Network Models

Assignment
Transportation

Intro to Modeling/Excel
How the Solver Works
Sensitivity Analysis
Objective

- Mini Course on Networks
  - Introduction to modeling
    - In Excel and AMPL
  - Intuitive description of solution approach
  - Intuitive description of sensitivity analysis
- Intuitive and visual context for covering technical aspects
Assignment Model

- Autopower Europe
  - Manufactures UPS for major installations
  - Four manufacturing plants
    - Leipzig, Germany
    - Nancy, France
    - Liege, Belgium
    - Tilburg, The Netherlands
  - One VP to audit each plant
Assignment Problem

- Who’s to visit whom?
  - VP’s expertise and plant’s needs
  - Available time and travel requirements
  - Language abilities
  - ...

15.057 Spring 03 Vande Vate
The Challenge

- Estimate costs (Done - Thoughts?)
- One VP to each plant
- One plant for each VP
- Minimize cost of assignments
## Estimated Assignment Costs

<table>
<thead>
<tr>
<th>VP</th>
<th>Leipzig</th>
<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finance (F)</td>
<td>24</td>
<td>10</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>Marketing (M)</td>
<td>14</td>
<td>22</td>
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<td>15</td>
</tr>
<tr>
<td>Operations (O)</td>
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<td>17</td>
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<tr>
<td>Personnel (P)</td>
<td>11</td>
<td>19</td>
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<td>13</td>
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## Assignments

<table>
<thead>
<tr>
<th>VP</th>
<th>Leipzig</th>
<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
<th>Plants Assigned</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>Marketing (M)</td>
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<tr>
<td>Operations (O)</td>
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<tr>
<td>Personnel (P)</td>
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</tbody>
</table>

## Cost of Assignments

<table>
<thead>
<tr>
<th>VP</th>
<th>Leipzig</th>
<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
<th>Total Cost</th>
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</thead>
<tbody>
<tr>
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<td>Marketing (M)</td>
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<td>0</td>
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<tr>
<td>Operations (O)</td>
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<tr>
<td>Personnel (P)</td>
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<tr>
<td>Total Cost</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
A Challenge

- Find best assignments
Building a Network Model

- In Excel
- Tools | Solver...

Set Target Cell: $F$28
By Changing Cells: $B$15:$E$18
The Constraints

- Each VP assigned to one plant
  - $F$15 = 1
  - $F$16 = 1
  - $F$17 = 1
  - $F$18 = 1
  - Shortcut: $F$15:$F$18 = 1

- Each plant assigned one VP
  - $B$19 = 1
  - $C$19 = 1
  - $D$19 = 1
  - $E$19 = 1
  - Shortcut: $B$19:$E$19 = 1
What’s Missing
Additional Constraints...

- Non-negativity
  - The variables cannot be negative
  - Handled separately

- Integrality
  - The variables should have integral values
  - We can ignore these because this is a network model!!!
Model Components

- Set Target Cell: Objective $F$28
  - The value we want to minimize/maximize
- Equal to: Min
  - Min for Minimize or Max for Maximize
- By Changing Cells:
  - Variables or Adjustables $B$15:$E$18
    - The values we can change to find the answer
- Subject to the Constraints ....
  - $B$19:$B$18 = 1
  - $F$15:$F$18 = 1
Excel Model

Set Target Cell: $F$28
Equal To: Max
Value of: 0
By Changing Cells: $B$15:$E$18
Subject to the Constraints:
$B$19:$E$19 = 1
$F$15:$F$18 = 1
Options

Solver Options

Max Time: 100 seconds
Iterations: 100
Precision: 0.000001
Tolerance: 5%
Convergence: 0.001

- Assume Linear Model
- Assume Non-Negative

Estimates:
- Tangent
- Quadratic

Derivatives:
- Forward
- Central

Search:
- Newton
- Conjugate
Limits

- **Max time**: Limits the time allowed for the solution process in seconds

- **Iterations**: Limits the number of interim calculations. (More details to come)
Precision

