Options (2)

Class 20
Financial Management, 15.414
Today

Options

- Option pricing
- Applications: Currency risk and convertible bonds

Reading

- Brealey and Myers, Chapter 20, 21
Options

Gives the holder the right to either buy (call option) or sell (put option) at a specified price.

- Exercise, or strike, price
- Expiration or maturity date
- American vs. European option
- In-the-money, at-the-money, or out-of-the-money
Option payoffs (strike = $50)

- **Buy a call**
- **Buy a put**
- **Sell a call**
- **Sell a put**
Valuation

Option pricing

How can we estimate the expected cashflows, and what is the appropriate discount rate?

Two formulas

➢ Put-call parity

➢ Black-Scholes formula*

* Fischer Black and Myron Scholes
Put-call parity

Relation between put and call prices

\[ P + S = C + PV(X) \]

- \( S \) = stock price
- \( P \) = put price
- \( C \) = call price
- \( X \) = strike price
- \( PV(X) \) = present value of $X = X / (1+r)^t$
- \( r \) = riskfree rate
Option strategies: Stock + put

- Buy stock
- Buy put
- Stock + put
Option strategies: Tbill + call

- Buy Tbill with FV = 50
- Buy call
- Tbill + call
Example

On Thursday, Cisco call options with a strike price of $20 and an expiration date in October sold for $0.30. The current price of Cisco is $17.83. How much should put options with the same strike price and expiration date sell for?

Put-call parity

\[ P = C + PV(X) - S \]

\[ C = $0.30, \quad S = $17.83, \quad X = $20.00 \]

\[ r = 1\% \text{ annually} \rightarrow 0.15\% \text{ over the life of the option} \]

\[ \text{Put option} = 0.30 + 20 / 1.0015 - 17.83 = $2.44 \]
Black-Scholes

Price of a call option

\[ C = S \times N(d_1) - X \times e^{-rT} \times N(d_2) \]

- \( S \) = stock price
- \( X \) = strike price
- \( r \) = riskfree rate (annual, continuously compounded)
- \( T \) = time-to-maturity of the option, in years

\[ d_1 = \frac{\ln(S/X) + (r + \sigma^2/2) \times T}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

- \( N(\cdot) \) = prob that a standard normal variable is less than \( d_1 \) or \( d_2 \)
- \( \sigma \) = annual standard deviation of the stock return
Cumulative Normal Distribution

- N(-2) = 0.023
- N(-1) = 0.159
- N(0) = 0.500
- N(1) = 0.841
- N(2) = 0.977
Example

The CBOE trades Cisco call options. The options have a strike price of $20 and expire in 2 months. If Cisco’s stock price is $17.83, how much are the options worth? What happens if the stock goes up to $19.00? 20.00?

Black-Scholes

\[ S = 17.83, \quad X = 20.00, \quad r = 1.00, \quad T = 2/12, \quad \sigma_{2003} = 36.1\% \]

\[ d_1 = \frac{\ln(S/X) + (r + \sigma^2/2) T}{\sigma\sqrt{T}} = -0.694 \]

\[ d_2 = d_1 - \sigma\sqrt{T} = -0.842 \]

Call price = \( S \times N(d_1) - X e^{-rT} N(d_2) = \$0.35 \)
Cisco stock price, 1993 – 2003
Cisco returns, 1993 – 2003
Cisco option prices

- Payoff (intrinsic value)
- Today's price (2 months)
### Option pricing

#### Factors affecting option prices

<table>
<thead>
<tr>
<th>Factor</th>
<th>Call option</th>
<th>Put option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock price (S)</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Exercise price (X)</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Time-to-maturity (T)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Stock volatility (σ)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Interest rate (r)</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Dividends (D)</td>
<td>–</td>
<td>+</td>
</tr>
</tbody>
</table>
Example 2

