Today’s Agenda

Why study facility location?
Issues to be modeled
Basic models
  Fixed charge problems
  Core uncapacitated and capacitated facility location models
Large-scale application (Hunt-Wesson Foods)
Logistics Industry

U.S. logistics industry: $900 billion - almost double the size of the high-tech industry: > 10 percent of the U.S. gross domestic product

11 per cent of Singapore's GDP with a growth of 9 per cent in year 2000

Singapore Logistics Enhancement & Applications Programme (LEAP) 2001

Global logistics: $3.43 trillion

1998, U.S. trucking industry revenues just under $200 billion

7.7 million trucks carried over 1 trillion ton miles of freight
Singapore Retail 21 Plan

**Re-invent the Retail Sector**
- E-enable the business
- Harness innovative retail technologies
- Adopt new retail concepts and business models

**Enhance Industry Efficiency**
- Integrate the value chain
- Strengthen shopping centre management
- Enhance the distribution structure
- Improve cost structure and backend support operations

**Raise Retailing Standards**
- Raise the professionalism of the retailing workforce
- Raise the image of the retail sector
- Promote service differentiation
  - Attract and retain staff

**Manage the Restructuring of HDB Retail Sub-Sector**
- Reduce redundant retail space
- Provide an enhanced package of assistance programmes
- Transform the mindset of HDB retailers
- Embark on cost management
- Identify regulatory measures

**Vision**
To be a World-class Centre of Retail Excellence
Basic Issue

Where to locate and how to size facilities?

How to meet customer demands from the facilities?

Which facility (facilities) serve each customer?

How much customer demand is met by each facility?

Facilities might be warehouses, retail outlets, wireless bay stations, communication concentrators
Some Elements of Cost & Service

Transportation Costs
  - Vehicles, Drivers, Fuel

Warehousing
  - Facility Construction/Rental, Handling Costs, Inventory

Customer Service
  - Service Time, Single Sourcing
System Trade-offs

Transportation Costs

Fixed Costs

Effect of More Facilities
System Trade-offs

Effect of “Individualized” Service (e.g., Direct Shipments)
Nature of Costs

**Fixed Costs**
- Facility construction/rental
- Vehicle purchases & rentals
- Personnel (drivers, managers)
- Fixed overhead

**Variable Costs**
- Inventory, handling, fuel
Hunt-Wesson Foods saves over $1 million per year
Restructuring North America operations, Proctor and Gamble reduces plants by 20%, saving $200 million/year
Many, many others (e.g., supplying parts to plants)
Facility Location Challenge
Modeling Issue

How do we model “lumpiness” of the costs (e.g., fixed costs)?

How do we model logical conditions (e.g., choice of warehouse locations)?
Modeling Fixed Costs

Flow $x \geq 0$
Flow $z > 0$

Incur fixed cost $F$ if either $x > 0$ or $z > 0$

Suppose $x + z \leq \frac{3}{2}$ (demand limitation)

Model

Minimize $Fy + \text{other terms}$
subject to $y = 1$ if either $x > 0$ or $z > 0$
Three Models (LP Relaxations)

Model 1
- \(x + z < 3/2\)
- \(x < 1, z < 1\)
- \(x + z < 2y\)
- \(x > 0, z > 0\)
- \(0 < y < 1\)

Model 2
- \(x + z < 3/2\)
- \(x < 1, z < 1\)
- \(x < y, z < y\)
- \(x > 0, z > 0\)
- \(0 < y < 1\)

Model 3
- \(x + z < 3y/2\)
- \(x < 1, z < 1\)
- \(x < y, z < y\)
- \(x > 0, z > 0\)
- \(0 < y < 1\)
Geometry (Weak Model)

Feasible region with $x \leq 1$, $y \leq 1$, $z \leq 1$, $x + z \leq 3/2$

$x + z \leq 3/2$

Feasible points

$x + z < 2y$
intersects at
$y = 3/4$
Geometry (Improved Model)

\[ x + z < \frac{3}{2} \]

intersects at
\[ x = z = y = \frac{3}{4} \& at \]
\[ y = 1 \] w/ x or z = 1
Geometry (Strong Model)

