RES.TLL-004 STEM Concept Videos, Fall 2013

Transcript – Flux and Gauss Law

What do rain, bees, and vectors have in common? They're all ways that MIT physicists visualize the concept of "flux." In this video, we'll explore flux as it relates to Gauss' law. If you've ever had trouble choosing the right Gaussian surface, getting a quantity out of an integral, or just wanted to know what flux is, get ready: this video is for you.

This video is part of the Derivatives and Integrals video series.
Derivatives and integrals are used to analyze the properties of a system. Derivatives describe local properties of systems, and integrals quantify their cumulative properties.

"Hello. My name is Peter Fisher. I am a professor in the physics department at MIT, and today I'll be talking with you about electric flux and Gauss' Law.

By this time in your course you've seen and used Gauss' Law quite a bit. You have a good handle on the electric field, what it does and how it works. We will also assume that you know how to take an area integral, a skill that is important to Gauss' Law but which is not taught in this video.

Our end goal is to improve your ability to use Gauss' Law. To do this, we'll help you develop an organized view of electric flux and what goes into it. We'll also spend a lot of time talking about symmetry and how to use it to your advantage.

We're going to start by thinking about what goes into flux and the interrelationships between these quantities, so that we can use flux more effectively.

As a refresher, here are the two equations we'll be working with today: the definition of flux and Gauss' Law. You may have seen these written with a double integral, or with slightly different notation, so take a quick look to make sure you're familiar with all the pieces.

To really use symmetry to our best advantage in Gauss' Law, we need to understand a few more basic things. We need to be able to use vector areas, to categorize charge distributions, and to determine when and how the two can work together.

The underlying reason we want to use symmetry is that integrals are difficult, significantly more difficult than derivatives. There are many situations in which we cannot determine the flux integral. Therefore, we seek to simplify.

The integrals we're dealing with are surface integrals, also called area integrals. There are two basic kinds, open and closed, as you can see on the screen. An open surface uses an open integral, and a closed surface uses a closed integral. Flux can be defined for any surface, but Gauss' Law uses only closed integrals.

When we take the integral, we use dA, a tiny piece of the area. dA is a vector piece of the area, so it has a direction to it as well. dA always points perpendicular to the surface. With closed surfaces we usually choose dA to point outward from the surface.

dA usually gets represented as two other differentials, such as dx times dy. Which two depends on the coordinate system you choose, with some being more complex than others. Sometimes we will need these, so it's good to have them. The good part is that if we use symmetry correctly, we can avoid having to do an integral at all.

Let's pursue our goal of not doing an integral. We have this integral of E dot dA. Let's write the dot product as a cosine. Under certain circumstances, we can remove the E from the integral. We can only do that when our electric field has uniform magnitude over the entire surface. You can see some examples on the bottom of the screen.

We can remove the cosine(theta) from our integral only when the angle between the area and the electric field doesn't change. The left-hand example shows a field that always points in the same direction, and an area that always points in the same direction. The middle example has a field whose direction changes, but it always points outward, just like the area vector from the cylinder points outward. It still works. In the right-hand example, the field has a constant direction, but the area vector does not. The angle between them changes, so we couldn't remove the cosine from the integral.

If we want to move both the field and the cosine out of the integral, we need to fulfill both conditions. The left-hand and center examples only fill one condition each. The example on the right has both, and would be ideal to use.

Before we move on, a reminder about flux and angles. Only electric field lines that actually pass through a surface provide flux. When the surface is parallel to the field, no flux is provided. This is particularly useful for some three-dimensional surfaces, such as the empty cylinder to the right. If we remember this, we can really simplify our integrals.

Another thing to remember: Flux is a scalar. As long as the field strength is the same, and the angle between the field and surface stays the same, the flux will be the same. We don't have to worry about the flux pointing in a particular direction, because it's just a number with no direction.

Now we know how we would like the field and the area to line up. Next we need to understand the circumstances that make that possible. We need to look at electric charge distributions and their symmetry.

Electric charge can come as a single point, as in an electron or proton, but it can also take other shapes. It might be stretched into a line, spread throughout a volume, or spread over a surface. Not a Gaussian surface - remember, that's something we make up to solve a problem. Charge is spread on a physical object. You can stretch the charge out into lines, spread it on thick wires or spherical shells, build a solid ball out of it, or even just smear it around on an object.

An object has symmetry when it has exactly similar parts that are either facing each other, or arranged around an axis. On the screen are some pictures of objects that have symmetry.

There are three categories that we'll investigate, and three types of symmetry to consider. Our hope is to have a highly symmetric charge distribution, with any of the three types of symmetry. That's when Gauss' Law is easiest to apply.

These examples have little or no symmetry to them, and are not the sort of thing for which we can use Gauss' Law.

These three charge distributions have visible symmetry, but it's not enough. We need a charge distribution so symmetric that the field it generates is also symmetric.

Let us show why. Let's say we have a charged cube, which is certainly a symmetric object. We can draw the field from it fairly easily. If we place a cubical Gaussian surface around the charge, we can see the problem: the field and the surface have different angles in different places. There's some symmetry here, but not enough to allow us to pull everything out of the integral.

