Managing Demand and Sales Dynamics in New Product Diffusion Under Supply Constraint

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Motivation

(Newspaper clipping about Apple Computer’s worldwide launch of the iPod Mini to strong demand)

• How will limited supply affect overall demand?
• How will the competition’s reaction affect demand?
• How long should Apple delay the global launch?
• Should Apple have delayed original launch?
Agenda

• Introduction
• Overview
• Related Literature
• Methodology
  – Model Formulation
  – Optimal Sales Plan
  – Supply-Constrained New Product Diffusion
  – Optimal Supply Decisions
• Discussion
• Critique
• Questions / Discussion
Introduction

Operations Literature
- Capacity sizing when launching a new product
- Specify an exogenous demand trajectory
- **Assumes:** *Capacity does not affect Demand*

Marketing Research
- Characterization of the demand process
- Social Diffusion Process including internal and external factors (price, advertising, population, etc.)
- **Assumes:** *Unconstrained Supply*

*How does a new product diffuse in the presence of a supply constraint?*
Overview

Joint Analysis of supply-related decisions and demand dynamics

• Improved characterization of the constrained demand and sales dynamics
  – Back-ordering vs. Lost Customers
  – Generalized diffusion model distinguishes between the demand process and sales process (min of demand and available supply)

• Improved Operational Planning
  – Capacity planning (Cost of backordering and lost customers vs. overcapacity)
  – Launch decision (MTS → delayed launch = preproduction)
  – Should we sell less than is currently demanded (given sufficient supply)?
(See Figure 1, page 189 in the Ho, Savin, and Terwiesch paper)
Related Literature

  - Widely used in marketing to forecast demand
  - New product demand follows patterns of social diffusion processes similar to those in epidemiology and the natural sciences (e.g. SIR epidemic model)
  - Bass diffusion Model Basics:
    - Potential Adopters subject to two means of communication:
      - External Influence (mass-media communication → advertising)
      - Internal Influence (word-of-mouth)

- Related Research
  - Jain, et al., 1991: Diffusion of telephone service in Israel
    - No competition = no customer losses
    - Capacity grows with backorders (assumed short lead time for capacity expansion)
    - Supply constraint is always binding → sales trajectory mirrors capacity
  - Kurawarwala and Matsuo, 1998
    - Model of procurement with Bass-like demand process with known parameters of internal and external influence, unknown market size
    - Extended newsvendor model
  - Fine and Li, 1988
    - Conditions for switching from one supply process to another during product life-cycle
    - Assume demand with symmetrical growth and decline stages
    - Assume that process switching will not influence the underlying demand dynamics (i.e. they assume that demand is exogenous to the model)
Methodology

• Model Formulation
• Optimal Sales Plan
• Supply-Constrained New Product Diffusion
• Optimal Supply Decisions
Model Formulation

The Firm:

• Introducing a new product (e.g. Mini-IPOD)
  – Short Lifecycle
  – Long lead times

• Key Decisions:
  – Capacity sizing (Assumes constant $c$ throughout the product life cycle)
  – Time to Market ($t_i \geq 0$)
  – Sales Plan $s(t)$
Notation

(For explanations of model notation, see Table 1 on page 191 of the Ho, Savin, and Terwiesch paper)
Customer Diffusion

\[ D(t) = S(t) + W(t) + L(t) \]

**Unconstrained Supply:**
\[ W(t) = 0, \ L(t) = 0 \implies D(t) = S(t) \]

**Else:**
\[ \frac{dL(t)}{dt} = lW(t) \]

(See Figure 2 on page 191 of the Ho, Savin, and Terwiesch paper)
Demand Process

Arrival of customers orders follows *Bass-like dynamics*:

$$\frac{dD(t)}{dt} = p[m - D(t)] + \frac{q}{m} S(t)[m - D(t)]$$

**External Influence:**
- Innovation Dynamics
- Advertising

**Internal Influence:**
- Interaction Dynamics
- Word-of-Mouth

- Assumes a uniqueness of the new product
  - New brand, new product category (movies, video game console, Pentium III)
- Allows for customer losses (i.e. Does not require monopoly)
  - Cross-brand or cross-category substitution

*Methodology* (5 of 9)
Production

Connecting Demand to the Supply Process:

Total Production: \[ R(t) = I(t) + S(t) \]