- Controls the precision of solutions.
- Is $1/3 \leq 0.3333?\ 0.333333?$
Quality of Solutions

- **Tolerance:** For integer problems. Later

- **Convergence:** For non-linear problems. Later
Review & Terminology

- Objective: Target Cell
- Equal to: Max or Min
- Variables: By Changing Cells
- Constraints: Constraints
  - LHS: Reference Cell - a function of the variables
  - RHS: Constraint - a constant (ideally)
- Options:
  - Assume Linear Model
  - Assume Non-negative
- Solve
What do you think?

- Realistic?
- Practical?
- Issues?
- Questions...
First Kind of Network Model

- Sum across row = Const.
- Sum down column = Const.

Each variable in two constraints:
- A “row” constraint
- A “column” constraint

<table>
<thead>
<tr>
<th>Estimated Assignment Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>VP</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>Finance (F)</td>
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<tr>
<td>Marketing (M)</td>
</tr>
<tr>
<td>Operations (O)</td>
</tr>
<tr>
<td>IT (I)</td>
</tr>
<tr>
<td>Personnel (P)</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Assignments</th>
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</thead>
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<td>VP</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
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<tr>
<td>Personnel (P)</td>
</tr>
<tr>
<td>VPs Assigned</td>
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</table>

<table>
<thead>
<tr>
<th>Cost of Assignments</th>
</tr>
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<tbody>
<tr>
<td>VP</td>
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<td>----</td>
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<tr>
<td>IT (I)</td>
</tr>
<tr>
<td>Personnel (P)</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
</tbody>
</table>
Influence of Optimization

- Changes focus of “negotiation” about assignments
  - from emotion and personal preferences
  - to estimation of cost
Motor Distribution

Netherlands

Amsterdam 500
The Hague 800
Antwerp 700
Tilburg 500
Liege 200
Nancy 900
Leipzig 400

Miles

0 50 100
Transportation Costs

<table>
<thead>
<tr>
<th>From Origin</th>
<th>To Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Leipzig</td>
</tr>
<tr>
<td>Amsterdam</td>
<td>120</td>
</tr>
<tr>
<td>Antwerp</td>
<td>61</td>
</tr>
<tr>
<td>The Hague</td>
<td>102.5</td>
</tr>
</tbody>
</table>

Unit transportation costs from harbors to plants

Minimize the transportation costs involved in moving the motors from the harbors to the plants
# A Transportation Model

## Autopower Transportation Model

### Unit Cost

<table>
<thead>
<tr>
<th>From/To</th>
<th>Leipzig</th>
<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amsterdam</td>
<td>$120.0</td>
<td>$130.0</td>
<td>$41.0</td>
<td>$59.5</td>
</tr>
<tr>
<td>Antwerp</td>
<td>$61.0</td>
<td>$40.0</td>
<td>$100.0</td>
<td>$110.0</td>
</tr>
<tr>
<td>The Hague</td>
<td>$102.5</td>
<td>$90.0</td>
<td>$122.0</td>
<td>$42.0</td>
</tr>
</tbody>
</table>

### Shipments

<table>
<thead>
<tr>
<th>From/To</th>
<th>Leipzig</th>
<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
<th>Total</th>
<th>Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amsterdam</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>500</td>
</tr>
<tr>
<td>Antwerp</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>700</td>
</tr>
<tr>
<td>The Hague</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>800</td>
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<tr>
<td>Total</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Required</td>
<td>400</td>
<td>900</td>
<td>200</td>
<td>500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Total Cost

<table>
<thead>
<tr>
<th>From/To</th>
<th>Leipzig</th>
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<th>Tilburg</th>
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<td>Amsterdam</td>
<td>-</td>
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<td>The Hague</td>
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<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Challenge

- Find a best answer
Building a Solver Model

Tools | Solver...

