

DIGITAL BUTTERWORTH FILTERS

1. Lesson 16 - 48 minutes

Design Examples

Analog Butterworth filter

$$|H_a(j\Omega)|^2 = \frac{1}{1 + (\frac{j\Omega}{\Omega_c})^{2N}}$$

$N$  increases

$$H_a(s)H_a(-s) = \frac{1}{1 + (\frac{s}{j\Omega_c})^{2N}}$$

poles at  $S_p = (-1)^k (j\Omega_c)$

$$S_p = e^{j\frac{\pi(2k+1)}{2N}} e^{j\frac{\pi}{2}} \Omega_c$$

Butterworth Circle (radius  $\Omega_c$ )

S-plane  $N=3$

Digital Filter Specs

$(1-\delta_p) \geq -1$  db  
 $\delta_s \leq -15$  db

Impulse Invariant Design

$$H(z) = \sum_{k=-\infty}^{\infty} h_a\left(\frac{z}{T} + j\frac{2\pi k}{T}\right)$$

Neglect Aliasing  $\Omega = \frac{\omega}{T}$

a.

Bilinear Transform Design ( $T=1$ )

$$\omega = 2 \arctan\left(\frac{\Omega}{2}\right)$$

Choose  $N=6$

$$1 + \left(\frac{j\frac{2\pi}{T}}{j\Omega_c}\right)^{2N} = 10^{-1}$$

$$\Omega_c T = 7032$$

$$H_a(s)H_a(-s) = \frac{T^2}{1 + (\frac{sT}{j7032})^{2N}}$$

$$\frac{T=1}{h_a(t)} \longleftrightarrow H_a(s)$$

$$h_a\left(\frac{t}{T}\right) \longleftrightarrow T H_a(sT)$$

$$h(n) = h_a(nT) = h_a(n)$$

Specifications:

$$20 \log_{10} |H_a(j2 \tan .1\pi)| \geq -1$$

$$20 \log_{10} |H_a(j2 \tan .15\pi)| \leq -15$$

b.

$$1 + \left[\frac{j2 \tan(.1\pi)}{j\Omega_c}\right]^{2N} = 10^{-1} \quad \textcircled{1}$$

$$1 + \left[\frac{j2 \tan(.15\pi)}{j\Omega_c}\right]^{2N} = 10^{-1.5} \quad \textcircled{2}$$

$$\Rightarrow N = 5.30466$$

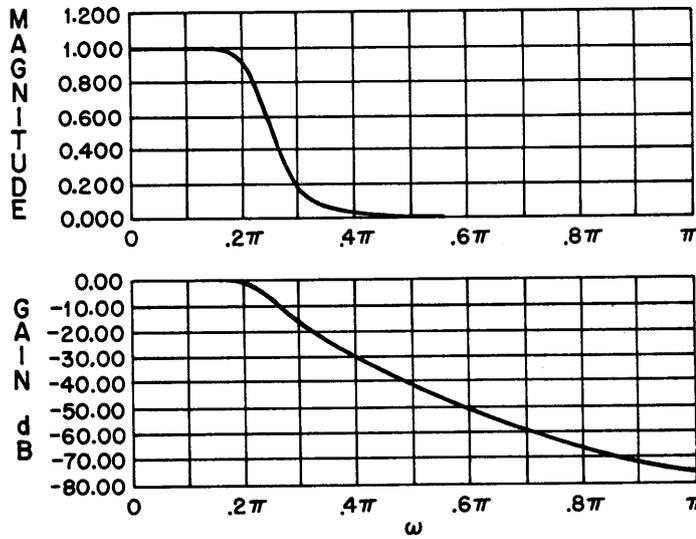
choose  $N=6$   $H_a(s)H_a(-s)$

Meet stopband  
exceed passband

$$1 + \left[\frac{j2 \tan(.15\pi)}{j\Omega_c}\right]^{2 \times 6} = 10^{-1.5}$$

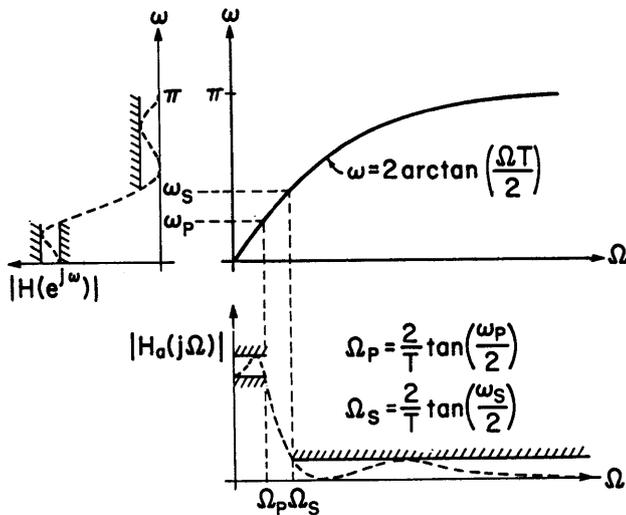
$$\Rightarrow \Omega_c = .76622$$

c.



