15.401 Finance Theory

MIT Sloan MBA Program

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Lectures 8–9: Forward and Futures Contracts

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Critical Concepts

- Motivation
- Forward Contracts
- Futures Contract
- Valuation of Forwards and Futures
- Applications
- Extensions and Qualifications

Readings:
- Brealey, Myers, and Allen Chapters 27
Motivation

- Your company, based in the U.S., supplies machine tools to customers in Germany and Brazil. Prices are quoted in each country’s currency, so fluctuations in the € / $ and R / $ exchange rates have a big impact on the firm’s revenues. How can the firm reduce (or ‘hedge’) these risks?

- Your firm is thinking about issuing 10-year convertible bonds. In the past, the firm has issued straight debt with a yield-to-maturity of 8.2%. If the new bonds are convertible into 20 shares of stocks, per $1,000 face value, what interest rate will the firm have to pay on the bonds?

- You have the opportunity to buy a mine with 1 million kgs of copper for $400,000. Copper has a price of $2.2 / kg, mining costs are $2 / kg, and you can delay extraction one year. How valuable is the option to delay? Is the mine a good deal?
Motivation


- Euro / $ (left scale)
- Real / $ (right scale)
Motivation

Hedging or Speculation?

Alternative Tools?
- Futures, forwards, options, and swaps
- Insurance
- Diversification
- Match duration of assets and liabilities
- Match sales and expenses across countries (currency risk)

Should Firms Hedge With Financial Derivatives?
- “Derivatives are extremely efficient tools for risk management”
- “Derivatives are financial weapons of mass destruction”
Motivation

View 1: Hedging is irrelevant (M&M)
- Financial transaction, zero NPV
- Diversified shareholders don’t care about firm-specific risks

View 2: Hedging creates value
- Ensures cash is available for positive NPV investments
- Reduces need for external finance
- Reduces chance of financial distress
- Improves performance evaluation and compensation

Examples:
- Homestake Mining
  Does not hedge because “shareholders will achieve maximum benefit from such a policy.”
- American Barrick
  Hedges aggressively to provide “extraordinary financial stability… offering investors a predictable, rising earnings profile in the future.”
- Battle Mountain Gold
  Hedges up to 25% because “a recent study indicates that there may be a premium for hedging.”
Motivation

Evidence*

- Random sample of 413 large firms
- Average cashflow from operations = $735 million
- Average PP&E = $454 million
- Average net income = $318 million

57% of Firms Use Derivatives In 1997

- Small derivative programs
- Even with a big move (3σ event), the derivative portfolio pays only $15 million and its value goes up by $31 million

Motivation

Basic Types of Derivatives

- **Forwards and Futures**
  A contract to exchange an asset in the future at a specified price and time.

- **Options** (Lecture 10)
  Gives the holder the right to buy (call option) or sell (put option) an asset at a specified price.

- **Swaps**
  An agreement to exchange a series of cashflows at specified prices and times.
Forward Contracts

Definition: A forward contract is a commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today.

- The price fixed now for future exchange is the forward price
- The buyer of the underlying is said to be “long” the forward

Features of Forward Contracts
- Customized
- Non-standard and traded over the counter (not on exchanges)
- No money changes hands until maturity
- Non-trivial counterparty risk
Forward Contracts

Example:
- Current price of soybeans is $160/ton
- Tofu manufacturer needs 1,000 tons in 3 months
- Wants to make sure that 1,000 tons will be available
- 3-month forward contract for 1,000 tons of soybeans at $165/ton
- Long side will buy 1,000 tons from short side at $165/ton in 3 months
Futures Contracts

Forward Contracts Have Two Limitations:
- Illiquidity
- Counterparty risk

Definition: A futures contract is an exchange-traded, standardized, forward-like contract that is marked to market daily. This contract can be used to establish a long (or short) position in the underlying asset.

Features of Futures Contracts
- Standardized contracts:
  - Underlying commodity or asset
  - Quantity
  - Maturity
- Exchange traded
- Guaranteed by the clearing house—no counter-party risk
- Gains/losses settled daily (marked to market)
- Margin required as collateral to cover losses
Futures Contracts

Example:
NYMEX crude oil (light) futures with delivery in Dec. 2007 at a price of $75.06 / bbl. on July 27, 2007 with 51,475 contracts traded

- Each contract is for 1,000 barrels
- Tick size: $0.01 per barrel, $10 per contract
- Initial margin: $4,050
- Maintenance margin: $3,000
- No cash changes hands today (contract price is $0)
- Buyer has a “long” position (wins if prices go up)
- Seller has a “short” position (wins if prices go down)
Futures Contracts

Payoff Diagram

- Long position (buy)
- Short position (sell)

Oil price, Dec07

-6 -4 -2 0 2 4 6

$70 $71 $72 $73 $74 $75 $76 $77 $78 $79 $80
Futures Contracts

Example. Yesterday, you bought 10 December live-cattle contracts on the CME, at a price of $0.7455/lb

- Contract size 40,000 lb
- Agreed to buy 40,000 pounds of live cattle in December
- Value of position yesterday:
  \[(0.7455)(10)(40,000) = 298,200\]
- No money changed hands
- Initial margin required (5%–20% of contract value)

Today, the futures price closes at $0.7435/lb, 0.20 cents lower. The value of your position is

\[(0.7435)(10)(40,000) = 297,400\]

which yields a loss of $800.
Futures Contracts

Why Is This Contract Superior to a Forward Contract?

