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PROFESSOR: We're going to look at scientific notation. When we write very, very big and very, very small numbers, it's hard. We don't want to write a number like that or a number like this, 125 million, because when we're doing multiplication and division with these numbers, it's hard because you've got all those extra 0's. But in reality, you don't have any more information in this number just because it happens to have a bunch of zeros in front.

So what we like to do is to have a shorthand way of saying a really small number or a really large number, and the way that we do that is we break down each of these numbers into powers of 10. So I'm going to write this number, 125 million, and I'm going to write it as 1.25, which is the smallest number that I can make out of this-- take these digits, 1.25. What would I have to multiply 1.25 to get 125 million?

AUDIENCE: 6.

PROFESSOR: Would I multiply it by 6? I would have to multiply it by 10 to the 6, but let's just write it out. 1.25 times-- it's going to be 100 million. If you have 100 million, and then you have 1.25 of 100 million, that means you have 125 million.

So in the same way, let's take this really small number, and we'll take the number itself, 5. What would we have to multiply by 5 in order to get this number? We have to multiply by-- go ahead, Nicki. I saw you doing it.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Well, it's going to be-- let's make it easier. Let's do this. This a 1/10, 1/100, and 1/1,000. So we have to-- if this is the number 5/1,000, we have to multiply 5 by the number 1/1,000, or 0.001. So I just take this small number that has just a 1 in it and some 0's, multiply it by a number. In the same case, same way I take a small number, 1.25, multiply it by something that has all the information about how many extra 0s there are, because all of these are just placeholders.

So in this case, I can rewrite this. Let's rewrite this one first. 1.25 times 100 million is a 1 followed by 1, 2, 3, 4, 5, 6, 7, 8 0s. So I can write that as 10 to the power of 8.

And just as a review, we'll do a little review over here. If you have 10 times 10, that's the same thing as 10 squared, just like if you had 5 times 5 that's the same thing as 5 squared. If I have 10 times 10 times 10, that is 1,000. 10 times 10 is 100. 100 times 10 is 1,000, but I can rewrite that as 10 to the power of 3, or 1,000. I just count the one and then how many 0s afterward-- 10 to the third.

Now I want you to practice with this. So we'll take this as much as we can. So I've represented this number, 125 million, as 1.25 times 10 to the eighth. Now, if I wanted to represent-- if I wanted to change back, what I could say is, well, 1.25 and then 10 to the eighth means multiply by 100 million. What I can think about is, if I have 1.25, if I multiply it by 10 to the eighth, I'm going to move the decimal place over eight times, because moving the decimal place over is the same thing as multiplying by 10. If you have 5 times 10, that's 50. So 5, move the decimal point over, it becomes 50.

Now, hopefully you guys will have seen this before a little bit. We'll give you lots of chance to practice, but if that's times 10 to the eighth, I'd write 1.25-- 1, 2, 3, 4, 5, 6, 7, 8-- I've moved the decimal point over eight places. And now I get back to my original number, but it's hard to work with-- 125 million.

Now, this way it works a little bit differently. To get 1/1,000, I had to take the number one, and I moved the decimal point over 1, 2, 3 places. So this number, 0.001, is written as 1 times 10 to the power of-- instead of moving to the right, we're moving to the left 1, 2, 3-- 10 to the power of minus 3, which 1 times any number is just that number, so that's really the same as 10 to the minus third. So then I can rewrite 5 times 10 to the minus third.

If I want to change it back, I just write five, and my decimal point is there. I'm going to move to the left three spaces-- 1, 2, 3. So I end up back with my original number, 0.005.

But let me just tell you a little story about how you can use these numbers to calculate. I may have already told this story to some of you during [INAUDIBLE], but let's write these numbers down again. If you won the lottery, and you won $6 million-- so this is multiplying and dividing powers of 10. If you won $6 million, that's the same thing as-- go ahead and write it in scientific notation, as a power of 10. So go ahead and write these things down.

