15.401 Finance Theory

MIT Sloan MBA Program

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Lectures 2–3: Present Value Relations

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Critical Concepts

- Cashflows and Assets
- The Present Value Operator
- The Time Value of Money
- Special Cashflows: The Perpetuity
- Special Cashflows: The Annuity
- Compounding
- Inflation
- Extensions and Qualifications

Readings:
- Brealey, Myers, and Allen Chapters 2–3
Key Question: What Is An “Asset”?  
- Business entity  
- Property, plant, and equipment  
- Patents, R&D  
- Stocks, bonds, options, …  
- Knowledge, reputation, opportunities, etc.

From A Business Perspective, An Asset Is A **Sequence** of Cashflows

\[
\text{Asset}_t \equiv \{ CF_t, CF_{t+1}, CF_{t+2}, \ldots \}
\]
Examples of Assets as Cashflows

- Boeing is evaluating whether to proceed with development of a new regional jet. You expect development to take 3 years, cost roughly $850 million, and you hope to get unit costs down to $33 million. You forecast that Boeing can sell 30 planes every year at an average price of $41 million.

- Firms in the S&P 500 are expected to earn, collectively, $66 this year and to pay dividends of $24 per share, adjusted to index. Dividends and earnings have grown 6.6% annually (or about 3.2% in real terms) since 1926.

- You were just hired by HP. Your initial pay package includes a grant of 50,000 stock options with a strike price of $24.92 and an expiration date of 10 years. HP’s stock price has varied between $16.08 and $26.03 during the past two years.
Valuing An Asset Requires Valuing A Sequence of Cashflows

- Sequences of cashflows are the “basic building blocks” of finance

\[
\text{Value of Asset}_t \equiv V_t(\text{CF}_t, \text{CF}_{t+1}, \text{CF}_{t+2}, \ldots)
\]

Always Draw A Timeline To Visualize The Timing of Cashflows
The Present Value Operator

What is \( V_t \)?
- What factors are involved in determining the value of any object?
  - Subjective?
  - Objective?
- How is value determined?

There Are Two Distinct Cases
- No Uncertainty
  - We have a complete solution
- Uncertainty
  - We have a partial solution (approximation)
  - The reason: synergies and other interaction effects
- Value is determined the same way, but we want to understand how
The Present Value Operator

Key Insight: Cashflows At Different Dates Are Different “Currencies”
- Consider manipulating foreign currencies

￥150 + £300 = ??
Key Insight: Cashflows At Different Dates Are Different “Currencies”

- Consider manipulating foreign currencies

\[ ¥150 + £300 = ??450 \]

- Cannot add currencies without first converting into common currency

\[ ¥150 + (£300) \times (153 ¥ / £) = ¥46,050.00 \]
\[ (¥150) \times (0.0065 £ / ¥) + £300 = £300.98 \]

- Given exchange rates, either currency can be used as “numeraire”
- Same idea for cashflows of different dates
The Present Value Operator

Key Insight: Cashflows At Different Dates Are Different “Currencies”

- Past and future cannot be combined without first converting them
- Once “exchange rates” are given, combining cashflows is trivial

A **numeraire** date should be picked, typically t=0 or “today”

- Cashflows can then be converted to **present value**

\[
V_0(CF_1, CF_2, CF_3, \ldots) = \left(\frac{\$1}{\$0}\right) \times CF_1 + \left(\frac{\$2}{\$0}\right) \times CF_2 + \cdots
\]
The Present Value Operator

Net Present Value: “Net” of Initial Cost or Investment
- Can be captured by date-0 cashflow $CF_0$

$$V_0(CF_0, CF_1, \ldots) = CF_0 + \left(\frac{\$1}{\$0}\right) \times CF_1 + \left(\frac{\$2}{\$0}\right) \times CF_2 + \cdots$$

- If there is an initial investment, then $CF_0 < 0$
- Note that any $CF_t$ can be negative (future costs)
- $V_0$ is a completely general expression for net present value

How Can We Decompose $V_0$ Into Present Value of Revenues and Costs?
The Present Value Operator

Example:

- Suppose we have the following “exchange rates”:

\[
\left( \frac{S_1}{S_0} \right) = 0.90, \quad \left( \frac{S_2}{S_0} \right) = 0.80
\]

