Overview: Time and Uncertainty

• Intertemporal Prices and Present Value

• Uncertainty

• Irreversible Investments and Option Value

Economics of Time: Some Issues

• Cash now versus cash payments in the future?
• Future payments are uncertain?
• When should we undertake a new project – now, later or never?
• How do we manage resources over time?
  – When do we end a profitable project?
  – How do we use up a non-renewable resource?
Intertemporal Prices

• Interest rate r, Today is t = 0:
  $1 invested today becomes $(1 + r) at t = 1
  $1 invested today becomes $(1 + r)^2 at t = 2, etc.

• Today’s price of ($1 at t = 1) is 1/(1+r)
  (i.e. $ 1/(1+r) invested today becomes $1 at t = 1)

• Today’s price of ($1 at t = 2) is 1/(1+r)^2, etc.

Present Value

• Present Value of a stream of cash flows is the value in today’s prices

\[ PV = C_0 + C_1/(1+r) + C_2/(1+r)^2 + ... + C_T/(1+r)^T \]

  where \( C_t \) is the (positive or negative) cash flow at time \( t \)

• PV Criterion: Invest in projects with \( PV > 0 \)

• \( r \) is “discount rate,” PV often computed for many values
Example

Consider two projects, A and B

<table>
<thead>
<tr>
<th>T = 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>r = 1%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project A</td>
<td>-200</td>
<td>50</td>
<td>50</td>
<td>120</td>
<td>15</td>
</tr>
<tr>
<td>Project B</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>-220</td>
<td>-15</td>
</tr>
<tr>
<td>Difference B - A</td>
<td>-30</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Timing of payments matters, with discount rate very important)
Choice under Uncertainty

• Another aspect of future cash flows is uncertainty. This is modeled via random variables with a distribution.

• How do you react to uncertainty?
  – Cover yourself; avoid big losses at all costs
  – Make decisions using average (mean) values, ignoring the randomness.
  – Take big risks, relishing in the thrill of the unknown ("the wonder of it all")

Risk Aversion

• Suppose you are offered a job with a financial firm, and there are two alternative compensation packages.
  A. $100,000 Salary
      $100,000 Bonus
      You expect to receive the bonus with probability .5.

  B. $W Salary only, where $W > 100,000.

• What is the smallest value of W that would cause you to take B over A?
Risk Aversion (continued)

• If your answer is
  $W = 150,000 = E(\text{package A})$, You are risk neutral
  
  $W < 150,000$, You are risk averse.

  $W > 150,000$, You are risk loving.

• Risk Premium: what you would pay to avoid facing the risk, e.g. $W = 130,000$ gives risk premium of
  
  $20,000 = E(\text{package A}) - W$. 
Figure 5.3 Risk Aversion

Figure 5.4 Risk Premium
Production Technology Choice

• Choice of a technology commits a firm to a production process
  – Risks arise from uncertainty in input prices
  – Risks arise from uncertainty in quantity or output prices

• Consider choosing a ‘high FC + low MC’ technology over a ‘low FC + high MC’ technology
  – This is a bet on high quantity or high output prices, enough to cover the high FC.
  – If substantial chance of low quantity or low prices, low FC choice is safer.
Example: Production Technology

- A risk neutral firm must choose between two available technologies
  - Technology 1: FC = 400 and MC = 9 (low FC + high MC)
  - Technology 2: FC = 4,000 and MC = 4 (high FC + low MC)
- Technology installed at cost FC today (year 0) and production occurs in year 1, with r = .1.
- In year 1, quantity is either 200 with probability p and 1000 with probability 1 – p
  - We consider p = .1, .2 and .5
- Price P = 12

Example: Production Technology (1)

- We must compute PV for each technology in each possible situation.
- For instance, with Technology 1
  - Q = 200: Variable profits: (P – MC)*200 = (12-9)*200 = 600
    - Present value with r = .1
      - $400 + 600 / (1+.1) = 145$
  - Q = 1000: Variable profits:(P – MC)*1000 = (12-9)*1000 = 3000
    - Present value with r = .1
      - $400 + 3000 / (1+.1) = 2,327$
- Technology 1: Expected Present Value at p = .2 and r = .1
  \[ \text{EPV} = .2 \times 145 + .8 \times 2,327 = 1,891 \]
Example: Production Technology (2)

• Expected Present Values

<table>
<thead>
<tr>
<th>Probability p</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>of Low Quantity</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Discount Rate r | | |
|-----------------|--|--|--|
| 10% Tech 1      | 2109 | 1891 | 1236 * |
| Tech 2          | 2691 * | 2109 * | 364 |
| 25% Tech 1      | 1808 | 1616 * | 1040 * |
| Tech 2          | 1888 * | 1396 | -160 |

* denotes preferred choice

Irreversibility and Option Value

• Many investment decisions are irreversible
  – Once committed, costs are (at least partially) sunk
• With uncertainty, there is a value to waiting
  – There is an ‘option’ value to flexibility – postponing decision while uncertainty resolves.
• Consider pricing with
  – Season tickets
  – Rent-to-buy arrangements
Example: Irreversibility

• Two possible technologies, B and V; it is uncertain which one will become the standard
• If you develop the right technology, then profits are 100. If not, your profits are 40 (since you have to license from someone else).
• Your market research suggests that there is a 80% probability that V will be the standard.
• How much do you want to pay to keep the B option alive until uncertainty resolves?

Example: Irreversibility (2)

• If you research only one technology, then you should research V, and your expected profits are:
  \[ \pi = 0.8 \times 100 + 0.2 \times 40 = 88 \]
• If you research both, then expected profits are:
  \[ \pi = 0.8 \times 100 + 0.2 \times 100 = 100 \]

• Value of keeping both options open is 12. This is what you are willing to pay.
Issues for Discussion

1. (“When to cut down the tree?” problem.) Suppose I have a process that is increasing in value, when do I halt it?

2. (“When to sell the oil?” problem.) Suppose we have a non-renewable resource, how do we best use it up?
When to cut down a tree?

- We assume that process initially increases rapidly in value and then slows down.

- Essential Logic: At any moment, you can halt the process and invest the value at interest r.

  Optimal to keep the process going when it’s value is growing at a rate greater than r, and halt it when the growth rate drops below r.

When to sell the oil?

- Consider two time periods: t = 0 and t = 1, fixed amount of oil to sell, price taker.

- Essential Logic: Sell when you get the highest profit for each unit. If you sell in both periods, marginal unit must have same profit PV, namely

  \[ P_0 - MC_0 = (P_1 - MC_1)/(1+r) \]

- Note, if MC flat or rising, P must rise.
- P – MC increases at rate of interest.
Take Away Points

• Money today and money tomorrow are different things. Present value is the correct way to combine such cash flows.
• People tend to be risk-averse. This is an important consideration for e.g. incentives.
• Flexibility has value (option value) which can be priced.