Production Rate: \[ r(t) = \begin{cases} c, & t < t^* \\ \frac{dD(t)}{dt}, & t \geq t^* \end{cases} \]

\[ t^* = \min(t \mid \frac{dD(t)}{dt} < c, \frac{d^2D(t)}{dt^2} < 0) \]
Choosing a Sales Rate: \( s(t) \)

**Objective:** maximize life-cycle discounted profits, with \( c \) and \( t_l \) fixed

**Profit Function:**

\[
P(c, t_l) = \max_{s(t) \geq 0} \left( \int_{t_l}^{+\infty} (a(t)s(t) - hI(t))e^{-\theta t} \, dt \middle| I(t_l) = ct_l \right), a(t) > 0
\]

Simplified by shifting the time origin to \( t_l \):

\[
\overline{P}(c, t_l) = \max_{s(t) \geq 0} \left( \int_0^{+\infty} (\bar{a}(t)\bar{s}(t) - h\bar{I}(t))e^{-\theta t} \, dt \middle| \bar{I}(0) = ct_l \right)
\]

\[
\bar{a}(t) = a(t + t_l), \quad \bar{s}(t) = s(t + t_l), \quad \bar{I}(t) = (t + t_l)
\]
Choosing a Launch Time: $t_l \geq 0$

Given: the optimal selling plan $s^*(t)$

Discounted pre-launch inventory costs

\[
h \int_{0}^{t_l} cte^{-\theta t} \, dt = \frac{hc}{\theta} \left( \frac{1}{\theta} (1 - e^{-\theta t}) - t_l e^{-\theta t} \right)
\]

Objective: Maximize Profits

\[
P^*(c) = \max_{t_l \geq 0} \left( P(c, t_l) - \frac{hc}{\theta} \left( \frac{1}{\theta} (1 - e^{-\theta t_l}) - t_l e^{-\theta t_l} \right) \right)
\]
Choosing Production Capacity: \( c \)

**Given:** the optimal selling plan \( s^*(t) \) and the launch time \( t_l \)

\[
\max_c = \left( P^*(c) - Hc \right)
\]
Optimal Sales Plan

Tactical Decision: choosing the sales rate $s(t)$ to maximize profits with $c$ and $t_l$ fixed

Profit Function:\n\[
P(c, t_l) = \max_{s(t) \geq 0} \left\{ \int_{0}^{+\infty} (a(t)s(t) - hI(t))e^{-\theta t} dt \right\}
\]

Proposition 1. For any profit margin $a(t) > 0$, holding cost $h > 0$, and launch time $t_l \geq 0$, the optimal sales rate is given by:
\[
s^*(t) = \begin{cases} 
    r(t), & W^*(t) > 0 \\
    \min(r(t), d^*(t)), & I^*(t) = 0, W^*(t) = 0 \\
    d^*(t), & I^*(t) > 0
\end{cases}
\]

Where: $d^*(t)$, $I^*(t)$ and $W^*(t)$ are optimal values $I^*(t)W^*(t) = 0$

Bottom Line: The firm should always favor the immediate sale.
Given the optimal sales plan:

• Specify the demand $D(t)$ and sales $S(t)$ dynamics and compare to the unconstrained Bass demand dynamics

• Obtain an expression for discounted profits IOT determine the optimal capacity $c$ and time to market $t_l$

Consider two cases:

• Patient Customers ($l = 0$)
• Impatient Customers ($l > 0$)
Patient Customers, \( L(t) = 0 \)

\[
D(t) = S(t) + W(t)
\]
\[
R(t) + ct_i = S(t) + I(t)
\]
\[
\frac{dD(t)}{dt} = p[m - D(t)] + \frac{q}{m}S(t)[m - D(t)]
\]
\[
\frac{dR(t)}{dt} = \begin{cases} 
  c, & t < t^* \\
  \frac{dD(t)}{dt}, & t \geq t^*
\end{cases}
\]
\[
\frac{dS(t)}{dt} = \begin{cases} 
  c, & W(t) > 0 \\
  \min\left( c, \frac{dD(t)}{dt} \right), & I(t) = 0, \\
  \frac{dD(t)}{dt}, & I(t) > 0 
\end{cases}
\]

Solve this set of equations given:
\[
t^* = \min(t \left| \frac{dD(t)}{dt} < c, \frac{d^2 D(t)}{dt^2} < 0 \right)\]
\[
D(0) = S(0) = R(0) = 0
\]

Analyzing the diffusion process for any chosen capacity \( c \) and launch time \( t_i \), produces three different regimes:

**Regime 1:** Unconstrained Diffusion (UD)

**Regime 2:** Initially Unconstrained Diffusion (IUD)

**Regime 3:** Initially Constrained Diffusion (ICD)
Unconstrained Diffusion (UD)

$c$ and $t_l$ are high enough to ensure that $W(t)=0$ for all $t$.