- What is the net present value of a project requiring a current investment of $10MM with cashflows of $5MM in Year 1 and $7MM in Year 2?

\[
NPV_0 = -S_0 + 5 \times 0.90 + 7 \times 0.80 = $0.10
\]

- Suppose a buyer wishes to purchase this project but pay for it two years from now. How much should you ask for?
The Present Value Operator

Example:
- Suppose we have the following “exchange rates”:

\[
\left( \frac{\$1}{\$0} \right) = 0.90 \quad , \quad \left( \frac{\$2}{\$0} \right) = 0.80
\]

- What is the net present value of a project requiring an investment of $8MM in Year 2, with a cashflow of $2MM immediately and a cashflow of $5 in Year 1?

\[
NPV_0 = 2 + 5 \times 0.90 \times 0 - 8 \times 0.80 = 0.10
\]

- Suppose a buyer wishes to purchase this project but pay for it two years from now. How much should you ask for?
The Time Value of Money

Implicit Assumptions/Requirements For NPV Calculations
- Cashflows are known (magnitudes, signs, timing)
- Exchange rates are known
- No frictions in currency conversions

Do These Assumptions Hold in Practice?
- Which assumptions are most often violated?
- Which assumptions are most plausible?

Until Lecture 12, We Will Take These Assumptions As Truth
- Focus now on exchange rates
- Where do they come from, how are they determined?
The Time Value of Money

What Determines The Growth of $1 Over T Years?

- $1 today should be worth more than $1 in the future (why?)
- Supply and demand
- **Opportunity cost of capital** $r$

\[
\begin{align*}
\text{$1$ in Year } 0 & = \text{$1$ in Year } 1 \\
& = \text{$1$ in Year } 2 \\
& \vdots \\
\text{$1$ in Year } T & = \text{$1$ in Year } T
\end{align*}
\]

- Equivalence of $1 today and any other single choice above
- Other choices are **future values** of $1 today
What Determines The Value Today of $1 In Year-T?

- $1 in Year-T should be worth less than $1 today (why?)
- Supply and demand
- Opportunity cost of capital $r$

\[
\frac{1}{1 + r} \text{ in Year 0} = \frac{1}{1 + r} \text{ in Year 1} \\
\frac{1}{(1 + r)^2} \text{ in Year 0} = \frac{1}{1 + r} \text{ in Year 2} \\
\vdots \\
\frac{1}{(1 + r)^T} \text{ in Year 0} = \frac{1}{1 + r} \text{ in Year } T
\]

- These are our “exchange rates” ($\frac{t}{0}$) or discount factors
The Time Value of Money

We Now Have An Explicit Expression for \( V_0 \):

\[
V_0 = CF_0 + \frac{1}{(1 + r)} \times CF_1 + \frac{1}{(1 + r)^2} \times CF_2 + \cdots
\]

\[
V_0 = CF_0 + \frac{CF_1}{(1 + r)} + \frac{CF_2}{(1 + r)^2} + \cdots
\]

- Using this expression, any cashflow can be valued!
- **Take positive-NPV projects, reject negative NPV-projects**
- Projects ranked by magnitudes of NPV
- All capital budgeting and corporate finance reduces to this expression
- However, we still require many assumptions (perfect markets)
The Time Value of Money

Example:

- Suppose you have $1 today and the interest rate is 5%. How much will you have in ...

  - 1 year … $1 \times 1.05 = $1.05
  - 2 years … $1 \times 1.05 \times 1.05 = $1.103
  - 3 years … $1 \times 1.05 \times 1.05 \times 1.05 = $1.158

- $1 today is equivalent to $1 \times (1 + r)^t$ in $t$ years

- $1$ in $t$ years is equivalent to $\frac{1}{(1+r)^t}$ today
The Time Value of Money

PV of $1 Received In Year $t$

- $r = 0.04$
- $r = 0.08$
- $r = 0.12$

Year when $1$ is received

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
The Time Value of Money

Example:
Your firm spends $800,000 annually for electricity at its Boston headquarters. Johnson Controls offers to install a new computer-controlled lighting system that will reduce electric bills by $90,000 in each of the next three years. If the system costs $230,000 fully installed, is this a good investment?