- Standardization makes futures liquid
- Margin and marking to market reduce default risk
- Clearing-house guarantee reduces counter-party risk

Diagram:
- Hedgers
- Futures Clearing Corp
- Speculators
What Determines Forward and Futures Prices?

- Forward/futures prices ultimately linked to future spot prices
- Notation:

<table>
<thead>
<tr>
<th>Contract</th>
<th>Spot at $t$</th>
<th>Forward at $F_{t,T}$</th>
<th>Futures at $H_{t,T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$S_t$</td>
<td>$F_{t,T}$</td>
<td>$H_{t,T}$</td>
</tr>
</tbody>
</table>

- Ignore differences between forward and futures price for now

\[ F_{t,T} \approx H_{t,T} \]

- Two ways to buy the underlying asset for date-$T$ delivery
  1. Buy a forward or futures contract with maturity date $T$
  2. Buy the underlying asset and store it until $T$
### Valuation of Forwards and Futures

**Date** | **Forward Contract** | **Outright Asset Purchase**
--- | --- | ---
0 | ▪ Pay $0 for contract with forward price $F_{0,T}$ | ▪ Borrow $S_0$
 |  | ▪ Pay $S_0$ for Asset
T | ▪ Pay $F_{0,T}$
 | ▪ Own asset | ▪ Pay back $S_0(1+r)^T$
 |  | ▪ Pay cumulative storage costs (if any)
 |  | ▪ Deduce cumulative “convenience yield” (if any)
 |  | ▪ Own asset
---
**Total Cost at T** | $F_{0,T}$ | $S_0(1+r)^T + \text{net storage costs}$

\[
F_{0,T} \approx H_{0,T} = (1 + r_f)^T S_0 + \text{FV}_T(\text{net storage costs})
\]

\[
\frac{F_{0,T}}{(1 + r)^T} \approx \frac{H_{0,T}}{(1 + r)^T} = S_0 + \text{PV}_0(\text{net storage costs})
\]
## Valuation of Forwards and Futures

<table>
<thead>
<tr>
<th>Date</th>
<th>Forward Contract</th>
<th>Outright Asset Purchase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Pay $0$ for contract with forward price $F_{t,T}$</td>
<td>Borrow $S_t$</td>
</tr>
<tr>
<td></td>
<td>Pay $F_{t,T}$</td>
<td>Pay $S_t$ for Asset</td>
</tr>
<tr>
<td>$T$</td>
<td>Own asset</td>
<td>Pay back $S_t(1+r)^{T-t}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pay cumulative storage costs (if any)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Deduce cumulative “convenience yield” (if any)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Own asset</td>
</tr>
</tbody>
</table>

**Total Cost at $T**

- Forward Contract: $F_{t,T}$
- Outright Asset Purchase: $S_0(1+r)^{T-t} + \text{net storage costs}$

\[
F_{t,T} \approx H_{t,T} = (1 + r_f)^{T-t}S_t + FV_T(\text{net storage costs})
\]

\[
\frac{F_{t,T}}{(1 + r)^{T-t}} \approx \frac{H_{t,T}}{(1 + r)^{T-t}} = S_t + PV_t(\text{net storage costs})
\]
Valuation of Forwards and Futures

What Determines Forward/Futures Prices?
- Difference between the two methods:
  - Costs (storage for commodities, not financials)
  - Benefits (convenience for commodities, dividends for financials)
- By no arbitrage (Principal P1), these two methods must cost the same

Gold
- Easy to store (negligible costs of storage)
- No dividends or benefits
- Two ways to buy gold for \( T \)
  - Buy now for \( S_t \) and hold until \( T \)
  - Buy forward at \( t \), pay \( F_{t,T} \) at \( T \) and take delivery at \( T \)
- No-arbitrage requires that
  \[
  F_{t,T} \approx H_{t,T} = (1 + r_f)^{(T-t)} S_t
  \]
Valuation of Forwards and Futures

Gasoline
- Costly to store (let $c$ be percentage cost per period)
- Convenience yield does exist (let $y$ be percentage yield per period)
- Not for long-term investment (like gold), but for future use
- Two ways to buy gasoline for $T$
  - Buy now for $S_t$ and hold until $T$
  - Buy forward at $t$, pay $F_{t,T}$ at $T$ and take delivery at $T$
- No-arbitrage requires that
  \[
  F_{t,T} \approx H_{t,T} = (1 + r_f + c - y)^{(T-t)} S_t
  \]
Valuation of Forwards and Futures

Financials

- Let underlying be a financial asset
  - No cost to store (the underlying asset)
  - Dividend or interest on the underlying
- Example: Stock index futures
  - Underlying are bundles of stocks, e.g., S&P, Nikkei, etc.
  - Futures settled in cash (no delivery)
  - Let the annualized dividend yield be \( d \); then:

\[
F_{t,T} \approx H_{t,T} = (1 + r_f - d)^{(T-t)} S_t
\]
Valuation of Forwards and Futures

Example:
Gold quotes on 2001.08.02 are
- Spot price (London fixing) $267.00/oz
- October futures (CMX) $269.00/oz
- What is the implied interest rate?

\[ F = S_0(1 + r_f)^{2/12} \]

\[ r_f = \left(\frac{F}{S_0}\right)^6 - 1 = 4.58\% \]
Valuation of Forwards and Futures

Example:
Gasoline quotes on 2001.08.02:
- Spot price is 0.7760
- Feb 02 futures price is 0.7330
- 6-month interest rate is 3.40%
- What is the annualized net convenience yield (net of storage costs)?

\[ 0.7330 = (0.7760)(1 + 0.0340 - y)^{6/12} \]

\[ y = 1.0340 - \left(\frac{0.7330}{0.7760}\right)^2 = 14.18\% \]
Example:

- The S&P 500 closed at 1,220.75 on 2001.08.02
- The S&P futures maturing in December closed at 1,233.50
- Suppose the T-bill rate is 3.50%
- What is the implied annual dividend yield?

\[
d = \left[ 1 + r_f - \left( \frac{F}{S_0} \right)^{12/4} \right] \\
= \left[ 1 + 0.0350 - \left( \frac{1233.50}{1220.75} \right)^{3} \right] = 0.33\%
\]
Applications

Index Futures Have Many Advantages

- Since underlying asset is a portfolio, trading in the futures market is easier than trading in cash market
- Futures prices may react quicker to macroeconomic news than the index itself
- Index futures are very useful for:
  - Hedging market risk in block purchases and underwriting
  - Creating synthetic index fund
  - Portfolio insurance
Applications

Example:
You have $1 million to invest in the stock market and you have decided to invest in the S&P 500. How should you do this?

- One way is to buy the S&P 500 in the cash market:
  - Buy the 500 stocks, weights proportional to their market caps

- Another way is to buy S&P futures:
  - Put the money in your margin account
  - Assuming the S&P 500 is at 1,000 now, number of contract to buy:
    (value of a futures contract is $250 times the S&P 500 index)

\[
\frac{$1,000,000}{250 \times 1,000} = 4
\]
Applications

Example (cont):

- As the S&P index fluctuates, the future value of your portfolio (in $MM) is given by the following table (ignoring interest payments and dividends):

<table>
<thead>
<tr>
<th>S&amp;P 500</th>
<th>Cash Portfolio</th>
<th>Futures Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>$0.90</td>
<td>$0.90</td>
</tr>
<tr>
<td>1,000</td>
<td>$1.00</td>
<td>$1.00</td>
</tr>
<tr>
<td>1,100</td>
<td>$1.10</td>
<td>$1.10</td>
</tr>
</tbody>
</table>

- Suppose you a diversified portfolio of large-cap stocks worth $5MM and are now worried about equity markets and would like to reduce your exposure by 25%—how could you use S&P 500 futures to implement this hedge?
  - (Short)sell 5 S&P 500 futures contracts (why 5?)
**Applications**

Example (cont):

- Compare hedged and unhedged portfolio (in $MM):

<table>
<thead>
<tr>
<th>S&amp;P 500</th>
<th>Cash Portfolio</th>
<th>Cash Plus Futures Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>$4.50</td>
<td>$4.50 + $0.125 = $4.625</td>
</tr>
<tr>
<td>1,000</td>
<td>$5.00</td>
<td>$5.00</td>
</tr>
<tr>
<td>1,100</td>
<td>$5.50</td>
<td>$5.50-$0.125=$5.375</td>
</tr>
</tbody>
</table>

- Fluctuations have been reduced
- As if 25% of the portfolio has been shifted to cash
Extensions and Qualifications

- Interest-rate, bond, and currency futures are extremely popular
- Single-stock futures are gaining liquidity
- Volatility futures recently launched (VIX)
Key Points

- Forward and futures contracts are zero-NPV contracts when initiated.
- After initiation, both contracts may have positive/negative NPV.
- Futures contracts are “marked to market” every day.
- Futures and forwards are extremely liquid.
- Hedging and speculating are important applications of futures/forwards.
Additional References


