RES.TLL-004 STEM Concept Videos, Fall 2013

Transcript – Diffusion and Fick's Law

How do neurons send chemical signals to neighboring neurons?
Why do you wear a jacket in the winter? Why do some animals have circulatory systems? These questions depend on random walks and diffusion.
In this video, using a very simple model, you will learn the fundamental difference between a regular and a random walk, and be able to predict the consequences of that difference for biophysical systems.

This video is part of the Probability and Statistics video series.

Many events and phenomena are probabilistic. Engineers, designers, and architects often use probability distributions to predict system behavior.

Hi, my name is Sanjoy Mahajan, and I'm a professor of Applied Science and Engineering at Olin College. Before watching this video, you should be familiar with moments of distributions and with concentration gradients.

After watching this video, you will be able to:

1. Describe the difference between regular and random walks.

2. And, explain the structure of Fick's law for flux.

Chapter 1: Regular and Random Walks
Between neurons, molecules travel by diffusion. They wander a bit, collide, change directions, wander back, collide again, and higgle and jiggle their way across the neural gap (the synaptic cleft).

Here is a diagram of it. This is the inside of one neuron, here is the inside of the other neuron, and here is the synaptic cleft in which there are molecules wandering across from the left neuron to the right where they are received and picked up and used to generate a signal in the second neuron.
An extremely simple model of this process, which has the merit of containing all the essential physics, is a molecule making a random walk on a one-dimensional number line:

To further simplify our life, this model molecule moves only at every clock tick, and sits peacefully waiting for the clock tick. At each clock tick, it moves left or right by one unit, with equal probability (50 percent) of moving in each direction.

Our molecule here, after a few ticks, has reached x=4. So the probability of finding it at x=4 is 1. What will happen to it in the next time ticks? After the next tick, the molecule is equally likely to be at 3 or 5. That changes the probability distribution to the following.

Thus, although we don't know exactly where it will be, we know that the expected value of x is still 4.

Pause the video here, and find the expected value after one more tick – that is, two ticks after it was known to be at 4.

You should have found that the expected value is still 4. Here is the probability distribution. It has a one-fourth chance to be at 2, a one-half chance to be at 4, and a one-fourth chance to be at 6.

Thus, the expected value of x is ¼ times 2 + ½ times 4 + ¼ times 6, which equals 4.
In short, the expected value never changes. Alone, it is thus not a good way of characterizing how the molecule wanders. We also need to characterize the spread in its position.

Thus, we use a higher moment, the second moment, the expected value of x-squared.
At first when the molecule was at x=4 right here, and it was for sure there, then the expected value of x^2 was just 4^2 or 16.
What about after one clock tick?
Pause the video here and work out $\langle x^2 \rangle$ after one tick.
You should have found that $\langle x^2 \rangle$ is now 17:
The molecule is equally likely to be here or here.
What happens after one more tick?
After one more tick the molecule has a ¼ chance of being at 2, a ½ chance of 4, and a ¼ chance at 6.

We find that the expected value of x-squared equals 18.
Hmm, it seems like the expected value increases by 1 with every clock tick. That's true in general, no matter how many ticks you wait, or where the molecule started.
Thus, for a molecule starting at the origin (at 0), the expected value of x-squared is just the number of clock ticks.
This equality is fascinating, because it contains the difference between this kind of walk, a random walk, and a regular walk. If the
molecule did a regular walk, moving one step every clock tick, without switching directions, then the number of clock ticks would be the expected value of x, not x^2.

The random walk is fundamentally different, and that fundamental difference will explain, among a vast number of physical phenomena, why you wear a jacket in the winter, and why some animals have circulatory systems.
Now, instead of speaking of counting clock ticks, let's measure actual time. Instead of counting units left

or right, let's measure actual distance. If each clock tick takes time tau, and each distance unit is lambda, instead of one, as before, then these relationships here change slightly to include the dimensions and units.
For the regular walk, x is lambda times the number of ticks.
The number of clock ticks is t/tau, so
the expected value of x squared is lambda squared times t/Tau here. And here we have lambda times T/Tau for the regular walk.
In the regular walk we can rewrite that as Lambda/Tau times T.
That lambda/tau here has a special name: the speed.
In a random walk, the constant of proportionality is lambda^2/tau.