Lighting System*

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* Assume the cost savings are known with certainty and the interest rate is 4%
The Time Value of Money

Example:

Lighting System

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</table>

\[
NPV = -230,000 + 86,538 + 83,210 + 80,010 = $19,758
\]

- Go ahead – project looks good!
The Time Value of Money

Example:
CNOOC recently made an offer of $67 per share for Unocal. As part of the takeover, CNOOC will receive $7 billion in ‘cheap’ loans from its parent company: a zero-interest, 2-year loan of $2.5 billion and a 3.5%, 30-year loan of $4.5 billion. If CNOOC normal borrowing rate is 8%, how much is the interest subsidy worth?

- Interest Savings, Loan 1: \(2.5 \times (0.08 - 0.00) = 0.2 \text{ billion}\)
- Interest Savings, Loan 2: \(4.5 \times (0.08 - 0.035) = 0.2 \text{ billion}\)

\[
PV = \frac{0.4}{(1.08)} + \frac{0.4}{(1.08)^2} + \frac{0.2}{(1.08)^3} + \frac{0.2}{(1.08)^4} + \ldots + \frac{0.2}{(1.08)^{30}}
\]

\[= 2.62 \text{ billion} \]
Special Cashflows: The Perpetuity

Perpetuity Pays Constant Cashflow $C$ Forever

- How much is an infinite cashflow of $C$ each year worth?
- How can we value it?

$$PV = \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \frac{C}{(1 + r)^3} + \cdots$$

$$(1 + r) \times PV = C + \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \cdots$$

$$r \times PV = C$$

$$PV = \frac{C}{r}$$
Special Cashflows: The Perpetuity

Growing Perpetuity Pays Growing Cashflow \( C(1+g)^t \) Forever

- How much is an infinite growing cashflow of \( C \) each year worth?
- How can we value it?

\[
PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots \\
\frac{(1+r)}{(1+g)} \times PV = \frac{C}{(1+g)} + \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \cdots \\
\left[\frac{(1+r)}{(1+g)} - 1\right] \times PV = \frac{C}{(1+g)} \\
\]

\[
PV = \frac{C}{r-g}, \quad r > g
\]
Special Cashflows: The Annuity

Annuity Pays Constant Cashflow $C$ For $T$ Periods

- Simple application of $V_0$

$$ PV = \frac{C}{1 + r} + \cdots + \frac{C}{(1 + r)^T} $$

$$(1 + r) \times PV = C + \frac{C}{1 + r} + \frac{C}{(1 + r)^{T-1}} $$

$$ r \times PV = C - \frac{C}{(1 + r)^T} $$

$$ PV = \frac{C}{r} - \frac{C}{r} \frac{1}{(1 + r)^T} $$
Special Cashflows: The Annuity

Annuity Pays Constant Cashflow C For T Periods

- Sometimes written as a product:

\[
PV = \frac{C}{r} - \frac{C}{r} \left(1 + \frac{1}{(1 + r)^T}\right) = C \times \frac{1}{r} \left[1 - \frac{1}{(1 + r)^T}\right]
\]

\[
ADF(r, T) \equiv \frac{1}{r} \left[1 - \frac{1}{(1 + r)^T}\right]
\]

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Annuity Pays Constant Cashflow C For T Periods

- Related to perpetuity formula

\[ \text{Perpetuity} - \text{Date-T Perpetuity} = \text{T-Period Annuity} \]
Special Cashflows: The Annuity

Example:

You just won the lottery and it pays $100,000 a year for 20 years. Are you a millionaire? Suppose that \( r = 10\% \).

\[
PV = 100,000 \times \frac{1}{0.10} \left( 1 - \frac{1}{1.10^{20}} \right)
\]
\[
= 100,000 \times 8.514 = 851,356
\]

- What if the payments last for 50 years?

\[
PV = 100,000 \times \frac{1}{0.10} \left( 1 - \frac{1}{1.10^{50}} \right)
\]
\[
= 100,000 \times 9.915 = 991,481
\]

- How about forever (a perpetuity)?

\[
PV = \frac{100,000}{0.10} = 1,000,000
\]
Compounding

Interest May Be Credited/Charged More Often Than Annually

- Bank accounts: daily
- Mortgages and leases: monthly
- Bonds: semiannually
- **Effective annual rate** may differ from **annual percentage rate**
- Why?

