Here is an example of a problem related to the Bernoulli process, which can be tricky, but is actually easy to answer if one makes good use of the fresh-start property.

Here is the setting.

Time is discrete, divided into slots.

We have a server that receives tasks to process.

Tasks received gets processed in the same time slot.

So slots are divided into busy ones-- those are the slots during which a task gets processed.

And idle slots-- these are the slots during which there is no task to be processed.

We assume that the process of job arrivals is described by a Bernoulli process with some known parameter p.

So, during each slot there is probability, p, that a job is present, and different slots are independent of each other.

We're interested in the first busy period of the server.

The first busy period starts at the first slot during which there is a job present.

And the busy period extends until just before the next idle slot.

For an example, it could be the case that the first slot is busy, in which case the busy period starts right here.

And the busy periods, in this example, extends for three time units.

It ends just before the next idle slot.

It could also be the case that the first slot is idle.

In that case, the busy period starts with the first busy slot that shows up.

It's this slot in this example.

And extends until just before the first idle slot that we observe.

So in this example, the busy period extends for four time steps.

What is the length of the first busy period?

Well, the length of the first busy period is a random variable.

So what we mean by this question is, what is the distribution of this random variable?

Here's how we go about it.

The process starts, we wait until a first busy slot appears.

This is a random time, which is actually the first arrival time.

And at that time, by our earlier discussion, the process will start fresh.

Starting from this next the slot here and going on forward, what we have is a Bernoulli process.

And at each slot, there's probability p that it is busy, and probability 1 minus p that it is idle.

Now, what is this slot here?

This is the first idle slot in this Bernoulli process that starts fresh at this particular time.

At each time step there is probability 1 minus p that the slot is idle.

Think now of idle slots as successes.

How long does it take until the first success?

We know that this is a geometric random variable with parameter equal to the probability of success.

Since we're thinking of the idle slot as being a success, the parameter, in this case, is going to be 1 minus p.

So, the length of this blue interval that starts at this slot and extends until the first idle slot has a geometric distribution with parameter 1 minus p.

But now notice that the length of this blue interval is exactly the same as the length of this red interval.

The red interval is just the same as the blue interval, but shifted by 1, but their lengths are the same.

And the length of the red interval is the length of the first busy period.

So we conclude that the first busy period is also a geometric random variable with parameter 1 minus p.