So that's 6 times 10 to the sixth. Again, that's a big number. Say you had 30 relatives that you wanted to divide that money in between. Is each person going to get more than $6 million or less than $6 million if you divvied it up?

AUDIENCE: Less than [INAUDIBLE].

PROFESSOR: You get less than $6 million. So if we took $6 million divided by 30 relatives-- this is dollars divided by relatives. Peter, do we need to keep this? Is it OK if I get rid of that?

AUDIENCE: Sample drawing.

PROFESSOR: Yeah, this was the sample drawing of the measurements that you made before. If we saw that and we were just in fifth grade, we'd say, OK, 6 million divided by 30. Now I have to add some more 0's, and move things around, and multiply, and blah, blah, blah, and everybody hates long division. It is a fact of life.

So the easy way to do it is, instead of going through to do this, you can rewrite things as scientific notation. So this problem then becomes 6 times 10 to the sixth-- we'll write it out-- dollars divided by-- do we write 30 as scientific notation?

AUDIENCE: [INAUDIBLE].

PROFESSOR: 3 times 10 to the first relatives. Here's how I want you to work with scientific notation when you're dividing things, and I'll give you an example of multiplying here in a second. So you take this many dollars divided by that many relatives. This is a mathematical expression. You can always just rewrite underneath it equals-- you're going to gather together the different parts. You're going to gather together first the numbers. So you're going to gather together 6 divided by 3, because when you have one thing divided by another, you can pull apart what you're dividing.

So 6 divided by 3-- and then I'm going to gather together-- put those in parentheses. I'm going to gather together the powers of 10. I'm going to say 10 to the sixth divided by 10 to the first, and then-- everybody should be writing this down. And then multiply by dollars per relative. And you gather together each one of those groups.

Now I'm going to move up here. That's a horrible marker. I'm just going to draw a line to let you know that I'm moving up here, and then you just simplify each one of those parts. What is 6 divided by 3? 2. So I can say equals 2 times-- now we've got this. Let's just rewrite it. 10 to the sixth divided by 10 to the first, and then we've got dollars per relative.

When you multiply and divide-- or when you divide powers of 10-- yeah? [INAUDIBLE], you have a question?

AUDIENCE: Yeah, but [INAUDIBLE]?

PROFESSOR: Yeah, I just wanted to keep it separate so that we could talk about this. So 10 to the sixth is 1 million, right? 10 to the first is 10. If you take 1 million divided by 10, the easy fast way to do it is, if you're looking at this fraction, you can cancel one of the 0's, and then it becomes 10-- it becomes 1 with 1, 2, 3, 4, 5 0's after it, but if you take a 1/10 of a million, you actually take a 100,000.

The quick and easy way to do that is to take each one of these exponents. You take two times, and you still write it's a power of 10, but if you divide exponents, you take the top exponent, and you subtract the bottom exponent. So that becomes 2 to the power of 10 to the 6 minus 1, and I want you to write that step out, because that's really, really important to write out the 6 minus 1 dollars per relative.

And then you've got plenty of paper in your lab notebooks. Feel free to use all of it. So take lots of space to write all of these things down. And that becomes 2 times-- What's 6 minus 1? 5. 2 times 10 to the fifth dollars per relative.

How do we change that back into something that makes sense? Well, that's a 2 followed by five 0's. So $200,000 per relative. That's going to make Aunt Susie pretty happy.

So when you divide powers of 10, you take the top exponent minus the bottom exponent, and it makes sense. $200,000 is less than $6 million. That's what we expected. So we got $200,000 per relative. So that's division. So dividing powers of 10 means you subtract the exponents. Let's write that out. Here are some of these rules.

So rules-- dividing powers of 10 involves subtracting exponents. So write this down. Put a box around it.

Now, let's continue our story. Say that Aunt Susie takes her $200,000, and she puts it into a bank account, and that bank account gets 5% interest every year. That's a pretty good bank account. And I want you to write all this stuff down just so that you can have it in your notebook so you can refer back to it. I'm going to get rid of this stuff.