Typical Compounding Conventions:

- Let $r$ denote APR, $n$ periods of compounding
- $r/n$ is per-period rate for each period
- Effective annual rate (EAR) is

$$ r_{EAR} \equiv \left( 1 + \frac{r}{n} \right)^n - 1 $$

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Compounding

Example:
Car loan—‘Finance charge on the unpaid balance, computed daily, at the rate of 6.75% per year.’
If you borrow $10,000, how much would you owe in a year?

Daily interest rate = 6.75 / 365 = 0.0185%
Day 1: Balance = 10,000.00 × 1.000185 = 10,001.85
Day 2: Balance = 10,001.85 × 1.000185 = 10,003.70
... …
Day 365: Balance = 10,696.26 × 1.000185 = 10,698.24

EAR = 6.982% > 6.750%
Inflation

What Is Inflation?
- Change in real purchasing power of $1 over time
- Different from time-value of money (how?)
- For some countries, inflation is extremely problematic
- How to quantify its effects?

\[
\begin{align*}
\text{Wealth } W_t & \Leftrightarrow \text{ Price Index } I_t \\
\text{Wealth } W_{t+k} & \Leftrightarrow \text{ Price Index } I_{t+k} \\
\text{Increase in Cost of Living} & \equiv I_{t+k}/I_t = (1 + \pi)^k \\
\text{“Real Wealth” } \tilde{W}_{t+k} & \equiv W_{t+k}/(1 + \pi)^k
\end{align*}
\]
“Real Wealth” \( \widetilde{W}_{t+k} \) \( \equiv \frac{W_{t+k}}{(1 + \pi)^k} \)

“Real Return” \( (1 + r_{\text{real}})^k \) \( \equiv \frac{\widetilde{W}_{t+k}}{W_t} \)

\[
= \frac{W_{t+k}}{W_t} \frac{1}{(1 + \pi)^k} = \frac{(1 + r_{\text{nominal}})^k}{(1 + \pi)^k}
\]

\( r_{\text{real}} \approx \frac{1 + r_{\text{nominal}}}{1 + \pi} - 1 \approx r_{\text{nominal}} - \pi \)
Inflation

For NPV Calculations, Treat Inflation Consistently

- Discount **real cashflows** using **real interest rates**
- Discount **nominal cashflows** using **nominal interest rates**
  - Nominal cashflows ⇒ expressed in actual-dollar cashflows
  - Real cashflows ⇒ expressed in constant purchasing power
  - Nominal rate ⇒ actual prevailing interest rate
  - Real rate ⇒ interest rate adjusted for inflation
Inflation

Example:
This year you earned $100,000. You expect your earnings to grow 2% annually, in real terms, for the remaining 20 years of your career. Interest rates are currently 5% and inflation is 2%. What is the present value of your income?

Real Interest Rate = \( \frac{1.05}{1.02} - 1 = 2.94\% \)

Real Cashflows

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>…</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cashflow</td>
<td>102,000</td>
<td>104,040</td>
<td>…</td>
<td>148,595</td>
</tr>
<tr>
<td>÷</td>
<td>1.0294</td>
<td>1.0294^2</td>
<td>…</td>
<td>1.0294^20</td>
</tr>
<tr>
<td>PV</td>
<td>99,086</td>
<td>98,180</td>
<td>…</td>
<td>83,219</td>
</tr>
</tbody>
</table>

Present Value = $1,818,674
Extensions and Qualifications

- Taxes
- Currencies
- Term structure of interest rates
- Forecasting cashflows
- Choosing the right discount rate (risk adjustments)
Key Points

- Assets are sequences of cashflows
- Date-t cashflows are different from date-(t+k) cashflows
- Use “exchange rates” to convert one type of cashflow into another
- PV and FV related by “exchange rates”
- Exchange rates are determined by supply/demand
- Opportunity cost of capital: expected return on equivalent investments in financial markets
- For NPV calculations, visualize cashflows first
- Decision rule: accept positive NPV projects, reject negative ones
- Special cashflows: perpetuities and annuities
- Compounding
- Inflation
- Extensions and Qualifications
Additional References
