Game Theory
for
Strategic Advantage

15.025

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Classic Examples

• Government
  – T-Bills, mineral rights (e.g. oil fields), assets (e.g. privatization)
  – Electromagnetic spectrum
  – Public construction contracts (e.g., California Highways)

• Internet
  – Display & keyword advertising, personal data (cookies)

• Real Estate
  – Development contracts
  – individual homes

• Stocks
  – IPOs, Repurchases, M&A

• Auctions in disguise
  – Patent races, Lobbying, Legal disputes, hiring
First-Price Auction

How *should* you bid?

Is bidding your total valuation $v_i$ a good strategy?

How much to shade?

New approach: types of your opponent
(i.e., when to win and when to lose)
Your Bids

First-Price Auction

Best Fit Line: bid = 0.83 * value
Setting Up the Problem

• You bid to maximize your expected payoff

• Make a projection about the other bidder’s strategy

• Presumably this strategy depends on the valuation that bidder has.

• Let $b_j(v_j)$ be your projection for the bid of the other bidder when their valuation is $v_j$. 
Bidders Problem

• Suppose your value is $v_i = v$.

• Choose a bid, $B$, to maximize expected profits.

• $E[\text{Profit}] = (v - B) * \text{Pr}(B \text{ is the highest bid})$

• $\text{Pr}(B \text{ is the highest bid}) = \text{Pr}(B > b_j(v_j))$
What is My Optimal Bid?

$\textbf{Key Calculation:}$

- who do I beat?
- who do I lose to?

$v_j^*(B) = B / \alpha$

$\text{Total Exp. Payoff}$

$(v-B)B/\alpha$
Bidder’s Problem Revisited

• So now you must choose $B$ to maximize
  \[ E[\text{Profit}] = (v - B)\frac{B}{a} \]

• Differentiate with respect to $B$
  \[ (v - 2B)/a = 0 \]
  \[ B = \frac{v}{2} \]

• If your opponent shades proportionally to his value $\Rightarrow$ bid half your value.
Equilibrium

• My rival is doing the same calculation as me.
  – If he conjectures that I bid \( \frac{1}{2} \) my value
  – He should bid \( \frac{1}{2} \) his value (for the same reasons)

• Therefore, in \textit{equilibrium}, we each bid half our value.

• More generally, with \( N \) bidders, bids = \( v^*(N-1)/N \)
Your Bids (3 bidders)

First-Price Auction

Best Fit Line: \( \text{bid} = 0.83 \times \text{value} \)

Equilibrium: \( \text{bid} = 0.67 \times \text{value} \)
Bayesian Nash Equilibrium

- Uncertainty over rival’s payoffs in this game
- Best-respond to expectation of your rival’s strategy
- Your rival does likewise
- Mutual best responses in this setting are called *Bayes-Nash Equilibrium*. 
M&A Auction Game

1) Want to acquire a large (2-division) company

2) You will bid for the company’s stock

3) The company’s true value = sum of two divisions’ values

4) Your firm has expertise in one area

5) Can estimate the value of one division / sector

6) Uncertain about the rest of the company
Wallets Game

1) Check how much cash is in your wallet.

2) That is your (perfect) estimate of 1 division.

3) I will randomly match you with 1 other bidder.

4) Bid for the company’s stock (= sum of wallets)
The Bidder’s Problem

- Your wallet contains $v$ dollars.
- The other bidder’s wallet contains $x$ dollars.

- You don’t know $x$, but it is randomly (uniformly) drawn from 0 to 100.
- The company is worth $v + x$.

- You conjecture a bidding strategy $b(x)$
- Choose a bid, $B$, to maximize expected profits:
  $u = v + x - B$ if you win and loser’s value is $x$
  $u = \text{zero}$ if you lose
Cautious Opponents

• Suppose your opponent thinks as follows:

1. “I am afraid the other wallet is empty.”

2. “I will never bid more than my wallet’s content.”

3. “So I’ll just bid \( b(x) = x \).”

• How do you respond to \( b(x) = x \)?

• What are your profits if you win against opponent \( x \)?

\[ v + x - B \]
How Should you Bid?

• Pr [win | B] = B / 100
• Maximize (v+x-B)B?
• Choose B = (v+x)/2 ... Don’t know x
• .... so I should bid (v+50)/2 = 25 + v/2? Right?
  When you win, x < B!
• Maximize (v+B/2-B)B
  \( v - B = 0 \) \( \Rightarrow B = v \)!!
  \( \Rightarrow \) Bid just your wallet’s content!
Lessons from Wallets

- Suppose your opponent bids aggressively ($a > 1$)
- Avoid the winner’s curse
- Suppose your opponent is overly cautious ($a < 1$)
- Take advantage of it!!
Seller Revenues

• Common-value auctions: revenue equivalence holds only under very special circumstances (symmetry)

• Open- or sealed bid? Are SPA and English auction still strategically equivalent?

• In general, winner’s curse $\Rightarrow$ English $>$ SPA $>$ FPA

• Instructive for the history of online ad auctions...