Assignment 3: Damped and Forced Oscillators (Midterm Week)

Preface: This problem set provides practice in understanding damped harmonic oscillator systems, solving forced oscillator equations, and exploring numerical solutions to differential equations.

1. **Equations of Underdamping**
   Using a force of 4 newtons, a damped harmonic oscillator is displaced from equilibrium by 0.2 meters. At $t = 0$ it is released from rest. The resultant displacement of the oscillator, from the equilibrium position, as a function of time, is shown in the figure below. Estimate, as well as you can using the given information, the following quantities:

   (a) The mass of the oscillator
   (b) The quality factor of the oscillator

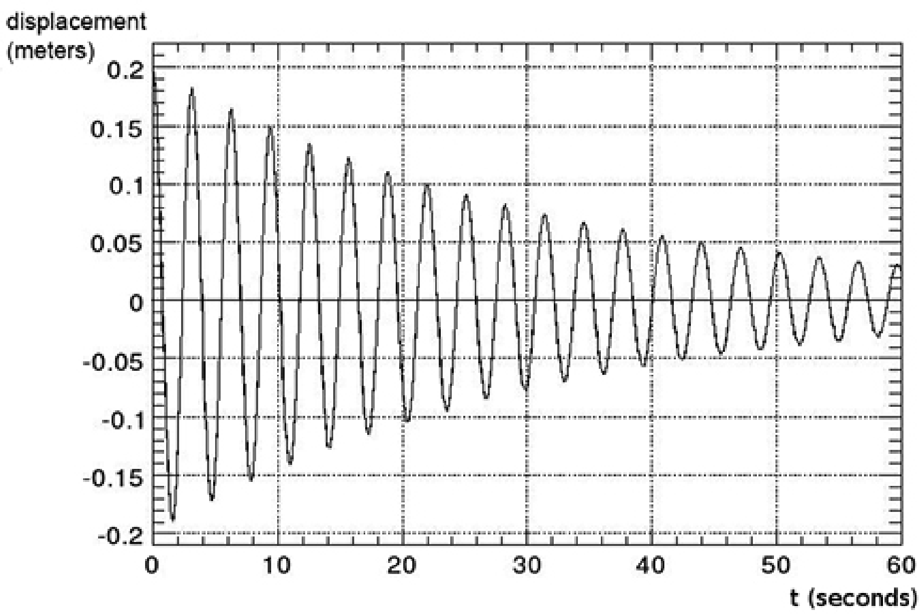

Figure 1: Damped Oscillator

2. **R factor**
   For an underdamped oscillator, Amy defines the "R factor" as

$$R = \pi \times \text{(Number of oscillation cycles it takes to reach } 1/e \text{ of the initial amplitude).} \tag{1}$$

   How does R compare to the quality factor Q for an underdamped oscillator? (We're considering a *very* weakly damped oscillator)

3. **Forced Oscillator**
A mass m is subject to a spring force $-kx$ and an external oscillating force $F(t) = F_0 \sin^2(\omega t)$. The mass begins at $x = 0$ from rest.

   (a) What is $x(t)$ given the above initial conditions? *Hint: You don't need to solve the equation from scratch if you put it in a form similar to what was shown in lecture.*

   (b) In terms of m and ω, what should k be in order for the motion to be at resonance? (*Hint: The answer is not* $k = m\omega^2$)

4. **Numerical Solution to Differential Equations - Part II**

   (a) What are the relationships between γ and ω_0 which determine whether a damped oscillator is underdamped, overdamped, or critically damped?

   (b) Using Euler's method (outlined in the previous assignment), produce *Mathematica* plots of $x(t)$ for underdamped, overdamped, and critically damped motion. Note, that the equation of motion for the damped oscillator can be written as

$$\ddot{x} = -2\gamma\dot{x} - \omega_0^2 x. \qquad (2)$$

   As your submission for the assignment, print out your code, and the plots
   Hint: I suggest you keep the parameters and initial conditions (and much of the code) from 5b in the previous assignment, and appropriately introduce the term $\gamma\dot{x}$, with γ set at the necessary value to obtain the various types of damped motions

5. **Ball in Bowl**

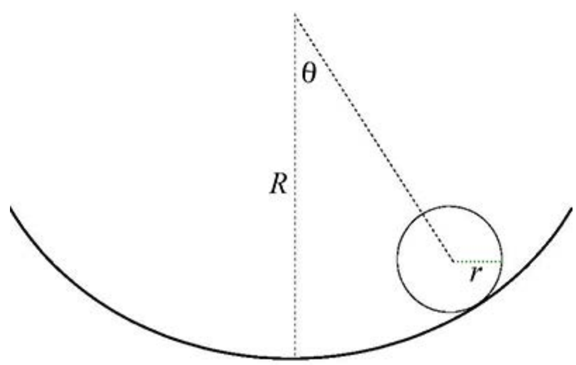

Figure 2: Ball in Bowl

A spherical ball of radius r and mass M, moving under the influence of gravity, rolls back and forth without slipping across the center of a bowl which is itself spherical with a larger radius R (Fig. 2). The position of the ball can be described by the angle θ between the vertical and a line drawn from the center of curvature of the bowl to the center of mass of the ball.

State (but do not answer) **three** precise physics questions we can ask about this system.

Note: You will be graded on the depth and the precision of your questions. As long as you ask questions, you will receive credit for the problem, but a question like "What is the value of the gravitational acceleration in the system?" would get less points than the question "If we introduce a constant frictional force F_{fr}, how much energy does the ball lose as it travels to its equilibrium at $\theta = 0$?"

Resource: Introduction to Oscillations and Waves
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