Set Target Cell: The cell holding the value you want to minimize (cost) or maximize (revenue)

Equal to: Choose Max to maximize or Min to minimize this

By Changing Cells: The cells or variables the model is allowed to adjust
Solver Model Cont’d

Subject to the Constraints: The constraints that limit the choices of the values of the adjustables.

Click on Add

- Cell Reference is a cell that holds a value calculated from the adjustables

- Constraint is a cell that holds a value that constraints the Cell Reference.

- $\leq$, $=$, $=>$ is the sense of the constraint. Choose one.
What are the Constraints?

- **Supply Constraints**
  - Amsterdam: \( G9 \leq H9 \)
  - Antwerp: \( G10 \leq H10 \)
  - The Hague: \( G11 \leq H11 \)

- **Demand Constraints**
  - Leipzig: \( C12 \Rightarrow C13 \)
  - Nancy: \( D12 \Rightarrow D13 \)
  - Liege: \( E12 \Rightarrow E13 \)
  - Tilburg: \( F12 \Rightarrow F13 \)

G9 is the total volume shipped from Amsterdam

Short cut:
\( G9:G11 \leq H9:H11 \)

C12 is the total volume shipped to Leipzig

Short cut:
\( C12:F12 \Rightarrow C13:F13 \)
The Model

Solver Parameters

Set Target Cell: $G$19
Equal To: Min
Value of: 0
By Changing Cells: $C9:F11$

Subject to the Constraints:

- $C12 \geq C13$
- $D12 \geq D13$
- $E12 \geq E13$
- $F12 \geq F13$
- $G10 \leq H10$
- $G11 \leq H11$
What’s Missing?
Options

Max Time: 100 seconds
Iterations: 100
Precision: 0.000001
Tolerance: 5%
Convergence: 0.001

- Assume Linear Model
- Assume Non-Negative
- Use Automatic Scaling
- Show Iteration Results

Estimates:
- Tangent
- Quadratic

Derivatives:
- Forward
- Central

Search:
- Newton
- Conjugate
How the Solver Works

- Not magic
- Quick and intuitive
- Not comprehensive
- Basic understanding of tool and terms
How the Solver works

A “Basic Feasible” solution. This choice of variables admits one and only one answer and the answer is non-negative.
A Basic Feasible Solution

Netherlands
- Amsterdam (500 miles)
- The Hague (800 miles)
- Antwerp (700 miles)

Belgium
- Liege (200 miles)
- Nancy (900 miles)

Germany
- Leipzig (400 miles)

Miles

0 50 100
More Technical Detail

Netherlands

Amsterdam
500
The Hague
800
Antwerp
700

Germany

Tilburg
500
Liege
200
Nancy
900

Leipzig
400

Belgium

Miles

0 50 100

Netherlands

Germany

Leipzig

Belgium

Miles

0 50 100
Mathematically*

- \( z \) are the basic variables
- \( y \) are the non-basic variables
- Write the constraints as
  \[
  Ax = Bz + Ny = b
  \]
- Fix the non-basic variables to \( y^* \)
- The unique solution for the basic variables
  \[
  x = B^{-1}(b - Ny^*)
  \]
- \( B \) must be invertible and so square
- Question: We have 7 constraints (3 ports, 4 plants) and only 6 basic variables. How so?

* For those who care to know
How Solver Works

Basic Feasible Solution because it is basic AND the basic variables satisfy the bounds ($\geq 0$)
Simple Improvement

Consider sending product from The Hague to Liege
Conserving Flow

Amsterdam

The Hague

Antwerp

Tilburg

Liege

Nancy
Conserving Flow

Costs
$122
$40
$162

Saves
$100
$90
$190

Net
$28 Reduced Cost
How Much Can We Save?

