Today’s Recitation: Discount Rates for Use in Capital Budgeting.

I. Quick Review of the CAPM.

II. Beta of Equity vs. Beta of Assets.

III. Beta of Assets vs. Beta of Project.

IV. Long-term Projects and Discount Rates.
I. Quick Review of the CAPM.

• CAPM gives us a theory of valuation (i.e. expected returns) for individually traded securities.

  → Linear relationship between expected return and security’s beta. Described by SML:

  \[ E[R_i] = R_f + \beta_i(E[R_m] - R_f). \]  

  (1)

• Beta of security \( i \)'s return is determined by:

  \[ \beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \]  

  (2)

  where \( R_m \) is the return on the market (often replaced with the value-weighted market return or S&P500 index).

  → Estimating this beta with historical data can be done using a linear regression:

  \[ R_i = a_i + b_i R_m + \epsilon_i. \]  

  (3)

  → \( b_i \) is an estimate of \( \beta_i \).

  → Problems with using historical beta?
I. Continued...

- Problem with using a long history of return data to estimate the equity beta...
  - On the one hand, using a longer estimation window is good because it reduces the estimation error in our predicted betas.
  - However, we run into a problem of changing betas.
  - It is reasonable to assume that a firm’s beta might change over time (e.g. firm undertakes projects in new industries, nature of industry changes, etc). In most cases, this process is slow and gradual (e.g. through organic growth), though it can occur quickly (e.g. some diversification mergers). In an example, we’ll see later that, when a company is leveraged, the beta of its stock can change even without underlying shifts in business risk.
  - Why does this create problems in estimation? Because it makes our beta estimates ambiguous in meaning. To illustrate, suppose that XYZ Corp had a beta of 2 in the 80s but, through mergers, lowered its beta to 1.5 in the 90s. In this case, it’s current beta is 1.5, which would roughly obtain using 90s data. Meanwhile, we’d likely get a beta estimate of about 1.75 (average of 1.5 and 2) if we used 80s and 90s data.
  - The main point is that you want to estimate betas using a long enough time series to reduce estimation error while not extending back so far that changing beta problems can cloud your results.
II. Beta of Equity vs. Beta of Assets.

• Beta is easiest to calculate for common stock because it is most actively traded and stock return data is readily available.  
  → Called beta of equity.

• Should we use the beta of equity to calculate the firm’s cost of capital? 
  → No. The beta of equity is only used to calculate the firm’s cost of equity. Unless the firm is all equity financed, the beta of its underlying assets will almost surely be different from its beta of equity. The same thing holds true for expected returns.

• Why? 
  → Most common reason is the presence of debt in the firm’s capital structure.  
  → How does debt make beta of assets different from beta of equity? Best seen with an example.
Fact: Holding business risk constant, the more levered a company (i.e. more debt), the riskier its equity and the higher its cost of equity.

↔ Intuition: So long as debt is roughly riskless, equity ends up picking up all the variability in firm value (including the variability that's correlated with the market). As more debt is issued, this variability per unit of equity value becomes larger which increases the beta of equity. This in turn increases the cost of equity.
• **Intuition:** Recall that interest payments on corporate debt (reflected by the cost of debt $E[R_D]$) reduce accounting profits. Subsequently, this lowers the company's tax burden. On the other hand, dividend payments (reflected by the cost of equity $E[R_E]$) do not reduce its accounting profits or tax burden (double-taxation: as profits for the firm, as dividend income for the investor). Through this difference in tax treatment, the government in some sense 'pays' part of the cost of debt but not some of the cost of equity. This payment is larger as the corporate tax rate increases. This explains the presence of a tax adjustment on the cost of debt with no similar adjustment for equity in the WACC formula.
III. Beta of Assets vs. Beta of Project.

- Of course, the firm’s cost of equity really only gives us a measure of the cost of financing *all* of the firm’s existing projects in their current state. That is, this gives us an idea of the cost of capital that’s appropriate for a project whose risk is the same as that of the firm’s typical project.

- This measure does not apply to all the firm’s projects.


  ↔ Other reasons to use comparables?

- Examples:

  - Company in industry X evaluates a project in industry Y. Can estimate project beta using beta of assets of pure-plays in industry Y.
  - Company in industry X evaluate a project in industry X and industry Y. Can estimate project beta using an average of own beta of assets and an average of beta of assets of pure-plays in industry Y. Alternatively, can estimate the beta of assets of a firm operating in both industries X and Y.
III. Continued...

• Keep in mind that methods 1 and 2 will give slightly different answers. Method 1 runs into the problem that highly (low) levered firms might have more (less) risky debt/equity than our firm (which would make for poor comparison). Meanwhile, method 2 makes the assumption that comparable firms have the roughly the same WACC which could ignore differences in the tax treatment of debt and equity if they have varying leverage ratios.

Method 2 is most commonly suggested.
IV. Long-term Projects and Discount Rates.

- What we’ve said about a project’s cost of capital:

... using the CAPM, we can find a single discount rate $E[R_A]$ and calculate the NPV of a long-lived project as:

$$NPV = C_0 + \frac{C_1}{(1 + E[R_A])} + \frac{C_2}{(1 + E[R_A])^2} + ... \quad (8)$$

*The only assumptions made here are that investors only care about portfolio expected return and variance and that markets are frictionless.*

- Is this true? Not quite. Remember that CAPM is a static theory of pricing. That is, it assumes all projects (and stocks, bonds, options, etc...) expire next period.

- When we apply the same discount rate over every period, we are not only assuming that each year’s cashflow is exposed to similar business risks but that later cashflows are progressively more exposed to these risks.
IV. Continued...

• **Illustration:** Suppose a company uses a single discount rate, say 15%, to evaluate a four year project with expected cashflows of 200 per year. It finds that the present value of each cashflow is given by:

i) \[ PV(C_1) = \frac{200}{1.15} = 173.91. \]

ii) \[ PV(C_2) = \frac{200}{(1.15)^2} = 151.23. \]

iii) \[ PV(C_3) = \frac{200}{(1.15)^3} = 131.50. \]

iv) \[ PV(C_4) = \frac{200}{(1.15)^4} = 114.35. \]

If the riskless rate is 8%, the company is indifferent to:

i) Exchanging \( C_1 \) for a riskless payment of 187.83 in year 1 (same PV).

ii) Exchanging \( C_2 \) for a riskless payment of 176.49 in year 2 (same PV).

iii) Exchanging \( C_3 \) for a riskless payment of 165.66 in year 3 (same PV).

iv) Exchanging \( C_4 \) for a riskless payment of 155.57 in year 4 (same PV).

These riskless payments are called the certainty equivalents (CEQs) for each of the respective cashflows.
IV. Continued...

• Notice that these CEQs fall noticeably over time. What does this mean?
  ↦ The company is willing to pay quite a bit more to avoid risk in year 4 cashflows (200-155.57=44.43) than in year 1 (200-187.83=12.17).
  ↦ This must mean that the year 4 cashflow is quite a bit riskier than that of year 1.

• Is this reasonable? Not always. The following example should highlight this: