We will now consider an operation that is, in some sense, the opposite of merging.

We have a node, and traffic arrives to that node.

And each time that we have an arrival, we flip a coin.

And with probability q, we send that arrival to one particular stream.

And with probability 1 minus q, we send the arrival to another stream.

So we get two streams that are formed by taking the original stream and splitting it into two pieces.

And as an example, these might be arrivals at a department store.

And one stream corresponds to the people who go to the clothes section of the department store, whereas the other stream corresponds to the people that go to all of the other sections of the store.

So let us now make our model a little more precise.

We have a Bernoulli process, which is independent across time.

We also use an independent coin flip to deal with each one of the arrivals.

But we will also make one additional assumption, namely that the Bernoulli process is also independent from the process of coin flips.

With this assumption in place, let us now continue, and let us draw a picture.

We have a Bernoulli process with parameter p, and arrivals get recorded at certain times.

Each time that there is an arrival, we will flip a coin.

And with probability q, the arrival will be sent to that stream.

With probability 1 minus q, the arrival will be sent to the other stream.

So one possible outcome of the experiment might be this one, where these two arrivals were sent to this stream and these two arrivals were sent to the top stream.

And we have these probabilities q and 1 minus q of sending the arrivals to one or the other stream.

What kind of process is this one?

We argue it is a Bernoulli process.

First, we need to check independence.

Here, the argument is more or less the same as in the case when we studied the merging of processes.

For example, if we look at two different slots and we ask, how is the event at that slot and at that slot determined?

Well, what happens in this slot is determined by whether we had an arrival here and what happened to the outcome of the coin flip at that time.

What happens in this slot is determined by whether we had an arrival here and what happened to the coin flip at that time.

Now, the coin flips are independent from the original Bernoulli process.

And for either the coin flips or the Bernoulli process, we have independence across time.

So all of the four random variables involved here that determine what happens in these two slots are independent of each other.

So what happens in this slot is a function of two random variables here, which are independent from the two random variables that determined what happens in that slot.

So what happens in these two slots are independent events.

And this argument goes through more generally when we consider multiple distinct slots.

So this is the argument for the independence of the different slots in this particular process.

And then during each slot, what happens is that we will have an arrival if and only if this process records an arrival, which happens with probability p, and the corresponding coin flip happens to send the arrival in this direction, which happens with probability q.

And so the conclusion is that this process is a Bernoulli process with parameter p times q.

By a similar argument, the other process that we obtain will also be Bernoulli but with probability p times 1 minus q.

And a final question-- are these two processes that we get after the splitting independent of each other?

This is a question that we can answer by reasoning intuitively.

If I tell you that there was an arrival in this slot, what can you infer from this?

Well, it tells me that there was an arrival in the original stream, which was sent here.

But since it was sent in this direction, it means that it was not sent in the other direction.

And so we do not have an arrival in this slot.

Knowing that we have an arrival here means that we do not have an arrival there.

So information about one of the streams gives us information about what happened in the other stream.

And therefore, we do not have independence.

So this is what happens when we split two Bernoulli processes.

And earlier we saw what happens when we merge two independent Bernoulli processes.

These two operations of merging and splitting are quite common in constructing more complex models using Bernoulli processes as the elements of those models.

They are often useful models either in transportation systems, where you have streams of traffic that merge or split, also in models of computer networks or any other kind of queueing system.

And these same operations of merging and splitting will also show up when we study the continuous time analog of the Bernoulli process, namely the Poisson process.