Call option with $X = 25$, $r = 3\%$

<table>
<thead>
<tr>
<th>Time to expire</th>
<th>Stock price</th>
<th>Std. deviation</th>
<th>Call option</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 0.25$</td>
<td>$18$</td>
<td>30%</td>
<td>$0.02$</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>30</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>30</td>
<td>7.26</td>
</tr>
<tr>
<td></td>
<td>$18$</td>
<td>50</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>50</td>
<td>2.57</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>50</td>
<td>7.75</td>
</tr>
<tr>
<td>$T = 0.50$</td>
<td>$18$</td>
<td>30</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>30</td>
<td>2.29</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>30</td>
<td>7.68</td>
</tr>
<tr>
<td></td>
<td>$18$</td>
<td>50</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>50</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>50</td>
<td>8.68</td>
</tr>
</tbody>
</table>
Option pricing

- 0 months
- 1 month
- 3 months
- 6 months

Stock price vs. Option price chart showing the relationship between stock price and option price for different time periods.
Using Black-Scholes

Applications

➤ Hedging currency risk

➤ Pricing convertible debt
Currency risk

Your company, headquartered in the U.S., supplies auto parts to Jaguar PLC in Britain. You have just signed a contract worth £18.2 million to deliver parts next year. Payment is certain and occurs at the end of the year.

The $ / £ exchange rate is currently $/£ = 1.4794.

How do fluctuations in exchange rates affect $ revenues? How can you hedge this risk?
s$/£, Jan 1990 – Sept 2001

Volatility
Full sample: 9.32%
After 1992: 8.34%
After 2000: 8.33%
After 2001: 7.95%
$ revenues as a function of $/£

$26.9 million

Exchange rate
Currency risk

Forwards

1-year forward exchange rate = 1.4513

Lock in revenues of 18.2 \times 1.4513 = $26.4 million

Put options*

S = 1.4794, \sigma = 8.3\%, T = 1, r = -1.8\%*

<table>
<thead>
<tr>
<th>Strike price</th>
<th>Min. revenue</th>
<th>Option price</th>
<th>Total cost (\times18.2 M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>$24.6 M</td>
<td>$0.012</td>
<td>$221,859</td>
</tr>
<tr>
<td>1.40</td>
<td>$25.5 M</td>
<td>$0.026</td>
<td>$470,112</td>
</tr>
<tr>
<td>1.45</td>
<td>$26.4 M</td>
<td>$0.047</td>
<td>$862,771</td>
</tr>
</tbody>
</table>

*Black-Scholes is only an approximation for currencies; r = r_{UK} - r_{US}
$ \text{revenues as a function of } s_{\$/£}$
Convertible bonds

Your firm is thinking about issuing 10-year convertible bonds. In the past, the firm has issued straight (non-convertible) debt, which currently has a yield of 8.2%.

The new bonds have a face value of $1,000 and will be convertible into 20 shares of stocks. How much are the bonds worth if they pay the same interest rate as straight debt?

Today’s stock price is $32. The firm does not pay dividends, and you estimate that the standard deviation of returns is 35% annually. Long-term interest rates are 6%.
Payoff of convertible bonds

Convertible into 20 shares

Convert if stock price > $50

(20 × 50 = 1,000)
Convertible bonds

Suppose the bonds have a coupon rate of 8.2%. How much would they be worth?

**Cashflows***

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>…</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>$82</td>
<td>$82</td>
<td>$82</td>
<td>$82</td>
<td>$1,082</td>
<td></td>
</tr>
</tbody>
</table>

**Value if straight debt:** $1,000

**Value if convertible debt:** $1,000 + value of call option

* Annual payments, for simplicity
Convertible bonds

Call option

\[ X = $50, \ S = $32, \ \sigma = 35\%, \ r = 6\%, \ T = 10 \]

Black-Scholes value = $10.31

Convertible bond

Option value per bond = \[ 20 \times 10.31 = $206.2 \]

Total bond value = \[ 1,000 + 206.2 = $1,206.2 \]

Yield = 5.47%*

*Yield = IRR ignoring option value