15.401 Finance Theory

MIT Sloan MBA Program

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Lecture 12: Introduction to Risk and Return

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Critical Concepts

- Motivation
- Statistical Background
- Empirical Properties of Stock Returns
- Anomalies

Readings
- Brealey, Myers, and Allen Chapters 7, 24.1, 24.4
Motivation

NPV and Other Valuation Techniques Need Cost of Capital

- Opportunity cost
- Required rate of return
- Risk-adjusted discount rate
- Determined by “the market”
- How???

Introduce Risk Into The Valuation Process

- How to measure risk
- How to estimate the required rate of return for a given level of risk
- Related questions:
  - How risky are stocks and what have their returns been historically?
  - Is the stock market “efficient”?
  - How can we gauge the performance of portfolio managers?
Statistical Background

Terminology

\[
\text{Return } R_{il} \equiv \frac{D_{it} + P_{it} - P_{it-1}}{P_{it-1}} = \frac{D_{it} + P_{it}}{P_{it-1}} - 1
\]

Expected Return \( \equiv \mathbb{E}[R_{it}] \)

Excess Return \( \equiv R_{it} - r_f \)

Risk Premium \( \equiv \mathbb{E}[R_{it}] - r_f \)
Statistical Background

Terminology

- **Mean, variance, standard deviation:**
  \[ \mu_i \equiv \mathbb{E}[R_{it}] \]
  \[ \sigma_i^2 \equiv \mathbb{E}[(R_{it} - \mu_i)^2] \]
  \[ \sigma_i = \sqrt{\sigma_i^2} \]

- **Sample estimators:**
  \[ \hat{\mu}_i = \frac{1}{T} \sum_{t=1}^{T} R_{it} \]
  \[ \hat{\sigma}_i^2 = \frac{1}{T-1} \sum_{t=1}^{T} (R_{it} - \hat{\mu}_i)^2 \]
  \[ \hat{\sigma}_i = \sqrt{\hat{\sigma}_i^2} \]
Statistical Background

Other Statistics

- **Median**
  - 50th percentile (probability of 1/2 that $R_t < \text{median}$)

- **Skewness**
  - Is the distribution symmetric?
  - Negative: big losses are more likely than big gains
  - Positive: big gains are more likely than big losses

- **Correlation**
  - How closely do two variables move together?

\[
\text{Cov}[R_{it}, R_{jt}] \equiv \mathbb{E}[(R_{it} - \mu_i)(R_{jt} - \mu_j)] \quad \text{Covariance}
\]

\[
\text{Corr}[R_{it}, R_{jt}] \equiv \frac{\mathbb{E}[(R_{it} - \mu_i)(R_{jt} - \mu_j)]}{\sigma_i \sigma_j} \quad \text{Correlation}
\]
Negatively Skewed Distribution
Examples of Correlation Between Two Random Variables

\( \rho = 0 \)

\( \rho = 0.5 \)

\( \rho = 0.8 \)

\( \rho = -0.5 \)
Statistical Background

Normal Distribution
- Bell-shaped, symmetric
- A model of randomness
- Central Limit Theorem

Confidence Intervals
If \( R \) is normally distributed, then ...
- 68% of observations fall within \( +/-1.00 \) std. deviations from mean
- 90% of observations fall within \( +/-1.65 \) std. deviations from mean
- 95% of observations fall within \( +/-1.96 \) std. deviations from mean
- 99% of observations fall within \( +/-2.58 \) std. deviations from mean
Empirical Properties of Stock Returns

What Characterizes U.S. Stock Returns?
- How volatile are stock returns?
- Are returns predictable?
- How does volatility change over time?
- What types of stocks have the highest returns?

What Properties Should Stock Prices Have In “Efficient” Markets?
- Random, unpredictable
- Prices should react quickly and correctly to news
- Investors cannot earn abnormal, risk-adjusted returns (or at least it shouldn’t be easy)
Empirical Properties of Stock Returns

Predictable Price Changes

$80

$70

$60

$50

$40

$30

$20
Empirical Properties of Stock Returns

Random Walks with Drift

$80$

$75$

$70$

$65$

$60$

$55$

$50$

0 10 20 30 40 50 60 70 80 90 100
Empirical Properties of Stock Returns

Four facts from history of U.S. financial markets:
1. Real interest rate has been slightly positive on average.
2. Return on more risky assets has been higher on average than return on less risky assets.
3. Returns on risky assets can be highly correlated to each other.
4. Returns on risky assets are (usually) serially uncorrelated.