Current Flows
Red flows increase
Green flows decrease

Costs
$122
$40
$162

Saves
$100
$90
$190

Net
$28

How much can we send to Liege?
How much do we save?
What’s the new answer?
New Answer

Costs
$122
$40
$162

Saves
$100
$90
$190

Net
$28

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The New Answer

Netherlands

Amsterdam

The Hague

Antwerp

Belgium

Tilburg

Germany

Nancy

Liege

Leipzig

Miles

0 50 100
What kind of solution is this?
An Optimal
Basic Feasible Solution
Summary

 Solver

► Finds a basic feasible solution
  ▪ Satisfies all the constraints
  ▪ Using these variables there is just one answer

► Computes reduced costs of non-basic variables one at a time
  ▪ How would increasing the new variable affect cost?

► Selects an entering variable
  ▪ Increasing this non-basic variable saves money

► Computes a leaving variable
  ▪ What basic variable first reaches zero?

► Repeats
Sensitivity Analysis

- How would the answer change if the data were a little different?
- Why is this important?
- Intuitive understanding
At what unit cost would Antwerp to Tilburg be attractive?
Price Sensitivity

At what unit cost would Antwerp to Tilburg be attractive?

The Hague  Amsterdam

$42

The Hague  Antwerp

$110

Antwerp  Tilburg

$40

Antwerp  Liege

$90

Liege  Tilburg

$90

Tilburg  Leipzig

$82

Nancy  Liege

$42

Nancy  The Hague

$40

The carrier would have to PAY us $8!

Costs:

$110
$90
$200

Savings:

$42
$40
$82

Net $118
## Autopower Transportation Model

### Unit Cost

<table>
<thead>
<tr>
<th>From/To</th>
<th>Leipzig</th>
<th>Nancy</th>
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</tr>
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<tbody>
<tr>
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<tbody>
<tr>
<td>Amsterdam</td>
<td>-</td>
<td>-</td>
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<tr>
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Reduced Costs are...

- The reduced cost of a variable is...
  The rate of change in the objective if we are forced to use some of that variable

- The reduced costs of basic variables are 0
Price Sensitivity: Basic Variables

Amsterdam
The Hague
Antwerp
Nancy
Tilburg
Leipzig

If the unit cost from Amsterdam to Liege increases by $10 does the answer change?
Checking Reduced Costs: Example

If the unit cost from Amsterdam to Liege increases by $10 does the answer change?

Example: Check the reduced cost of Antwerp to Liege

$51 = $41 + $10

If the unit cost from Amsterdam to Liege increases by $10 does the answer change?
Check All Reduced Costs

Amsterdam
The Hague
Antwerp
Liege
Tilburg
Leipzig
Nancy

Why only these two?
Value of Price Sensitivity?
Resource Sensitivity

- How would the objective value change if we had more of a resource
- Tells us the marginal value of that resource
- If the optimal solution doesn’t use all of the resource, then...
Resource Sensitivity

Amsterdam 500

The Hague 700

Antwerp 800

Tilburg 500

Leipzig 401

Liege 200

Nancy 900

What would happen if we needed 1 more engine in Leipzig?
Infeasible

Supply
- Amsterdam 500
- Antwerp 700
- The Hague 800
- Total 2,000

Demand
- Leipzig 400+1
- Nancy 900
- Liege 200
- Tilburg 500
- Total 2,000+1
What would happen to cost if we moved supply for 1 engine from Amsterdam to Antwerp?
What’s the value of supply in Antwerp relative to supply in Amsterdam?
Value of Resource Sensitivity
A Special Feature

- We can eliminate any one of the constraints in this problem without changing the answers!

- Why?
Redundant Constraint

- **Supply**
  - Amsterdam 500
  - Antwerp 700
  - The Hague 800
  - Total 2,000

- **Demand**
  - Leipzig 400
  - Nancy 900
  - Liege 200
  - Tilburg 500
  - Total 2,000

- Know shipments from
  - Amsterdam
  - Antwerp

- And they provide...
  - Leipzig 200
  - Nancy 600
  - Liege 0
  - Tilburg 400
That means...