Even with $t_l = 0$, capacity $c$ could be sufficient to ensure unconstrained Bass diffusion is preserved

• What is the **smallest capacity** level required?

  or

• Given $c$, what is the **earliest launch time**?
Unconstrained Diffusion (UD)

Determining the **smallest capacity** required to sustain UD.

\[ \tau_+ = \max(\tau | c = d_{Bass}(\tau)) \]

\[ c = \frac{pm(q+p)^2 \exp((p+q)\tau_+)}{(q+p \exp((p+q)\tau_+))^2} \]

\[ \tau_+ = \frac{1}{p+q} \ln\left(\frac{q}{p}\right) + \frac{1}{p+q} \ln\left(\frac{1 + \sqrt{1 - \frac{c}{c_o^*}}}{1 - \sqrt{1 - \frac{c}{c_o^*}}} \right) \]

\[ c_o^* = m(p+q)^2 / 4q \]

\[ D_{Bass}(\tau_+) = \frac{m(q-p)}{2q} + \frac{m(p+q)}{2q} \sqrt{1 - \frac{c}{c_o^*}} \]

For \( t_l = 0 \), the smallest capacity required

\[ c_s^*(p, q, m) \]

is determined as the capacity \( c \) where

\[ c \tau_+ = D_{Bass}(\tau_+) \]

It follows:

\[ c_s^*(p, q, m) < c_o^*(p, q, m) \]

**UD** (2 of 4)
Unconstrained Diffusion (UD)

\[ c_s^*(p,q,m) < c_o^*(p,q,m) \]

(See Figure 3 on page 195 of the Ho, Savin, and Terwiesch paper)
Unconstrained Diffusion (UD)

Determining the \textit{earliest launch time} that sustains UD.

**Lemma 1.** For a given \( c \), unconstrained diffusion is sustained IFF:

\[
t_l > t_l^*(c)
\]

The critical launch time is a non-increasing function of \( c \):

\[
\frac{\partial t_l^*(c)}{\partial c} \leq 0
\]

Provides the level of preproduction that avoids any supply shortages over the entire life cycle.
Initially Unconstrained Diffusion (IUD)

For a given $c$ and $0 < t_l < t_l^*$ pre-launch inventory is insufficient to support Bass diffusion over the entire life cycle of the product.

Given a finite amount of inventory at $t = 0$:

- It is possible to sustain an UD for a finite duration
- The diffusion process goes through 3 phases:
  - Initial Unconstrained Bass Diffusion (UP1)
  - Constrained Diffusion (CP)
  - Second Unconstrained Bass Diffusion (UP2)
Initially Unconstrained Bass Diffusion (UP1)

Demand and Sales are identical and increasing: \( s(t) = d(t), \frac{ds(t)}{dt} > 0 \)

\[
D(t) = S(t) = pm \left[ \frac{\exp((p+q)t) - 1}{q + p \exp((p+q)t)} \right]
\]

\( W(t) = 0 \)

This phase lasts until \textit{production + inventory} can no longer sustain the unconstrained diffusion. We can define the ending time of this phase:

\[
\tau_1 = \min \left\{ \tau : c(\tau + t_i) = m \left( 1 - \frac{q + p}{q + p \exp((p+q)\tau)} \right) \right\}
\]
Constrained Bass Diffusion (CP)

Customers are waiting and sales rate is equal to capacity:

\[ W(t) > 0, \frac{dS}{dt} = c \]