These cases are ideal: a point charge, a spherical shell, an infinite sheet of charge, or an infinite cylindrical shell of charge. No doubt you've used Gauss' Law with these charge distributions before. How can we describe such distributions and find others like them?

All three of those have a special quality: if you move in two directions, they look exactly the same. No matter how far you move across the plane in the x or y directions, it looks identical. Move along the cylinder or around it, and it looks the same. Rotate around the sphere around the equator or the poles, and you can't tell the difference.

We can also add similar objects together or scale them however we like. By integrating the fields from a lot of shells, we can create a solid object, just like these Russian nesting dolls.

Of course, perfectly symmetric charge distributions are rare in real life. This is probably because infinitely long cylinders are so hard to find. Thus we often use Gauss' Law in cases where the charge distributions are close to one of our symmetric cases. We can usually obtain excellent estimates of the true electric field in this way.

Two approximations are particularly common: when the object is so large that we can treat it as being infinite, or when we are so close to the object that its large-scale features become unimportant. In these cases, we often treat the object as being an infinite plane.

Here's an example of the first approximation. Compare the distance from your point of interest to the object, to the size of the object itself. If the smallest dimension of the object is ten times your distance from it, you can rely on a very good estimation from Gauss' Law, even if your plane isn't really infinite.

To use the other approximation, we want a situation where the curvature is very small in the place we care about. For example, it would be very difficult for us to find the field at this point. However, if we zoom in, we can reach a point where the surface looks like a flat plane. Finding an estimate for the field at this point will be much easier.

Now we have the background we need. It's time to put all the pieces together and pick our Gaussian surface.

We'll do a quick review in case you missed something, and give you an example or two. After that, it's your turn to pick the best surface for a given situation.

In order to simplify our integrals as much as possible, we seek out situations with as much symmetry as we can get. The symmetry in the charge distribution isn't something we can control, but our choice of surface will determine whether there is symmetry in how the electric field meets the Gaussian surface. And that's always the key: you choose the surface to make the problem easier for you. Finally, it's important to remember that in some cases, Gauss' Law won't be helpful, and it's good to look for another method.

Here are three charge distributions: a long cylinder, a sphere, and a flat plane. To maximize the effects of symmetry, we choose surfaces that match up well with the charge distributions. To find the field near a cylinder, we surround it with another cylinder. To find the field near a sphere, we encase it in a sphere. To find the field near a charged plane, we use a box-shaped surface. Sometimes people will use a cylindrical surface instead, and count the flux that goes through the top of the cylinder. Both approaches are valid.

Here is your first challenge: choose the Gaussian surface that will best fit this charge distribution. We want to know the electric field at a certain distance from the center of this solid sphere.

What surface would you choose?

A sphere inside our existing sphere will be easiest. This example helps us remember that the Gaussian surface can be inside an object. The area of the surface will be the usual four thirds pi r cubed. We'll need to remember to use only the charge inside our volume, not the whole charge of the sphere.

Here's challenge number two. We would like to know the electric charge contained in a thundercloud. Let's say that the inside of the cloud has an electric field that looks like this.

What surface would you choose?

A box-shaped surface will take advantage of the Cartesian symmetry in this electric field. Because of the alignment between the sides of the box and the electric field, they will not contribute to the flux integral. We can use just the area of the top and bottom of the box, and the field at those locations, to find the flux. This is a common technique.

Here's challenge number three: finding the electric field at a certain distance from the center of charged disc.

What surface would you choose?

No surface will work! The problem is that there's not enough symmetry in this situation to pick an appropriate surface.

If we can't get an exact solution, we would like to approximate the field. Unfortunately, if we look at the distances involved, we are not close enough to the disc to make a good approximation.

In the end, this is not a good Gauss' Law problem. It is solved much more easily through other techniques, such as integration and Coulomb's Law. An example can be found in the supplemental materials for this video.

Here is the final challenge. A top and side view of the field are shown.

What surface would align well with this field?

A cylindrical surface would work very well with this circular field. You can see that the field is perpendicular to the area vector at all points. Our dot product is now giving us a cosine of ninety degrees at all times. This means a zero value for the flux at all points. We don't even need to determine the area of the cylinder.

If our flux is zero, the total charge inside must also be zero. We might guess that positive and negative charges together could create this field, but as it turns out, they cannot. Fields that look like this are created by a changing magnetic field, using Faraday's Law, which you will learn more about later in your course.

There are some times when you'll just have to integrate; there's no way around it. Not every situation is perfectly symmetric. When you come to these situations, look for the following things to help you out. First, an area vector that you can define easily. Second, an electric field that you can easily find or that is defined for you in a problem. Third, see if you can create a surface where either the magnitude of the field or its angle is uniform. The more of these you can take advantage of, the better.

Let's review.

The more symmetry we can take advantage of in our problem, the easier time we'll have with Gauss' Law and flux. Ideally, we want a situation where the charge, the field it creates, and the Gaussian surface we choose all have the same symmetry, so that we can simplify the integral as much as possible.

For the final segment of this video, we wanted to answer a common question about electromagnetism. What is flux? What does it mean? How can we understand it?

We took a video camera into the physics department at MIT to find out how physics professors and students think about flux. Here's what we found out.

Thank you for watching this video. We hope that it has improved your understanding of flux and Gauss' Law. Good luck in your future exploration of electromagnetism.