- We can arbitrarily set the (relative) value of one constraint to 0. (the one we throw away)
- Set the shadow price or marginal value of supply in Amsterdam to 0, then the shadow price of supply in Antwerp is -$67.5.
- Why negative?
- If we had extra supply, where would we want it? Amsterdam or Antwerp?
Internally Consistent

Given the Shadow Prices

<table>
<thead>
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<th>Constraint</th>
<th>Shadow Price</th>
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<tr>
<td>Amsterdam</td>
<td>-</td>
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<tr>
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</tr>
<tr>
<td>The Hague</td>
<td>-17.5</td>
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<td>Leipzig</td>
<td>120</td>
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<tr>
<td>Nancy</td>
<td>107.5</td>
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<tr>
<td>Liege</td>
<td>41</td>
</tr>
<tr>
<td>Tilburg</td>
<td>59.5</td>
</tr>
</tbody>
</table>

Example: Antwerp-Tilburg

Reduced Cost = Cost - Origin - Dest.

\[ 118 = 110 - (-67.5) - 59.5 \]

\[ = 110 + 8 \]

We can calculate the Reduced Costs

<table>
<thead>
<tr>
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<th>Cost</th>
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</tr>
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</table>
Summary

- Solver can tell us at what price a non-basic (inactive) variable will be attractive through the Reduced Cost.
- Solver can tell us how changes in the price of a basic variable affect the solution.
- Solver can tell us the value of a resource via the Shadow Price or Marginal Value.
Sensitivity Info From Solver

![Solver Results dialog box]

Solver found a solution. All constraints and optimality conditions are satisfied.

- Keep Solver Solution
- Restore Original Values

Reports
- Answer
- Sensitivity
- Limits

Buttons:
- OK
- Cancel
- Save Scenario...
- Help
# Sensitivity Report: Price

## Adjustable Cells

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
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<th>Reduced Cost</th>
<th>Objective Coefficient</th>
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<th>Allowable Decrease</th>
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</table>
### Sensitivity Report: Resource

#### Constraints

<table>
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<th>Allowable Decrease</th>
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</table>
Value

- If our proposal comes up non-basic, reduced cost tells us how much harder we have to work to make it attractive.
- If we are unsure of prices, price sensitivity can tell us whether it is worth refining our estimates of the values.
- Marginal values can help us target investments in capacity.
Caveats

- Sensitivity Analysis is pretty nerdy stuff
- Technical difficulties
- Only meaningful for changes to a single value
- Only meaningful for *small* changes
- Doesn’t work for Integer Programming
- Can always just change the values and re-solve, but...
# Bad Example

## Autopower Transportation Model

<table>
<thead>
<tr>
<th>From/To</th>
<th>Leipzig</th>
<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
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<tbody>
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## Shipments

<table>
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<tr>
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<th>Nancy</th>
<th>Liege</th>
<th>Tilburg</th>
<th>Total</th>
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## Total Cost

<table>
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<th>Liege</th>
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</tbody>
</table>

15.057 Spring 03 Vande Vate
Sensitivity

Moving one unit from Liege to Tilburg should save $120

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
<th>Final Value</th>
<th>Shadow Price</th>
<th>Constraint R.H. Side</th>
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<tr>
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<tr>
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Solver says the price is not good for 1 unit
## Autopower Transportation Model

### Unit Cost

<table>
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### Shipments

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<th>Tilburg</th>
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<td>$500</td>
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### Total Cost

<table>
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<th>Liege</th>
<th>Tilburg</th>
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</tbody>
</table>
Thursday

- Spreadsheet Models
  - 03ShortestPathModel.xls
  - 04TransshipmentModel.xls
  - 05SingaporeElectricGeneratorModel.xls

- Download the free student version of the AMPL/CPLEX 8.0 System from www.ampl.com
  - http://www.ampl.com/cm/cs/what/ampl/DOWNLOADS/cplex71.html#new

- AMPL Example Model
  - TransportationModel.mod

- Access Database
  - TransportationData.mdb