Given:

\[ D(\tau_1) = S(\tau_1) = D_1 \]

\[ D(t) = m - (m - D_1) \exp \left[ - \left( \left( p + q \frac{D_1}{m} \right)(t - \tau_1) + \frac{qc (t - \tau_1)^2}{2m} \right) \right], \]

\[ S(t) = D_1 + c(t - \tau_1), \]

\[ W(t) = -c(t - \tau_1) + (m - D_1) \left( 1 - \exp \left( - \left( p + q \frac{D_1}{m} \right)(t - \tau_1) - \frac{qc(t - \tau_1)^2}{2m} \right) \right) \]

This phase lasts until there are no customers waiting. Ending time of this phase:

\[ \tau_2 = \min \left( t | t > \tau_1, W(t) = 0 \right) \]
Second Unconstrained Bass Diffusion (UP2)

Demand and Sales are identical and decreasing: $s(t) = d(t)$, $ds(t)/dt < 0$

$$D(t) = S(t) = m - \frac{(m - D_2)(p + q)}{q - \frac{q}{m} D_2 + (p + \frac{q}{m} D_2) \exp((p + q)t - \tau_2)}$$

$W(t) = 0$

$D_2 = D(\tau_2)$

**Lemma 2.** Peak Demand and Sales Rates

$$\tau_{\text{max}}^D \Rightarrow d(\tau_{\text{max}}^D)$$

$$\tau_{\text{max}}^S \Rightarrow s(\tau_{\text{max}}^S)$$

for all values of production capacity
Initially Constrained Diffusion (ICD)

\[ t_l = 0 \text{ and } c < \text{initial inflow of potential adopters (pm). Initial constrained diffusion later replaced by unconstrained Bass process.} \]

Behaves much like the 2\textsuperscript{nd} and 3\textsuperscript{rd} phase of the IUD regime.

\[
W(t) > 0, \quad 0 < t < \tau_2 \\
W(t) = 0, \quad t \geq \tau_2
\]

**Lemma 3. Demand and Sales Dynamics in ICD Regime**

\[
\tau_{\text{max}}^D \implies d(\tau_{\text{max}}^D)
\]

Maximum sales rate is equal to \( c \).

\[
D(t) = m \left( 1 - \exp \left[ - \left( pt + \frac{qc t^2}{m} \frac{1}{2} \right) \right] \right),
\]

\[
S(t) = ct,
\]

\[
W(t) = -ct + m \left( 1 - \exp \left( - \left( pt + \frac{qct^2}{2m} \right) \right) \right),
\]

\[
\tau_2 = \min(t | t > 0, W(t) = 0)
\]
Impatient Customers, $l > 0$

\[
W(t, l) = -\frac{c}{l} (1 - \exp(-l(t - \tau_1))) + (m - D_1) \exp(-l(t - \tau_1)) \\
\times \left(1 - \exp\left(-\left(\tilde{p}(t - \tau_1) + \frac{qc(t - \tau_1)^2}{2m}\right)\right)\right) + (m - D_1) \exp(-l(t - \tau_1)) \\
\times \left(l \sqrt{\frac{2\pi m}{qc}} \exp \frac{mp^2}{2qc}\right) \\
\times \left(\Phi\left(\sqrt{\frac{qc}{m}}(t - \tau_1) + \sqrt{\frac{m}{qc}\tilde{p}}\right) - \Phi\left(\sqrt{\frac{m}{qc}\tilde{p}}\right)\right)
\]

\[
D(t, l) = m - (m - D_1) \\
\times \exp\left[-\left((p + q \frac{D_1}{m})(t - \tau_1) + \frac{qc (t - \tau_1)^2}{2m}\right)\right], \\
S(t, l) = D_1 + c(t - \tau_1), \\
L(t, l) = D(t, l) - S(t, l) - W(t, l)
\]

**Proposition 2.** NP diffusion dynamics subject to customer loss behave as outlined by Lemmas 1-3. Unconstrained phases unchanged, constrained phases described:

Enables firm to track the fraction of lost customers at any time.
Proposition 3. The length of the constrained phase is given by:

\[ T_c(l) = \tau_2(l) - \tau_1 \]

and is a decreasing function of \( l \):

\[ \frac{\partial T_c(l)}{\partial l} < 0 \]

Suggests that the duration of the constrained phase decreases as customer impatience increases.

