

Discrete-time Signals and Systems

Discrete-time Signals and Systems

An Operator Approach

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Preface

This book aims to introduce you to a powerful tool for analyzing and designing systems – whether electronic, mechanical, or thermal.

This book grew out of the ‘Signals and Systems’ course (numbered 6.003) that we have taught on and off to MIT’s Electrical Engineering and Computer Science students.

The traditional signals-and-systems course – for example [17] – emphasizes the analysis of continuous-time systems, in particular analog circuits. However, most engineers will not specialize in analog circuits. Rather, digital technology offers such vast computing power that analog circuits are often designed through digital simulation.

Digital simulation is an inherently discrete-time operation. Furthermore, almost all fundamental ideas of signals and systems can be taught using discrete-time systems. Modularity and multiple representations, for example, aid the design of discrete-time (or continuous-time) systems. Similarly, the ideas for modes, poles, control, and feedback.

Furthermore, by teaching the material in a context not limited to circuits, we emphasize the generality of these tools. Feedback and simulation abound in the natural and engineered world, and we would like our students to be flexible and creative in understanding and designing these systems.

Therefore, we begin our ‘Signals and Systems’ course with discrete-time systems, and give our students this book. A fundamental difference from most discussions of discrete-time systems is the approach using operators. Operators make it possible to avoid the confusing notion of ‘transform’. Instead, the operator expression for a discrete-time system, and the system’s impulse response are two representations for the same system; they are the coordinates of a point as seen from two different coordinate systems. Then a transformation of a system has an active meaning: for example, composing two systems to build a new system.

How to use this book

Aristotle was tutor to the young Alexander of Macedon (later, the Great). As ancient royalty knew, a skilled and knowledgeable tutor is the most effective teacher [3]. A skilled tutor makes few statements and asks many questions, for she knows that questioning, wondering, and discussing promote long-lasting learning. Therefore, questions of two types are interspersed through the book:

questions marked with a ► in the margin: These questions are what a tutor might ask you during a tutorial, and ask you to work out the next steps in an analysis. They are answered in the subsequent text, where you can check your solutions and my analysis.

numbered questions: These problems, marked with a shaded background, are what a tutor might ask you to take home after a tutorial. They give further practice with the tool or ask you to extend an example, use several tools together, or resolve paradoxes.

Try lots of questions of both types!

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