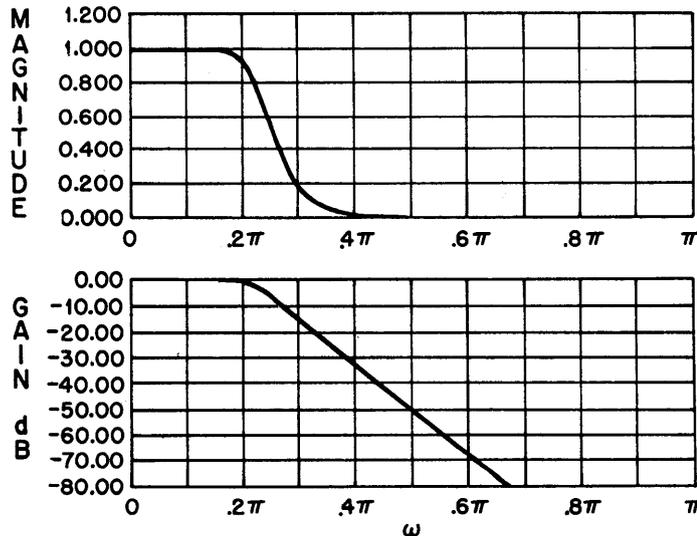
Frequency response of sixth-order digital Butterworth filter obtained by using impulse invariance.

d.



The bilinear transformation.

e.



Frequency response of sixth-order digital Butterworth filter obtained by using the bilinear transformation.

f.

---

## 2. Comments

---

In this lecture we discuss a design example utilizing the two techniques of impulse invariance and the bilinear transformation. We concentrate specifically on the design of a digital filter which is obtained from an analog Butterworth filter. The steps involved are to (1) transform the specifications on the digital filter to the appropriate specifications on the analog filter (2) design the analog filter based on these specifications (3) transform the analog filter to the desired digital filter. The specific example sketched out in the lecture is carried through in more detail in the text.

In both the impulse invariant and bilinear design procedures, a parameter  $T$  appears. This parameter is, in fact, unnecessary, as is illustrated in the lecture. Consequently, it is often convenient to choose this parameter to be unity.

## 3. Reading

---

Text: Appendix B1 (page 845), example 7.3 section 7.1.1 (pg. 411) and example 7.4 section 7.1.2 (page 419). You may also want to read Appendix B.2 in which Chebyshev filters are discussed, and Appendix B.3 in which elliptic filters are discussed.

## 4. Problems

---

### Problem 16.1

Let  $|H(j\Omega)|^2$  denote the squared magnitude function for an analog Butterworth filter of order 5 with a cutoff frequency  $\Omega_c$  of  $2\pi \times 10^3$ . Determine and indicate in the  $s$ -plane the poles of the system function  $H(s)$ . Assume that the system is stable and causal.

### Problem 16.2

We want to design a digital lowpass filter with a passband magnitude characteristic that is constant to within 0.75 dB for frequencies below  $\omega = 0.2613\pi$  and stopband attenuation of at least 20 dB for frequencies between  $\omega = 0.4018\pi$  and  $\pi$ . Determine the poles of the lowest order Butterworth analog transfer function which when mapped to a digital filter using impulse invariance will meet the

---

specifications. Indicate also how you would proceed to obtain the transfer function of the digital filter.

Problem 16.3

With the same specifications as in Problem 16.2 determine the poles of the lowest order Butterworth analog transfer function which when mapped to a digital filter using the bilinear transformation will meet the specifications. Indicate how you would proceed to obtain the transfer function of the digital filter.

Problem 16.4\*

The filter designed in Problem 16.3 is to be converted to a highpass filter using a lowpass to highpass transformation as discussed in section 7.2. In the filter designed in Problem 16.3 the pass band edge occurred at  $\omega = 0.2613\pi$  and the stopband edge at  $\omega = 0.4018\pi$ . The new highpass filter is to have the passband edge at  $\omega = 0.4018\pi$ . Specify how to obtain the transfer function of the highpass filter from the lowpass filter of Problem 16.3, and determine the frequency of the stopband edge.

MIT OpenCourseWare  
<http://ocw.mit.edu>

Resource: Digital Signal Processing  
Prof. Alan V. Oppenheim

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.