The CAPM

Class 11
Financial Management, 15.414
Today

The CAPM

- Measuring risk
- Systematic vs. diversifiable risk
- The trade-off between risk and return

Reading

- Brealey and Myers, Chapter 8.2 – 8.5
Review

Diversification

➤ Diversification reduces risk, as long as stocks aren’t perfectly correlated with each other.

➤ Portfolio variance depends primarily on the covariances among stocks, not the individual variances. Risk common to all firms cannot be diversified away.

➤ Investors should try to hold portfolios that maximize expected return for a given level of risk. The tangency portfolio is the best portfolio.
Diversification

If correlation = 1.0

If correlation = 0.4

If correlation = 0.0

Std dev of portfolio

Number of stocks

0% 2% 4% 6% 8% 10% 12%

1 11 21 31 41 51 61 71 81 91

If correlation = 1.0

If correlation = 0.4

If correlation = 0.0
Optimal portfolios

- Mean returns:
  - GM: 2.4%
  - IBM: 1.8%
  - Motorola: 1.2%
  - 0.6%
  - 0.0%

- Riskfree asset: 0.0%

- Efficient frontier:

- Tangency portfolio:
  - GM
  - IBM
  - Motorola

- Std dev:
  - 0.0% to 16.0%
The CAPM

Capital Asset Pricing Model

- Stock prices are affected by firm-specific and marketwide risks. Investors care only about risk that is non-diversifiable.

- A stock’s non-diversifiable risk is measured by beta, the slope when the stock is regressed on the market:

  \[ R_i = \alpha + \beta R_M + \epsilon \]

- Expected, or required, returns are a linear function of betas:

  \[ E[R_i] = r_f + \beta_i E[R_M - r_f] \]

  Market risk premium

For example, a stock with \( \beta = 2 \) is twice as risky as the market, so investors require twice the risk premium.
Slope = \( E[R_M] - r_f \)

\[ \text{Slope} = E[R_M] - r_f \]
Beta

Regression slope

How sensitive is the stock to overall market movements? How much does the stock go up or down when other stocks go up or down?

\[ R_i = \alpha + \beta R_M + \varepsilon \]

\( \varepsilon \) = firm-specific return  
('diversifiable,' 'idiosyncratic,' or 'unsystematic' risk)

\( \beta \) = sensitivity to market returns  
('systematic,' 'non-diversifiable,' or 'macroeconomic' risk)

\( R^2 \) = explained variance  
(fraction of variance explained by market returns)
Regressions in Excel
Gillette vs. Total U.S. market return

Monthly returns

$\beta = 0.81$

$R^2 = 0.19$
NASDAQ vs. Total U.S. market return

Monthly returns
$\beta = 1.57$
$R^2 = 0.77$
### Betas, 1960 – 2001

#### Size-sorted portfolios

<table>
<thead>
<tr>
<th>Decile</th>
<th>$\beta$</th>
<th>$R^2$</th>
<th>Decile</th>
<th>$\beta$</th>
<th>$R^2$</th>
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<tbody>
<tr>
<td>Smallest</td>
<td>1.58</td>
<td>0.60</td>
<td>Smallest</td>
<td>1.27</td>
<td>0.49</td>
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<tr>
<td>2</td>
<td>1.45</td>
<td>0.76</td>
<td>2</td>
<td>1.25</td>
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<tr>
<td>3</td>
<td>1.45</td>
<td>0.81</td>
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<td>1.26</td>
<td>0.75</td>
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<td>4</td>
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<td>1.22</td>
<td>0.79</td>
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<tr>
<td>5</td>
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<td>0.86</td>
<td>5</td>
<td>1.18</td>
<td>0.80</td>
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<tr>
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<td>1.27</td>
<td>0.90</td>
<td>6</td>
<td>1.13</td>
<td>0.85</td>
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<tr>
<td>7</td>
<td>1.22</td>
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<tr>
<td>8</td>
<td>1.16</td>
<td>0.95</td>
<td>8</td>
<td>1.04</td>
<td>0.91</td>
</tr>
<tr>
<td>9</td>
<td>1.05</td>
<td>0.96</td>
<td>9</td>
<td>1.02</td>
<td>0.95</td>
</tr>
<tr>
<td>Largest</td>
<td>0.92</td>
<td>0.97</td>
<td>Largest</td>
<td>0.96</td>
<td>0.97</td>
</tr>
</tbody>
</table>
CAPM

