**Problem 1** For the following questions/statements just give TRUE or FALSE answers. Do not derive the answers.

Consider a G/G/1 queueing system. The arrival rate is \( \lambda \) and service rate is \( \mu > \lambda \).

A. Let \( L_{10} \) be the steady state number of customers in positions 1 to 10 (customer in service is assumed to be in position 1). Let \( S_{10} \) be the steady state time that a typical customer was in one of the positions 1-10. Then the Little’s Law holds, namely \( \mathbb{E}[L_{10}] = \lambda \mathbb{E}[S_{10}] \).

B. The distributional law holds for \( L_{10} \) and \( S_{10} \) when the scheduling policy is
   
   (i) First-In-First-Out
   
   (ii) Last-In-First-Out

C. Suppose that the system has instead two servers (that is we have G/G/2 queueing system). Then the probability that the system is empty is \( 1 - \rho \), where \( \rho = \frac{\lambda}{2\mu} \).

**Problem 2** Consider an M/G/1 queueing system where arrival rate \( \lambda = 1 \) and service time with a mixed distribution. Namely, with probability 1/2 it is exponential with rate 2 and with probability 1/2 it is exponential with rate 3. Assume the system operates under the First-In-First-Out policy.

A. Compute the traffic intensity \( \rho \) of this system.

B. Suppose the revenue obtained from a customer is \( e^{-cy} \) if the customer waited \( y \) time units. Compute the expected steady state revenue when \( c = 1 \).

C. Extra credit. Suppose for every customer who waited \( y \) time units the cost \( e^{cy} \) is paid. What is the largest \( c_0 \) for which the expected cost is finite? Is \( c_0 < 1.9 \)?

**Problem 3** M/D/1 Queueing system with feedback. Namely each served customer comes back to the queue with some probability \( p \) and leaves the system with probability \( 1 - p \). The service time is assumed to be deterministic with value \( d \) both for the initial and returning customers. The arrival process is Poisson with rate \( \lambda \).
• Under which conditions on $\lambda, d, p$ is there a steady-state regime? Do not prove this, just provide a right answer.

• Compute the expected number of customers in the queue.
  
  HINT: Observe that the number in the system is policy invariant for all non-preemptive work conserving scheduling policies.

Useful facts

The expected waiting time in M/G/1 queueing system under First-Come-First Serve policy is

$$E[W] = \frac{\lambda E[X^2]}{2(1 - \rho)},$$

and the Laplace Transform of the waiting time is

$$\phi_W(s) = \frac{(1 - \rho)s}{\lambda \beta(s) - \lambda + s},$$

where $\lambda$ is the arrival rate, $X$ is service time, $\rho = \lambda E[X]$ and $\beta(s)$ is the Laplace Transform of the service time distribution.