Quiz For Lecture # 11

European Call Option using Black-Scholes/Merton

Consider a European call option on a stock when there are ex-dividend dates in two months and five months. The dividend on each ex-dividend date is expected to be $0.50. The current share price is $50 and the strike price is $50. The stock price volatility is 20% per annum and the risk free rate is 8.329% per annum, the volatility is continuously compounded, the interest rate is a simple interest rate, the time to maturity is six months. Calculate the call option's price and delta using Black-Scholes/Merton.

(Reminder: Showing the essential steps along the way will enhance your chances of partial credit in case you make an error.)

\[ r = n \cdot \ln \left( 1 + \frac{r_f}{n} \right) \]
\[ 0.08 = 1 \cdot \ln (1+0.08329/1) \]

\[ d_y = 0.5 \cdot e^{-0.08 \cdot \frac{2}{12}} + 0.5 \cdot e^{-0.08 \cdot \frac{5}{12}} = 0.9770 \]

\[ c_0 = (S_0 - d_y) \cdot N(d_1) - Ke^{-r_f T} N(d_2) \]

where

\[ d_1 = \frac{\ln \left( \frac{S_0 - d_y}{K} \right) + \left( r_f + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]

\[ S_0 = 50, \ K = 50, \ r_f = 0.08, \ \sigma = 0.2, \ T = 0.5. \]

We can use the discounted dividend of 0.977 and deduct it from the spot price of the stock. Using the B-S/M formula on a dividend-paying stock,

\[ d_1 = \frac{\ln \left( \frac{50 - 0.977}{50} \right) + \left( 0.08 + \frac{0.2^2}{2} \cdot \frac{6}{12} \right)}{0.2 \cdot \sqrt{\frac{6}{12}}} = 0.2140 \]
\[ d_2 = d_1 - 0.2 \sqrt{0.5} = 0.0726 \]

\[ c_0 = (50 - 0.977) \cdot N(0.2140) - 50 \cdot e^{-0.08 \cdot 0.5} \cdot N(0.0726) \]

\[ c_0 = 0.0726 \]

\[ \text{This question is taken from a former midterm exam. It was worth 25\% of the entire midterm.} \]
This implies that: \( N(d_1) = 0.58, N(d_2) = 0.53 \)

so that the call price \( c_0 \) from the B-S/M formula is:

\[
c_0 = (50-0.977) \cdot 0.58 - 50 \cdot 0.53 \cdot e^{-0.08 \times 0.5} = 3.26.
\]