Risk and return (1)

Class 9
Financial Management, 15.414
Today

Risk and return

- Statistics review
- Introduction to stock price behavior

Reading

- Brealey and Myers, Chapter 7, p. 153 – 165
Road map

Part 1. Valuation

Part 2. Risk and return

Part 3. Financing and payout decisions
Cost of capital

DCF analysis

\[ \text{NPV} = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5} + \ldots \]

\(r = \text{opportunity cost of capital}\)

The discount rate equals the rate of return that investors demand on investments with comparable risk.

Questions

➤ How can we measure risk?

➤ How can we estimate the rate of return investors require for projects with this risk level?
Examples

In November 1990, AT&T was considering an offer for NCR, the 5th largest U.S. computer maker. How can AT&T measure the risks of investing in NCR? What discount rate should AT&T use to evaluate the investment?

From 1946 – 2000, small firms returned 17.8% and large firms returned 12.8% annually. From 1963 – 2000, stocks with high M/B ratios returned 13.8% and those with low M/B ratios returned 19.6%. What explains the differences? Are small firms with low M/B ratios riskier, or do the patterns indicate exploitable mispricing opportunities? How should the evidence affect firms’ investment and financing choices?
Background

The stock market

➢ Primary market
   New securities sold directly to investors (via underwriters)
   Initial public offerings (IPOs)
   Seasoned equity offerings (SEOs)

➢ Secondary market
   Existing shares traded among investors
   Market makers ready to buy and sell (bid vs. ask price)
   Market vs. limit orders

NYSE and Amex: Floor trading w/ specialists
NASDAQ: Electronic market
Combined: 7,022 firms, $11.6 trillion market cap (Dec 2002)
Background

Terminology

➢ **Realized return**

\[
    r_t = \frac{D_t + P_t - P_{t-1}}{P_{t-1}} = \left(\frac{D_t}{P_{t-1}}\right) + \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) \quad \text{(DY + cap gain)}
\]

➢ **Expected return** = best forecast at beginning of period

\[
    E[r_t] = \frac{E[D_t] + E[P_t - P_{t-1}]}{P_{t-1}}
\]

➢ **Risk premium**, or expected excess return

Risk premium = \( E[r_t] - r_f \)
Statistics review

Random variable \((x)\)

- **Population parameters**
  
  mean = \(\mu = E[x]\)
  
  variance = \(\sigma^2 = E[(x - \mu)^2]\), standard deviation = \(\sigma\)
  
  skewness = \(E[(x - \mu)^3] / \sigma^3\)

- **Sample of \(N\) observations**
  
  sample mean = \(\bar{x} = \frac{1}{N} \sum_{i} x_i\)
  
  sample variance = \(s^2 = \frac{1}{N - 1} \sum_{i} (x_i - \bar{x})^2\), standard deviation = \(s\)
  
  sample skewness
Example

µ = 0, σ = 1, skew = 0

µ = 0, σ = 2, skew = 0

[Probability density function: shows probability that x falls in a given range]
Example

$\mu = 0, \sigma = 2, \text{skew } = 1.3$

$\mu = 0, \sigma = 2, \text{skew } = 0$
Statistics review

Other statistics

- **Median**
  
  50th percentile: \( \text{prob} (x < \text{median}) = 0.50 \)

- **Value-at-Risk (VaR)**
  
  How bad can things get over the next day (or week)?
  
  1st or 5th percentile: \( \text{prob} (x < \text{VaR}) = 0.01 \) or 0.05
  
  ‘We are 99% certain that we won’t lose more than $Y in the next 24 hours’
Example

$\mu = 0$, $\sigma = 2$, skew = -1.3
Statistics review

Normal random variables

Bell-shaped, symmetric distribution

\[ x \sim N(\mu, \sigma^2) \]
\[ x \text{ is normally distributed with mean } \mu \text{ and variance } \sigma^2 \]

‘Standard normal’
mean 0 and variance 1 [or \( N(0, 1) \)]

Confidence intervals
68% of observations fall within \( +/-1 \) std. deviation from mean
95% of observations fall within \( +/-2 \) std. deviations from mean
99% of observations fall within \( +/-2.6 \) std. deviations from mean
Statistics review

Estimating the mean

Given a sample $x_1, x_2, \ldots, x_N$

Don’t know $\mu$, $\sigma^2$ $\Rightarrow$ estimate $\mu$ by sample average $\bar{x}$
estimate $\sigma^2$ by sample variance $s^2$

How precise is $\bar{x}$?

std dev ($\bar{x}$) $\approx s / \sqrt{N}$

95% confidence interval for $\mu$

\[
\bar{x} - 2 \frac{s}{\sqrt{N}} \hspace{2cm} \bar{x} \hspace{2cm} \bar{x} + 2 \frac{s}{\sqrt{N}}
\]
Application

From 1946 – 2001, the average return on the U.S. stock market was 0.63% monthly above the Tbill rate, and the standard deviation of monthly returns was 4.25%. Using these data, how precisely can we estimate the risk premium?

> **Sample:** \( \bar{x} = 0.63\%, \ s = 4.25\%, \ N = 672 \) months

> **Std dev (\( \bar{x} \)) = \( \frac{4.25}{\sqrt{672}} \) = 0.164%**

> **95% confidence interval**

  Lower bound = \( 0.63 - 2 \times 0.164 = 0.30\% \)
  Upper bound = \( 0.63 + 2 \times 0.164 = 0.96\% \)

Annual (\( \times 12 \)): \( 3.6\% < \mu < 11.5\% \)
Statistics review

Two random variables

How do \( x \) and \( y \) covary? Do they typically move in the same direction or opposite each other?

**Covariance** = \( \sigma_{x,y} = \mathbb{E}[(x – \mu_x)(y – \mu_y)] \)

If \( \sigma_{x,y} > 0 \), then \( x \) and \( y \) tend to move in the same direction
If \( \sigma_{x,y} < 0 \), then \( x \) and \( y \) tend to move in opposite directions

**Correlation** = \( \rho_{x,y} = \frac{\text{covariance}}{\text{stdev}_x \cdot \text{stdev}_y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y} \)

\(-1 \leq \rho_{x,y} \leq 1\)
Correlation

\[ \rho = 0 \]

\[ \rho = 0.5 \]

\[ \rho = 0.8 \]

\[ \rho = -0.5 \]
Properties of stock prices

Time-series behavior

➤ How risky are stocks?

➤ How risky is the overall stock market?

➤ Can we predict stock returns?

➤ How does volatility change over time?
# Stocks, bonds, bills, and inflation

## Basic statistics, 1946 – 2001

Monthly, %

<table>
<thead>
<tr>
<th>Series</th>
<th>Avg</th>
<th>Stdev</th>
<th>Skew</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.32</td>
<td>0.36</td>
<td>0.82</td>
<td>-0.84</td>
<td>1.85</td>
</tr>
<tr>
<td>Tbill (1 yr)</td>
<td>0.38</td>
<td>0.24</td>
<td>0.98</td>
<td>0.03</td>
<td>1.34</td>
</tr>
<tr>
<td>Tnote (10 yr)</td>
<td>0.46</td>
<td>2.63</td>
<td>0.61</td>
<td>-7.73</td>
<td>13.31</td>
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<tr>
<td>VW stock index</td>
<td>1.01</td>
<td>4.23</td>
<td>-0.47</td>
<td>-22.49</td>
<td>16.56</td>
</tr>
<tr>
<td>EW stock index</td>
<td>1.18</td>
<td>5.30</td>
<td>-0.17</td>
<td>-27.09</td>
<td>29.92</td>
</tr>
<tr>
<td>Motorola</td>
<td>1.66</td>
<td>10.02</td>
<td>0.01</td>
<td>-33.49</td>
<td>41.67</td>
</tr>
</tbody>
</table>
Stocks, bonds, bills, and inflation, $1 in 1946
Tbill rates and inflation

- Inflation
- Tbill rate

Mar-51 Mar-60 Mar-69 Mar-78 Mar-87 Mar-96
10-year Treasury note

Jan-46  Jan-51  Jan-56  Jan-61  Jan-66  Jan-71  Jan-76  Jan-81  Jan-86  Jan-91  Jan-96  Jan-01
Motorola monthly returns, 1946 – 2001
U.S. monthly stock returns

Histogram
Motorola monthly returns

Histogram

Return

-27%  -21%  -15%  -9%  -3%  3%  9%  15%  21%  27%
Scatter plot, GM vs. S&P 500 monthly returns

correlation = 0.622
Scatter plot, S&Pₜ vs. S&Pₜ₋₁ daily

correlation = 0.084
Scatter plot, $S&P_t$ vs. $S&P_{t-1}$ monthly

Correlation = 0.057
Volatility of U.S. stock market

[Monthly std dev = std dev of daily returns during the month × \sqrt{21}]
Properties of stock prices

Cross-sectional behavior

- What types of stocks have the highest returns?
- What types of stocks are riskiest?
- Can we predict which stocks will do well and which won’t?
Size portfolios, monthly returns

Firms sorted by market value (deciles)

Avg returns (% monthly)

0.40 0.60 0.80 1.00 1.20 1.40

Small 2 3 4 5 6 7 8 9 Big
Size portfolios in January

Avg returns (% monthly)

Firms sorted by market value (deciles)
M/B portfolios, monthly returns

Firms sorted by Mkt / Bk equity (deciles)
Momentum portfolios, monthly returns

Firms sorted by past 12-month return (deciles)
Time-series properties

Observations

- The average annual return on U.S. stocks from 1926 – 2001 was 11.6%. The average risk premium was 7.9%.

- Stocks are quite risky. The standard deviation of monthly returns for the overall market is 4.5% (15.6% annually).

- Individual stocks are much riskier. The average monthly standard deviation of an individual stock is around 17% (or 50% annually).

- Stocks tend to move together over time: when one stock goes up, other stocks are likely to go up as well. The correlation is far from perfect.
Time-series properties

Observations

➢ **Stock returns are nearly unpredictable.** For example, knowing how a stock does this month tells you very little about what will happen next month.

➢ **Market volatility changes over time.** Prices are sometimes quite volatile. The standard deviation of monthly returns varies from roughly 2% to 20%.

➢ **Financial ratios like DY and P/E ratios vary widely over time.** DY hit a maximum of 13.8% in 1932 and a minimum of 1.17% in 1999. The P/E ratio hit a maximum of 33.4 in 1999 and a minimum of 5.3 in 1917.
Cross-sectional properties

Observations

- **Size effect**: Smaller stocks typically outperform larger stocks, especially in January.

- **January effect**: Average returns in January are higher than in other months.

- **M/B, or value, effect**: Low M/B (value) stocks typically outperform high M/B (growth) stocks.

- **Momentum effect**: Stocks with high returns over the past 3- to 12-months typically continue to outperform stocks with low past returns.