Key insight

For a diversified investor, beta measures a stock’s contribution to portfolio risk. Beta, not variance, is the appropriate measure of risk.

The required return on a stock equals:

\[ E[R_i] = r_f + \beta_i \ E[R_M - r_f] \]
Security Market Line

Slope = $E[R_M] - r_f$

Market portfolio ($\beta = 1$)

$\beta = 0$

$\beta = 0.5$

$\beta = 1.5$

Stock's expected return

Stock's beta

0% 5% 10% 15% 20% 25%

0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Example 1

Using monthly returns from 1990 – 2001, you estimate that Microsoft has a beta of 1.49 (std err = 0.18) and Gillette has a beta of 0.81 (std err = 0.14). If these estimates are a reliable guide for their risks going forward, what rate of return is required for an investment in each stock?

\[ E[R_i] = r_f + \beta_i (E[R_M] - r_f) \]

Tbill rate = 1.0%; market risk premium is around 4 – 6%.

**Expected returns**

**Gillette:**  \[ E[R_{GS}] = 0.01 + (0.81 \times 0.06) = 5.86\% \]

**Microsoft:**  \[ E[R_{MSFT}] = 0.01 + (1.49 \times 0.06) = 9.94\% \]
Example 2

Over the past 40 years, the smallest decile of firms had an average monthly return of 1.33% and a beta of 1.40. The largest decile of firms had an average return of 0.90% and a beta of 0.94. Over the same time period, the riskfree rate averaged 0.43% and the market risk premium was 0.49%. Are the average returns consistent with the CAPM?

\[ E[R_i] = r_f + \beta_i (E[R_M] - r_f) \]

Tbill rate = 0.43%; market risk premium is 0.49%.

How far are average returns from the CAPM security market line?
Size portfolios, 1960 – 2001

Average returns vs. CAPM

<table>
<thead>
<tr>
<th>Decile</th>
<th>Avg return</th>
<th>$\beta$</th>
<th>$r_f + \beta_i E[R_M - r_f]$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest</td>
<td>1.33</td>
<td>1.40</td>
<td>1.15</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>1.06</td>
<td>1.33</td>
<td>1.11</td>
<td>-0.06</td>
</tr>
<tr>
<td>3</td>
<td>1.13</td>
<td>1.34</td>
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<td>0.01</td>
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<td>1.14</td>
<td>1.28</td>
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<td>0.05</td>
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<td>7</td>
<td>1.04</td>
<td>1.14</td>
<td>1.02</td>
<td>0.02</td>
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<td>8</td>
<td>1.10</td>
<td>1.09</td>
<td>0.99</td>
<td>0.11</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>1.03</td>
<td>0.97</td>
<td>0.03</td>
</tr>
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<td>0.90</td>
<td>0.94</td>
<td>0.93</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

Difference = Avg. return – CAPM prediction
Size portfolios, 1960 – 2001
Example 3

You are choosing between two mutual funds. Over the past 10 years, BlindLuck Value Fund had an average return of 12.8% and a β of 0.9. EasyMoney Growth Fund had a return of 17.9% and a β of 1.3. The market’s average return over the same period was 14% and the Tbill rate was 5%.