Exact representation!

\[ x + z \leq \frac{3y}{2}, \]
\[ x < y, \quad z < y \]
intersects at \( y = 1 \)
Core (Uncapacitated) Facility Location

Minimize Fixed + Routing Costs

Subject to

Meet customer demand from facilities

Assign customer only to open facility
Parameters:

Core Facility Location Model

Demand $d_i$ for each customer $i$
Fixed cost $F_j$ for each facility location $j$
Cost $c_{ij}$ of routing all customer $i$ demand to facility $j = \text{per unit cost times demand } d_i$
Decisions: Core Facility Location Model

Where do we locate facilities?

\[ y_j = 1 \text{ if we locate facility at location } j \]

Fraction of service that customer \( i \) receives from facility \( j \) (\( x_{ij} \))
Network Representation
3 Customers, 4 Facilities

Customers

Facilities

\[ d_1 \rightarrow 1 \]
\[ d_2 \rightarrow 2 \]
\[ d_3 \rightarrow 3 \]

\[ x_{ij} \]

\[ y_j \]
Facility Location Costs

\[ c_{11}x_{11} + c_{12}x_{12} \]
\[ + c_{13}x_{13} + c_{14}x_{14} \]
\[ + \ldots \]
\[ + c_{31}x_{31} + c_{32}x_{32} \]
\[ + c_{33}x_{33} + c_{34}x_{34} \]
\[ + F_1y_1 + F_2y_2 + F_3y_3 + F_4y_4 \]
### Constraints: Tabular Representation

#### Facilities (Locations)

<table>
<thead>
<tr>
<th></th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{21}$</td>
<td>$x_{22}$</td>
<td>$x_{23}$</td>
<td>$x_{24}$</td>
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<tr>
<td>$x_{31}$</td>
<td>$x_{32}$</td>
<td>$x_{33}$</td>
<td>$x_{34}$</td>
<td></td>
</tr>
</tbody>
</table>

$= 1$

#### Customers

$\begin{align*}
x_{11} & \leq y_1, & x_{12} & \leq y_2, & x_{13} & \leq y_3, & x_{14} & \leq y_4 \\
x_{21} & \leq y_1, & x_{22} & \leq y_2, & x_{23} & \leq y_3, & x_{24} & \leq y_4 \\
x_{31} & \leq y_1, & x_{32} & \leq y_2, & x_{33} & \leq y_3, & x_{34} & \leq y_4
\end{align*}$
Model
(Uncapacitated Facilities)

Minimize $\sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j$

Subject to

$\sum_j x_{ij} = 1$ for all customers $i$

$x_{ij} \leq y_j$ for all customers $i$

$x_{ij} \geq 0$ and facilities $j$

$y_j = 0$ or $1$ for all facilities $j$
Open at most three of facilities 1, 6 and 8-11

\[ y_1 + y_6 + y_8 + y_9 + y_{10} + y_{11} \leq 3 \]

Assign each customer to a single facility

\[ x_{11} \text{ integer, } x_{12} \text{ integer, etc.} \]
Open a facility at location 3 only if we open one at location 7

\[ y_3 \leq y_7 \]

**Note:** Power of using integer variables to model logical restrictions
Modeling Enhancements

Multiple products
Facility capacities and operating ranges (min and max throughput if open)
Multi-layered distribution networks
Service restrictions
  - Single sourcing
  - Timing of deliveries
Inventory positioning and control
Alternate Model (Uncapacitated Facilities)

Minimize \( \sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j \)

Subject to

\( \sum_j x_{ij} = 1 \) for all customers \( i \)
\( \sum_i x_{ij} \leq n y_j \) for all facilities \( j \)
\( x_{ij} \geq 0 \) for all pairs \( i,j \)
\( y_j = 0 \) or 1 for all facilities \( j \)
Alternate Model (Uncapacitated Facilities)

Minimize \( \sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j \)

Subject to

\( \sum_j x_{ij} = d_i \) for all customers \( i \)

\( \sum_i x_{ij} \leq (\sum_i d_i) y_j \) for all facilities \( j \)

\( x_{ij} \geq 0 \) for all pairs \( i,j \)

