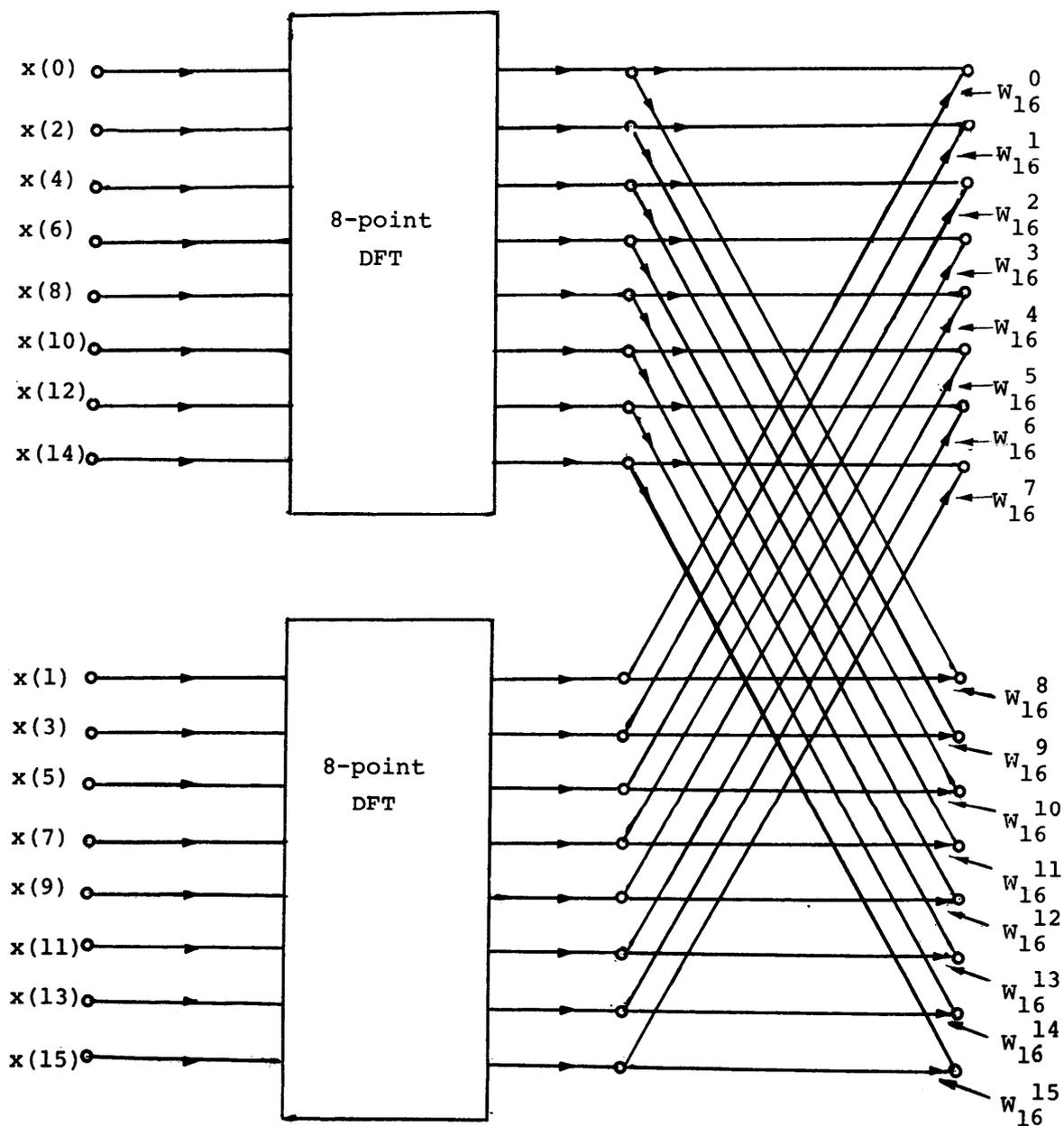


Figure S18.1-1

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### Solution 18.2

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From the expression for the inverse DFT it follows that

$$N x^*(n) = \sum_{k=0}^{N-1} X^*(k) e^{-j\left(\frac{2\pi}{N}\right)kn}$$

Thus by using as the input to the DFT program the complex conjugate of  $X(k)$ , the output sequence will be  $N$  times the complex conjugate of  $x(n)$ .

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### Solution 18.3

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(a)  $\frac{N}{2}$

(b) In proceeding from array  $(m-1)$  to array  $m$  we are combining  $2^{(m-1)}$  point DFT's to form  $2^m$  point DFT's. Thus the coefficients are successive powers of  $W_M$  where  $M = 2^m$ . Thus these coefficients are  $W_M^k$  where  $k = 0, 1, 2, \dots, \left(\frac{M}{2} - 1\right)$  or, since

$$W_M = (W_N)^{N/M}$$

The powers of  $W_N$  involved in computing the  $m^{\text{th}}$  array from the  $(m-1)$ st array are

$$W_N^{Nk/M} \quad k = 0, 1, 2, \dots, (M/2 - 1)$$

(c)  $2^{(m-1)}$

(d)  $2^m$  for  $1 \leq m \leq (\log_2 N) - 1$ . For the last array ( $m = \log_2 N$ ) there are no two butterflies utilizing the same coefficients.

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### Solution 18.4

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With  $g(n) = x_1(n) + j x_2(n)$ ,

$$G(k) = X_1(k) + j X_2(k)$$

With  $X_1(k)$  and  $X_2(k)$  expressed in terms of their real and imaginary parts,

$$X_1(k) = X_{1R}(k) + j X_{1I}(k)$$

$$X_2(k) = X_{2R}(k) + j X_{2I}(k)$$

$G(k)$  can be written as

$$G(k) = [X_{1R}(k) - X_{2I}(k)] + j [X_{2R}(k) + X_{1I}(k)]$$

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Now, since  $x_1(n)$  and  $x_2(n)$  are real,  $X_{1R}(k)$  and  $X_{2R}(k)$  are even and  $X_{1I}(k)$  and  $X_{2I}(k)$  are odd. Thus,

$$G_{ER}(k) = X_{1R}(k)$$

$$G_{OR}(k) = -X_{2I}(k)$$

$$G_{EI}(k) = X_{2R}(k)$$

$$G_{OI}(k) = X_{1I}(k)$$

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