This constant lambda squared/Tau, which has dimensions of length squared/time, is the diffusion constant D.
Let's see how "fast" a random walk goes, in comparison with a regular walk.

Suppose that the molecule has to cross the narrow gap between two neurons, a synaptic cleft, which has width L. If we wait long
enough, until <x^2> is roughly L^2, the molecule is likely to have crossed the gap.
How long do we wait on average? Until "t" here is about L^2 / D.
Thus, the "speed" of the random walk is something like the distance divided by this time t, and that time t is the distance squared divided by the diffusion constant. So this speed is the diffusion constant divided by distance.
Again we see the fundamental difference between a regular and a random walk. A regular walk has a constant speed here of lambda over tau, as long as lambda and tau don't change. In contrast, in a random walk, the speed is inversely proportional to the gap L.

Chapter 2: Fick's Law

This result explains the structure of Fick's Law for the flux of stuff. Flux is particles per area per time.
Flux, say's Fick's law, equals the diffusion constant times the concentration gradient dn/dx, where n is the concentration.
How are the flux and diffusion velocity connected?
Well flux is also equal to the concentration n times the speed. And here the speed is D/L.
But where did the dx here and the dn here come from? What do those have to do with n and L?
Imagine two regions. One with concentration n1 and another with concentration n2, separated by a distance delta x.

So this is the concentration of neurotransmitter here at one end and concentration of neurotransmitter here at the other end of say, a gap. We could use this same model for oxygen in a circulatory system.
Then the flux in one direction is this and in the reverse direction, it's this.
The net flux is n2-n1 times D/delta x.
So we've explained the d and the delta x in Fick's law over here. What about the dn? Well n2-n1 is the

difference in n, or just dn, so this piece here is dn. This is dx, and this is D.
So we arrive at Fick's Law based on the realization that flux is concentration times speed, and the speed here in a random walk is the diffusion constant divided by L.
And that's why you wear a coat, rather than a thin shirt, in the winter. The thin shirt has a dx of maybe 2 mm. But the winter coat may be 2 cm thick.

That reduces the heat flux by a factor of 10 through your coat compared to the shirt. And you can stay warm just using the heat produced by your basal metabolism – about 100 Watts.
For our final calculations, let's return to the neurotransmitter and then discuss circulatory systems.
How long would it take a neurotransmitter molecule to diffuse across a 20 nm synaptic cleft? The diffusion constant for a typical
neurotransmitter molecule wandering in water, which is mostly what's in between neurons, is about 10^-10 m^2/s.

Pause the video here and make your estimate of the time.

You should have found that the time is about 4 microseconds.

Is that time short or long? It's short, because it's much smaller than, say, the rise time of a nerve signal or the timing accuracy of
nerve signals. Over the short distance of the synaptic cleft, diffusion is a fast and efficient way to transport molecules.

How does this analysis apply to a circulatory system? Imagine a big organism, say you or me, but without a circulatory system.

How long would oxygen need to diffuse from the lung to a leg muscle say, one meter away?

That's where the oxygen is needed to burn glucose and produce energy. Oxygen, a small molecule, has a slightly higher diffusion constant than a neurotransmitter molecule does – D is roughly 1x10^-9 meters squared/sec.
Pause the video and make your estimate of the diffusion time.

You should have found that the diffusion time is roughly…10 to the ninth seconds!

That's roughly 30 years! Over long distances, diffusion is a lousy method of transport!

That's why we need a circulatory system. Using a dense network of capillaries, the circulatory system brings oxygen-rich blood near every cell...and only then, when the remaining distance is tiny, does it let diffusion finish the job!

To Review

In this video, we saw how a random walk, which is the process underlying diffusion, is fundamentally different from a regular walk, and how that difference explains the structure of Fick's law and allows us to estimate diffusion times.
The moral is that we live and breathe based on the random walk, whose physics we can understand with a simple number line and moments of distributions.