Which fund is better?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Avg return</th>
<th>β</th>
<th>( r_f + \beta_i E[R_M - r_f] )</th>
<th>Dif</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>14.0%</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BlindLuck</td>
<td>12.8</td>
<td>0.9</td>
<td>13.1</td>
<td>-0.30</td>
</tr>
<tr>
<td>EasyMoney</td>
<td>17.9</td>
<td>1.3</td>
<td>16.7</td>
<td>1.20</td>
</tr>
</tbody>
</table>

[‘Dif’ is referred to as the fund’s ‘alpha’]
Example 3

The graph illustrates the relationship between average return and beta. The line on the graph represents the concept of 'Easy Money' which is represented by the point on the high end of the beta scale, corresponding to a high average return. The 'Market' is represented by the point at beta 1, which is the midpoint of the graph. 'Blind Luck' is positioned lower on the scale, indicating a lower average return compared to the market. The graph helps to visualize how different beta levels are associated with varying degrees of risk and return.
CAPM

Applications

➢ **Measures and quantifies ‘risk’**
  One stock or project is riskier than another stock or project if it has a higher $\beta$.

➢ **Valuation**
  The CAPM provides a way to estimate the firm’s cost of capital (risk-adjusted discount rate).*

➢ **Evaluating a stock or mutual fund’s risk-adjusted performance**
  The CAPM provides a benchmark.

* Graham and Harvey (2000) survey CFOs; 74% of firms use the CAPM to estimate the cost of capital.
Observation 1

Portfolios

A portfolio’s beta is a weighted average of the betas of the individual stocks.

Stocks 1, …, N
Portfolio return: \( R_P = w_1 R_1 + w_2 R_2 + \ldots + w_N R_N \)

Individual stocks
\[
\begin{align*}
R_1 &= \alpha_1 + \beta_1 R_M + \varepsilon_1 \\
R_2 &= \alpha_2 + \beta_2 R_M + \varepsilon_2 \\
&\vdots \\
R_N &= \alpha_N + \beta_N R_M + \varepsilon_N
\end{align*}
\]

Portfolio
\[
R_P = \alpha_P + \beta_P R_M + \varepsilon_P
\]

What happens to the residual variance when more stocks are added?

avg of \( \beta_1, \ldots, \beta_N \)
Observation 1

Example

Two groups of stocks

Group 1: $\beta = 0.5$
Group 2: $\beta = 1.5$

All stocks have a standard deviation of 40%. The market portfolio has standard deviation of 20%.

How does portfolio beta and residual risk change as the portfolio gets more and more stocks?
Hypothetical portfolios vs. market portfolio

\[ \beta = 0.5, \ N = 10 \]

\[ \beta = 0.5, \ N = 50 \]

\[ \beta = 1.5, \ N = 10 \]

\[ \beta = 1.5, \ N = 50 \]
Diversification

Group 1: \( \beta = 0.5 \)

Group 2: \( \beta = 1.5 \)
Observation 2

Total variance vs. beta risk

Two assets can have the same total variance, but much different $\beta$’s. Investors should care only about systematic, beta, risk.

$$\text{var}(R_i) = \beta^2 \text{var}(R_M) + \text{var}(\varepsilon_i)$$

Which stock is riskier?

**Stock 1:** $\text{std}(R_1) = 0.40$, $\beta = 0.5$

**Stock 2:** $\text{std}(R_2) = 0.40$, $\beta = 1.5$
Observation 3

Assets can have negative risk!

A stock’s $\beta$ is less than 0 if the stock is negatively correlated with the market portfolio.
If the market goes down, it goes up.

Such a stock contributes negatively to portfolio risk.
The stock is better than riskfree!

Examples
Various derivative securities; return from a short sale of stock
Observation 4

Tangency portfolio

The CAPM implies that the market portfolio should be the tangency portfolio. The market portfolio will have the highest risk-return trade-off (or Sharpe ratio) of any possible portfolio.

You cannot gain by stock-picking. Competition among investors ensures that stock prices are efficient; the only way to earn a higher rate of return is to take more risk.

Portfolio advice
Buy an index fund (like Vanguard 500)