I know this is a lot of math going on today, but once we get used to it, it'll come as second nature. So now Aunt Susie invests in a 5% per year account. So how much money is she going to get at the end of the year? How much extra money?

So what is 5% of $200,000? That's our question. Now, eventually we're going to learn a problem solving technique of how to think about equations and things, but for right now, let's just translate this into a mathematical statement. So say what is-- that's x. That's what we want to call. What is 5%? How do we write 5% as a decimal? Go ahead, Lauren.

AUDIENCE: 5/100.

PROFESSOR: 5?

AUDIENCE: 0.05.

PROFESSOR: So 5/100 because it's 5 per cent. 5 per cent is-- cent is about 100. So 5/100. So we can say 5 divided by 100, or we can also say 0.05-- 5%. So those are tenths. Those are hundredths. So you've got 5/100.

What is 5/100 of 200,000? We've just translated this sentence into math. What is-- x is 5/100 of or times 200,000. So we can rewrite that x equals 0.05 times 200,000.

Now let's rewrite this as scientific notation. How do we write 0.05 in scientific notation? [INAUDIBLE]?

AUDIENCE: [INAUDIBLE] 5 times [INAUDIBLE] 10 to the 2 [INAUDIBLE] 10 to the negative 2.

PROFESSOR: So 5 times 10 to the 2 is 5 times 100, which is 500.

AUDIENCE: 10 to the negative 2.

PROFESSOR: We want to say 5 times 10 to the negative 2. So this number is less than 1. It's a fraction of 1. And we also multiply times-- and this one we said was 2 times 10 to the fifth. So again, we want-- now I forgot to put in our units here, because here we're saying this is dollars, and a percent is just a fraction of this quantity. So it doesn't have a unit, but we will know that our answer is going to be, well, how much money is she going to have extra? So our unit should be in dollars.

Now we do the exact same thing. We gather together the numbers. So 5 times 2-- gather those together. And then we multiply by 10 to the minus 2 times 10 to the 5, and I want you to just write it like that. Put those two numbers right next to each other, and then the unit is still dollars.

All we're doing is just gathering together different parts. Let's go up here. We'll rewrite that as x equals. Now, what is 5 times 2?

AUDIENCE: 10.

PROFESSOR: 10. So let's not try to jump any steps. We're going to write that as scientific notation. We'll just write that as 10 times-- now, when you multiply powers of 10, you want to add the exponents. So 10 to the minus 2 times 10 to the 5 becomes 10 to the minus 2 plus 5. And then the unit is still dollars.

So that then becomes 10. Let's simplify this. What is minus 2 plus 5? Hang on one second.

AUDIENCE: Three.

PROFESSOR: Three. If you go negative 2, and then you add on 5 more, you end up back at positive 3. So 10 to the third dollars. How can we rewrite 10 times 10 to the third in terms of scientific notation?

AUDIENCE: [INAUDIBLE].

PROFESSOR: How can we rewrite this as scientific notation? We could say 10 times 10 to the third.

AUDIENCE: [INAUDIBLE].

PROFESSOR: We can-- what's that?

AUDIENCE: Is it 1,000?

PROFESSOR: Well, it's 10 times 10 to the third. 10 to the third is 1,000, so it's 10,000, but in order to keep it all straight let's just do 10 to the first because 10 is just 10 to the power of 1 times 10 to the third dollars. Again, you multiply the exponent, and you multiply the powers of 10, and you've got 10 to the 1 plus 3 dollars. And then we simplify that, and that's just 10 to the 4 dollars.

If we rewrite that, that's a 10-- or a 1 with 3, 4 0's after it. So $10,000. So at the end of one year, Aunt Susie is going to have an additional $10,000 on top of her.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah, where's that bank? That's a good question. So again, your second rule is multiplying powers of 10 involves addition of exponents. That's her second rule. Most of the time we're only going to be doing division and multiplication.