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<td>Avg</td>
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<td>Inflation</td>
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<td>Tbill (1 yr)</td>
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<td>Tnote (10 yr)</td>
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<td>VW stock index</td>
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<td>EW stock index</td>
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<td>Motorola</td>
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NYSE, Amex, NASDAQ: 6,700 firms, $16.4 trillion market cap
Empirical Properties of Stock Returns

Total Return of Stocks, Bonds, Bills and Inflation 1946 – 2001

- cpi
- tbill
- 10 note
- vw
- ew

Dec-45 Dec-53 Dec-61 Dec-69 Dec-77 Dec-85 Dec-93 Dec-01
Empirical Properties of Stock Returns

Interest Rates 1953 – 2001

- 1-yr Tbill
- 10-yr Tbond

Jun-53 Jan-63 Jun-72 Jan-82 Jun-91 Jan-01
Empirical Properties of Stock Returns

Total Returns, 10-Year U.S. T-Bond, 1946 – 2001
Empirical Properties of Stock Returns

Empirical Properties of Stock Returns

Total Returns, Motorola 1946 – 2001
Empirical Properties of Stock Returns

Scatterplot, VWRETD Today vs. Yesterday, 1980 – 1999
Empirical Properties of Stock Returns

Scatterplot, S&P 500 This Month vs. Last Month, 1926 to 1997
Empirical Properties of Stock Returns

Empirical Properties of Stock Returns

Anomalies: The Size Effect, 1964 – 2004

Firms sorted by MARKET CAPITALIZATION

Firms sorted by MARKET CAPITALIZATION

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Anomalies: The Value Premium, 1964 – 2004

Firms sorted by PRICE / BOOK EQUITY

- Low
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- High
Anomalies: Momentum, 1964 – 2004

Firms sorted by PAST 12-MONTH RETURN

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Firms sorted by last year's OPERATING ACCRUALS

*Operating income minus operating cashflows

Average Annual Returns, 1 – 5 Years After IPO

- 1st year: 1.6%
- 2nd year: 3.6%
- 3rd year: 5.0%
- 4th year: 4.0%
- 5th year: 11.6%

Non-issuers

IPOs

Average Annual Returns, 1 – 5 Years After SEO

- 1st year: 6.6%
- 2nd year: 0.3%
- 3rd year: 7.5%
- 4th year: 9.1%
- 5th year: 11.8%

Non-issuers
SEO
Anomalies: Takeover Announcements

Stock price of TARGET

Image by MIT OpenCourseWare.
Anomalies: Performance of Mutual Funds

Key Points

Observations

- The average annual return on U.S. stocks from 1926 – 2004 was 11.2%. The average risk premium was 7.8%.
- **Stocks are quite risky.** The standard deviation of returns for the overall market is 4.5% monthly (16.4% annually).
- **Individual stocks are much riskier.** The average monthly standard deviation of an individual stock is around 17% (or 50% annually).
- **Stocks tend to move together over time:** when one stock goes up, other stocks are likely to go up as well. The correlation is far from perfect.
- **Stock returns are nearly unpredictable.** For example, knowing how a stock does this month tells you very little about what will happen next month.
- **Market volatility changes over time.** Prices are sometimes quite volatile. The standard deviation of monthly returns varies from roughly 2% to 20%.
- **Financial ratios like DY and P/E ratios vary widely over time.** DY hit a maximum of 13.8% in 1932 and a minimum of 1.17% in 1999. The P/E ratio hit a maximum of 33.4 in 1999 and a minimum of 5.3 in 1917.
Key Points

Anomalies:

- **Size Effect**: Smaller stocks typically outperform larger stocks, especially in January.
- **January Effect**: Returns in January tend to be abnormally high.
- **Value Effect**: Low P/B (value) stocks typically outperform high P/B (growth) stocks.
- **Momentum**: Stocks with high returns over the past 12 months typically continue to outperform stocks with low past returns.
- **Accruals and Issuances**: Stocks with high past accruals and/or recent stock offerings typically underperform stocks with low past accruals and no stock offerings.
Additional References