Impatience = fn (Competition)

Proposition 4. The fraction of customers lost is given by:

\[ f(l) = \frac{(m - D_i) \left(1 - \exp\left[-\left(\frac{m}{2} + \frac{q_c T^2_c(l)}{m}\right)\right]\right)}{m} - cT_c(l) \]

\[ \frac{\partial f(l)}{\partial l} > 0 \]

and is an increasing function of \( l \):
Optimal Supply Decisions

**Given:** characterizations of the demand and sales dynamics

**Tactical Decision:** choose the optimal capacity and time to launch

**Profit Function:**

\[
P(c, t_l) = \int_{0}^{+\infty} (as(t) - hI(t))e^{-\theta t} dt
\]

**Proposition 5.** Performs the integration resulting in a “seemingly” complex equation for profit function which can be used for computing the optimal capacity and time to market.

pp 200-201
Optimal Time to Market

(See Figure 5a and 5b on page 201 of the Ho, Savin, and Terwiesch paper)

• For fixed capacity, optimal $t_l$ increases with both $p$ and $q$
• Optimal $t_l$ is more sensitive to $q$ than $p$
• Implies it is more important to estimate $q$ well
Optimal Time to Market (2)

(See Figure 6 on page 202 of the Ho, Savin, and Terwiesch paper)

- For fixed $h$, optimal $t_l$ shortens as $c$ increases
- As the $h$ increases, the optimal $t_l$ decreases for the same level $c$
- Suggests that firms may want to substitute capacity with pre-production by delaying product launch
Optimal Capacity Size

(See Figure 7 on page 202 of the Ho, Savin, and Terwiesch paper)

- Optimal capacity $c^{opt}$ increases with both $p$ and $q$
- $c^{opt}$ exhibits a saturation effect as the speed of diffusion increases
Optimal Capacity Size (2)

- $c^{opt}$ is a decreasing function of $H$ (capacity holding cost)
- Higher inventory costs $h$ push $c^{opt}$ down for the same level of $H$

(See Figure 8 on page 203 of the Ho, Savin, and Terwiesch paper)
Value of Endogenizing Demand

(See Figure 9a on page 203 of the Ho, Savin, and Terwiesch paper)

- Value of endogenizing demand can be significant
- Value goes up and then down as both $p$ and $q$ increase
  - Slow diffusion $\Rightarrow$ dynamics less likely to be constrained
  - Rapid diffusion $\Rightarrow$ heavily constrained $\Rightarrow$ lost customers
Value of Endogenizing Demand (2)

(See Figure 9b on page 204 of the Ho, Savin, and Terwiesch paper)
Value of Endogenizing Demand (3)

(See Figure 10 on page 204 of the Ho, Savin, and Terwiesch paper)

- Value of endogenizing demand can be significant when capacity costs $H$ are relatively high
- As $H$ decreases, the optimal capacity increases, demand diffusion becomes more Bass-like
Discussion

- It is important to include supply constraints in the estimation of diffusion parameters
- Increase in pre-production (delaying product launch) can act as a substitute for capacity
- Shows how optimal time to market and capacity vary with diffusion parameters
  - Timing and capacity are more sensitive to imitation (q) than innovation (p)
  - Optimal capacity exhibits saturation effect as the speed of diffusion increases
- Value of endogenizing demand in supply-related decisions can be substantial
- Informs operational decision-making
  - Develop more accurate forecasts of demand
  - Challenges assumption that demand forecasts merely serve as inputs to operations planning processes and are not affected by supply decisions
  - Suggests it is optimal to pre-produce
- **Future Research**
  - Estimation of diffusion parameters
  - Using marketing mix variables to influence diffusion
  - Waiting time dynamics
Major Contributions

(to the Operations and Marketing literature)

• Developed closed-form expressions of demand and sales dynamics in a Bass-like diffusion environment with a supply constraint

• Integrated capacity, time to market, and sales plan into a unified decision hierarchy

• Endogenized demand dynamics in determining the optimal capacity in a constrained diffusion environment
Critique

• Aggressive agenda, but…
  – Needs a more robust application
• 52 equations in ~18 pages
  – Didn’t provide the intuition for some of the math
• Proof of key “finding” $s^*(t)$ not included
• Missing graphs (3a-3b) referred to in paper
• Relies on many constant parameters that would not be constant in real life (e.g. loss rate)
• Many other factors “rolled up” in innovation parameter (e.g. price, competition, etc.)
• Offers some operationally useful ideas
Discussion