\( y_j = 0 \text{ or } 1 \) for all facilities \( j \)
Model (Capacitated Facilities)

Minimize $\sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j$

Subject to

$\sum_j x_{ij} = d_i$ for all customers $i$

$x_{ij} \leq d_i y_j$ for all $i, j$ pairs

$\sum_i x_{ij} \leq \text{CAP}_j y_j$ for all facilities $j$

$x_{ij} \geq 0$ for all $i, j$ pairs

$y_j = 0$ or $1$ for all facilities $j$
### Locations

<table>
<thead>
<tr>
<th>Customers</th>
<th>Locations</th>
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<td>$x_{32}$</td>
</tr>
</tbody>
</table>

\[ K_1 y_1 \leq d_1 \]
\[ K_2 y_2 \leq d_2 \]
\[ K_3 y_3 \leq d_3 \]

\[ \text{plus cell constraints } x_{ij} \leq d_i y_j \]
Network Representation

Customers

Facilities

$X_{ij}$ $Y_j$

$d_1$ $d_2$ $d_3$

$K_1$ $K_2$ $K_3$ $K_4$
Solution Approaches

- **Heuristics**
  - Add, drop, and/or exchange
  - Linear programming relaxation
  - Bounding (Lagrangian relaxation)

- **Optimization methods**
  - Large-scale mixed integer programming
  - Benders decomposition
  - Lagrangian relaxation (e.g., dualize capacity constraints to give uncapacitated facility location subproblem)
Hunt-Wesson Foods
Ingredients

- Multiple products
- Multiple plants
- Many DCs, many customers
- Site selection and sizing
- Customer service levels
- Complex costs
Flows

i \rightarrow j \rightarrow k

14 plants
45 DC Choices
121 Customer Zones
17 Product Groups $p$
Data Preprocessing

49 product-plant combinations
(from 14x17 = 238)

682 DC-customer zone combinations
(from 45x121 = 5,445 possibilities)
Data Preprocessing

23,513 product-plant-DC-customer combinations
(from 49x682 = 33,418 possibilities)
System Requirements

Data easy to acquire
Inexpensive/quick to run
Easily updated
User-oriented
Flexible (what if capabilities)
Measurable benefits
Indices

\( p = \text{products} \)
\( i = \text{plants} \)
\( j = \text{distribution centers} \)
\( k = \text{customer zones} \)
Decision Variables

\( x_{pijk} = \text{amount of product } p \text{ shipped from plant } i \text{ to customer zone } k \text{ through DC } j \)

\( z_j = 1 \text{ if DC } j \text{ open} \)

\( y_{jk} = 1 \text{ if DC is sole source of customer zone } k \)
Constraints

\[ \sum_{jk} x_{pijk} \leq S_{pi} \]
\[ \sum_i x_{pijk} = D_{pk}y_{jk} \]
\[ \sum_j y_{jk} = 1 \]
\[ V_jz_j \leq \sum_{pk} D_{pk}y_{jk} \leq V_jz_j \]
\[ x_{pijk} \geq 0 \]
\[ z_j, y_{jk} = 0 \text{ or } 1 \]

+ Configuration Constraints on y,z
Objective

\[ \sum_{pijk} c_{pijk} x_{pijk} \quad \text{Transportation Cost} \]

\[ + \sum_{j} f_{j} z_{j} \quad \text{Fixed DC Cost} \]

\[ + \sum_{j} v_{j} \sum_{pk} D_{pk} y_{jk} \quad \text{Variable DC Cost} \]
Model Development

Aggregation of Data
Preselection of Certain Decisions in Large Applications
Choice of Models

Why integer instead of linear programming?
Power of Integer Programming

- Fixed costs
- Bounding # of facilities
- Precedence constraints
- Mandatory service constraints
  - Sole sourcing
- Service timing
Stages in Model Development

Probationary analysis
Analyzing results
  Sensitivity analysis
  What if analysis
  Priority analysis
Today’s Lessons

Facility location and distribution important in practice

Geometry of fixed cost modeling

Model choice is important in problem solving

Strong vs. weak forcing constraints

Optimization models are able to solve large-scale practical problems