Simulation: Conley Fisheries

Summer 2003
Outline

- Motivating the need for simulation
- The Conley Fisheries problem
- Random number generators
- Sampling from discrete and continuous distributions
- Simulating the Conley Fisheries Problem in Excel (without Crystal Ball)
- Analyzing the Simulation Results…
The Need for Simulation

Consider the following simple mathematical model:

Profits = total revenue – total cost

If we know the data with certainty (as we supposedly do when preparing financial statements based on historical data), calculating profits is trivial…

However, calculating future profits requires that we consider the uncertainties associated with revenues and costs (i.e. revenues and cost are random variables)

These uncertainties result in a large number of profit scenarios…

Just looking at a single profit figure based on expected values totally ignores the fact that actual results may deviate significantly from the expected value of profits.
In general, we see that profits, \( P \), is a function of two random variables: total revenue, \( R \), and total cost, \( C \):

\[
P = f(R, C)
\]

Since \( P \) is a function of random variables \( R \) and \( C \), then \( P \) is also a random variable.

Deriving the probability distribution for functions of random variables is impossible in most practical cases.

Using simulation, we can calculate the distribution of the random variable of interest empirically by generating a large number of scenarios for the underlying r.v.s.

In this case, we can generate scenarios for \( P \) by generating random values of \( R \) and \( C \), and then construct the empirical distribution of \( P \) (frequency table/Histogram).

The probability distribution of \( P \) gives us the detailed information we need to answer many questions....
Conley Fisheries Inc.: “one boat problem”

For the Conley Fisheries example, please see Chapter 5 in the course textbook.
Equivalent Distribution for Q

<table>
<thead>
<tr>
<th>Quantity (Q in lbs)</th>
<th>Probability</th>
<th>CDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>1,000</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>2,000</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>3,000</td>
<td>0.08</td>
<td>0.18</td>
</tr>
<tr>
<td>3,500</td>
<td>0.82</td>
<td>1.00</td>
</tr>
</tbody>
</table>

E(Q) = 3240 lbs
Since P and Q are independent we have that:

E(R) = (3.65 $/lb) (3240 lbs) - 10000 = $1826

*Notice that the units of E(R) are $ = ($/lb) (lbs)

In practice, calculating the expected value of a simulated random variable is not straightforward...
General procedure to generate random values for a r.v.

i. Generate a uniform random value, v, in the interval [0,1]

ii. Use v to calculate a random value, x, for the probability distributions of interest using the inverse of the CDF:

\[ x = F^{-1}(v) \]
1. Generate a random value, q, for quantity (r.v. Q)
2. Generate a random value, p, for price (r.v. P)
3. Calculate Rockport profits: \( qp - 10,000 \) ($)

To perform 1 and 2 we use a random number generator (a uniform \([0,1]\) r.v.) to:

i. Generate a random value, \( v \), in the interval \([0,1]\)

ii. Use \( v \) to calculate a random value, \( x \), for the probability distributions of interest using the inverse of the CDF:

\[ x = F^{-1}(v) \]
Generating a random value for Q

1. Generate a random value, \( v \), in the interval \([0,1]\) (e.g., using Excel RAND() we get \( v=0.0345 \))

2. Generate a random value, \( d \), by reading the CDF table for \( d \) backwards (implementing this in Excel is easy!):

   In this case, \( d=1000 \) since \( F(0)=0.02 < v \leq 0.05=F(1000) \)

3. Calculate \( q=\min(3500,d)=1000 \) lbs.

(Important!: Notice that we arbitrarily assign the upper end of the interval if \( v=F() \). In theory, this is a probability zero event. However, this can happen in practice as computers have finite architectures! So, if \( v=.0500 \), then \( d=1000 \), not the next higher value)
Reading the CDF for Q
1. Generate a random value, v, in the interval [0,1] (e.g. using Excel RAND() we get v=0.4321)

2. Read the CDF table for Z N(0,1) backwards or use Excel NORMINV(). In this case z= -0.17103

3. Use the z formula to calculate a value for X N(3.65 $/lb, 0.20 $/lb): \( x = \mu + z\sigma = 3.65 + (-0.17103)(0.20) = 3.62^* \)

4. Calculate P = max (0,x)= 3.62 $/lb

(* Notice that x can be calculated using NORMINV() if we give 3.65 and .20 as the mean and SD arguments instead of 0 and 1, i.e. N[3.65,.20].)
A Random Value for P

Cumulative Distribution Function of the Random Variable P (Price)

F(P) vs. Price

<table>
<thead>
<tr>
<th>F(P)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>3.25</td>
</tr>
<tr>
<td>0.5</td>
<td>3.65</td>
</tr>
<tr>
<td>1.0</td>
<td>4.05</td>
</tr>
</tbody>
</table>
Out of Sight, Out of Mind

- Remember that simulations are driven by the decision maker’s understanding.
- If you don’t consider key sources of uncertainty, you can’t identify their contributions.
- If you don’t include branches in your decision tree or outcomes on the branches, they won’t be considered in your decision.
Fault Tree Example

Car won’t start

- Low battery
- Starting system fault
  - Switches defective
    - Ignition
    - Starter relay
  - Seat belt problem
    - Not fastened
- Fuel system fault
- Ignition system fault
- Other engine fault
- Vandal-ism
- All others
- Heavy object on seat
Fault Tree Example

Car won’t start

- Low battery
- Fuel system fault
- Other engine fault
- All others
## Results

<table>
<thead>
<tr>
<th>Students</th>
<th>Battery</th>
<th>Starting system</th>
<th>Fuel system</th>
<th>Ignition system</th>
<th>Engine</th>
<th>Mischief</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>26%</td>
<td>20%</td>
<td>19%</td>
<td>14%</td>
<td>7%</td>
<td>5%</td>
<td>8%</td>
</tr>
<tr>
<td>Pruned</td>
<td>43%</td>
<td>31%</td>
<td>12%</td>
<td></td>
<td></td>
<td></td>
<td>14% (47%)</td>
</tr>
<tr>
<td>Car Mechanics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full</td>
<td>41%</td>
<td>11%</td>
<td>10%</td>
<td>25%</td>
<td>5%</td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>Pruned</td>
<td>48%</td>
<td>23%</td>
<td>7%</td>
<td></td>
<td></td>
<td></td>
<td>22% (45%)</td>
</tr>
</tbody>
</table>
Summary and Look Ahead

- Simulation is useful to understand the impact of uncertainty on the distribution of expected outcomes.
- Fortunately, computers can be a big help in generating large numbers of scenarios.
- Computers can’t think by themselves; they need you to tell them what to think about (out of sight, out of mind).
- Prepare the Gentle Lentil case for tomorrow but nothing to hand in. Try to use Crystal Ball, but you are not required to hand anything in.