Let us now examine what conditional probabilities are good for.

We have already discussed that they are used to revise a model when we get new information, but there is another way in which they arise.

We can use conditional probabilities to build a multi-stage model of a probabilistic experiment.

We will illustrate this through an example involving the detection of an object up in the sky by a radar.

We will keep our example very simple.

On the other hand, it turns out to have all the basic elements of a real-world model.

So, we are looking up in the sky, and either there's an airplane flying up there or not.

Let us call Event A the event that an airplane is indeed flying up there, and we have two possibilities.

Either Event A occurs, or the complement of A occurs, in which case nothing is flying up there.

At this point, we can also assign some probabilities to these two possibilities.

Let us say that through prior experience, perhaps, or some other knowledge, we know that the probability that something is indeed flying up there is 5% and with probability 95% nothing is flying.

Now, we also have a radar that looks up there, and there are two things that can happen.

Either something registers on the radar screen or nothing registers.

Of course, if it's a good radar, probably Event B will tend to go together with Event A. But it's also possible that the radar will make some mistakes.

And so we have various possibilities.

If there's a plane up there, it's possible that the radar will detect it, in which case Event B will also happen.

But it's also conceivable that the radar will not detect it, in which case we have a so-called miss.

And so a plane is flying up there, but the radar missed it, did not detect it.

Another possibility is that nothing is flying up there, but the radar does detect something, and this is a situation that's called a false alarm.

Finally, there's the possibility that nothing is flying up there, and the radar did not see anything either.

Now, let us focus on this particular situation.

Suppose that Event A has occurred.

So we are living inside this particular universe.

In this universe, there are two possibilities, and we can assign probabilities to these two possibilities.

So let's say that if something is flying up there, our radar will find it with probability 99%, but will also miss it with probability 1%.

What's the meaning of this number, 99%?

Well, this is a probability that applies to a situation where an airplane is up there.

So it is really a conditional probability.

It's the conditional probability that we will detect something, the radar will detect the plane, given that the plane is already flying up there.

And similarly, this 1% can be thought of as the conditional probability that the complement of B occurs, so the radar doesn't see anything, given that there is a plane up in the sky.

We can assign similar probabilities under the other scenario.

If there is no plane, there is a probability that there will be a false alarm, and there is a probability that the radar will not see anything.

Those four numbers here are, in essence, the specs of our radar.

They describe how the radar behaves in a world in which an airplane has been placed in the sky, and some other numbers that describe how the radar behaves in a world where nothing is flying up in the sky.

So we have described various probabilistic properties of our model, but is it a complete model?

Can we calculate anything that we might wish to calculate?

Let us look at this question.

Can we calculate the probability that both A and B occur?

It's this particular scenario here.

How can we calculate it?

Well, let us remember the definition of conditional probabilities.

The conditional probability of an event given another event is the probability of their intersection divided by the probability of the conditioning event.

But this doesn't quite help us because if we try to calculate the numerator, we do not have the value of the probability of A given B. We have the value of the probability of B given A. What can we do?

Well, we notice that we can use this definition of conditional probabilities, but use it in the reverse direction, interchanging the roles of A and B. If we interchange the roles of A and B, our definition leads to the following expression.

The conditional probability of B given A is the probability that both events occur divided by the probability, again, of the conditioning event.

Therefore, the probability that A and B occur is equal to the probability that A occurs times the conditional probability that B occurs given that A occurred.

And in our example, this is 0.05 times the conditional probability that B occurs, which is 0.99.

So we can calculate the probability of this particular event by multiplying probabilities and conditional probabilities along the path in this tree diagram that leads us here.

And we can do the same for any other leaf in this diagram.

So for example, the probability that this happens is going to be the probability of this event times the conditional probability of B complement given that A complement has occurred.

How about a different question?

What is the probability, the total probability, that the radar sees something?

Let us try to identify this event.

The radar can see something under two scenarios.

There's the scenario where there is a plane up in the sky and the radar sees it.

And there's another scenario where nothing is up in the sky, but the radar thinks that it sees something.

So these two possibilities together make up the event B.

And so to calculate the probability of B, we need to add the probabilities of these two events.

For the first event, we already calculated it.

It's 0.05 times 0.90.

For the second possibility, we need to do a similar calculation.

The probability that this occurs is equal to 0.95 times the conditional probability of B occurring under the scenario where A complement has occurred, and this is 0.1.

If we add those two numbers together, the answer turns out to be 0.1445.

Finally, a last question, which is perhaps the most interesting one.

Suppose that the radar registered something.

What is the probability that there is an airplane out there?

How do we do this calculation?

Well, we can start from the definition of the conditional probability of A given B, and note that we already have in our hands both the numerator and the denominator.

So the numerator is this number, 0.05 times 0.99, and the denominator is 0.1445, and we can use our calculators to see that the answer is approximately 0.34.

So there is a 34% probability that an airplane is there given that the radar has seen or thinks that it sees something.

So the numerical value of this answer is somewhat interesting because it's pretty small.

Even though we have a very good radar that tells us the right thing 99% of the time under one scenario and 90% under the other scenario.

Despite that, given that the radar has seen something, this is not really convincing or compelling evidence that there is an airplane up there.

The probability that there's an airplane up there is only 34% in a situation where the radar thinks that it has seen something.

So in the next few segments, we are going to revisit these three calculations and see how they can generalize.

In fact, a large part of what is to happen in the remainder of this class will be elaboration on these three ideas.

They are three types of calculations that will show up over and over, of course, in more complicated forms, but the basic ideas are essentially captured in this simple example.