The coefficients n-choose-k that we calculated in the previous segment are known as the binomial coefficients.

They are intimately related to certain probabilities associated with coin tossing models, the so-called binomial probabilities.

This is going to be our subject.

We consider a coin which we toss n times in a row, independently.

For each one of the tosses of this coin, we assume that there is a certain probability, p, that the result is heads, which of course, implies that the probability of obtaining tails in any particular toss is going to be 1 minus p.

The question we want to address is the following.

We want to calculate the probability that in those n independent coin tosses, we're going to observe exactly k heads.

Let us start working our way towards the solution to this problem by looking first at a simple setting and then move on.

So let us answer this simple question.

What is the probability that we observe this particular sequence?

Of course here we take n equal to six, and we wish to calculate this probability.

Now, because we have assumed that the coin tosses are independent, we can multiply probabilities.

So the probability of this sequence is equal to the probability that the first toss is heads times the probability that the second toss is tails, which is 1 minus p, times the probability that the third toss is tails, which is 1 minus p, times the probability of heads, times the probability of heads, times the probability of heads.

And by collecting terms, this is p to the 4th times 1 minus p to the second power.

More generally, if I give you a particular sequence of heads and tails, as in this example, and I ask you, what is the probability that this particular sequence is observed, then by generalizing from this answer or from the derivation of this answer, you see that you're going to get p to the power number of heads.

And the reason is that each time that there's a head showing up in this sequence, there's a corresponding factor of p in this numerical answer.

And then there are factors associated with tails.

Each tail contributes a factor of 1 minus p.

And so we're going to have here 1 minus p to a power equal to the number of tails.

Now, if I ask you about the probability of a particular sequence and that particular sequence has happened to have exactly k heads, what is the probability of that sequence?

Well, we already calculated what it is.

It is the previous answer, except we use the symbol k instead of just writing out explicitly "number of heads." And the number of tails is the number of tosses minus how many tosses resulted in heads.

Now, we're ready to consider the actual problem that we want to solve, which is calculate the probability of k heads.

The event of obtaining k heads can happen in many different ways.

Any particular k-head sequence makes that event to occur.

Any particular k-head sequence has a probability equal to this expression.

The overall probability of k heads is going to be the probability of any particular k-head sequence, times the number of k-head sequences that we have.

Now, the reason why we can carry out this argument is the fact that any k-head sequence has the same probability.

Otherwise, we wouldn't be able to write down an answer which is just the product of two terms.

But because every k-head sequence has the same probability, to find the overall probability, we take the probability of each one of them and multiply it with the number of how many of these we have.

So to make further progress, now we need to calculate the number of possible k-head sequences.

How many are there?

Well, specifying a k-head sequence is the same as the following.

You think of having n time slots.

These time slots corresponds to the different tosses of your coin.

And to specify a k-head sequence, you need to tell me which ones of these slots happen to contain a head.

You need to tell me k of those slots.

So in other words, what you're doing is you're specifying a subset of the set of these n slots, a subset that has k elements.

You need to choose k of the slots out of the n and tell me that those k slots have heads.

That's the way of specifying a particular k-head sequence.

So what's the number of k-head sequences?

Well, it's the same as the number of ways that you can choose k slots out of the n slots, which is our binomial coefficient, n-choose-k.

Therefore, the answer to our problem is this expression here, times n-choose-k, which is shown up here.

At this point, we can pause and consider a simple question to check your understanding of the binomial probabilities.