Managing Customer Relationships
Through Price and Service Quality

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04/22/2004

Agenda

- Problem raised by the paper
- Model description
  - Individual customer behavior
  - Aggregate customer behavior
- System behavior in steady state
- Profit optimization
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Imagine that you are a computer hotline…
- People call you when they have questions about their computers
- They pay for this service

You want to increase your profit, which parameters can you adjust?
- Price of the service
- Number of people who answer the questions (“service quality”)

How do you play on these parameters?
- Decreasing price and increasing service quality can augment the number of customer and increase my profit

But…
- More customers can lead to more delays and thus more customers who defect
- Lower price can lead to less profits
General context

- Subscription-based, capacity-constrained service
- Customers choose the depth of their relationship with the company based on their level of satisfaction
- Pricing and service quality affect this level of satisfaction
- The intuitive reasoning:
  - Service quality \rightarrow Customer base and profit
  - Pricing \downarrow
- But sometimes:
  - Service quality \rightarrow Customer base
  - Price \downarrow profitable

Complex relationship between pricing, service quality and profitability
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Customers interact with the company whenever they choose to do so. \( \tilde{w}_k \) evolves with:

\[
\tilde{w}_k = \alpha_k w_k + (1 - \alpha_k) \tilde{w}_{k-1}
\]

\( x \) is a random variable with distribution

\[
F(x \mid \tilde{w}_k) = \tilde{F}_k
\]
Interactions model

$p_s$  subscription fee to have access to the service during a period $[pT; (p+1) T]$

$p_u$  usage fee. To be paid each time the service is used

(See Figure 1 on page 9 of the Bitran, et al. paper)
Rate of the interaction with the customer

Customers use the service at the rate $\eta : \eta$ (level of satisfaction, $p_u$)
Utility of customer: $(\nu(\eta) - p_u) + c(w)$

Customers will renew their subscription when:

$$T \left[ \eta \left( \nu(\eta) - p_u \right) - \eta \left( \bar{c}(\bar{w}) \right) \right] - p_s > 0 \quad \text{with} \quad \bar{c}(\bar{w}) = \int_0^\infty c(x) dF(x \mid \bar{w})$$

We can then define $b$, the expected net utility per unit of time:

$$b \left( \eta; p_s, p_u, \tilde{F}(\cdot) \right) = T \left[ \eta \left( \nu(\eta) - p_u \right) - \eta \left( \bar{c}(\bar{w}) \right) \right] - p_s$$

The customer will choose $\eta^* = \arg \max(b)$
And he will defect whenever: $b \left( \eta^*; p_s, p_u, \tilde{F}(\cdot) \right) = b^* < b_{\text{min}}$
Dynamic of customer-company interactions

(See Figure 2 on page 10 of the Bitran, et al. paper)
Results

Monotonocity of customer's usage rate:
\[ \eta^* \downarrow \text{ with } p_u \uparrow \]
\[ \eta^* \downarrow \text{ with } \bar{c} \uparrow \]
\[ \eta^* \downarrow \text{ with } \tilde{w} \uparrow \]

Monotonocity of the customer's utility:
\[ b^* \downarrow \text{ with } p_u \text{ and } p_s \uparrow \]
\[ b^* \downarrow \text{ with } \bar{c} \uparrow \]
\[ b^* \downarrow \text{ with } \tilde{w} \uparrow \]

The company can use \( \tilde{w} \) as a concrete and manageable measure of customer utility \( b^* \)
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Customers stay with the company if \( b^* \geq b_{\text{min}} \)
That corresponds to \( \tilde{w} \leq \tilde{w}_{\text{max}} \) with \( b^*(\tilde{w}_{\text{max}}) = b_{\text{min}} \)

We can construct a Markov process where each state is described by the number of previous interactions and the customer's current level of satisfaction represented by \( \tilde{w} \).
To discretize the set of level of satisfaction we use:
\[
I = [0; \tilde{w}_{\text{max}}] = [0; u_1] \cup [l_1; u_2] \cup ... [l_s; u_s]
\]

The states of the Markov process are \((i, k)\) where after the \( k \)th interaction, \( \tilde{w} \in [l_i; u_i] \).
Markov process representation

(See Figure 3 on page 14 of the Bitran, et al. paper)
State transition probability

- Recall that:

\[ \tilde{w}_k = \alpha_k w_k + (1 - \alpha_k) \tilde{w}_{k-1} \]

- In term of probability:

\[
\Pr(\tilde{w}_{k+1} \leq x \mid \tilde{w}_k = y) = \Pr\left( w_k \leq \frac{x - (1 - \alpha_k)y}{\alpha_k} \right) = F\left( \frac{x - (1 - \alpha_k)y}{\alpha_k} \right)
\]

Where \( F \) is the true distribution of the company service quality

- If we assume that \( \tilde{W} \) is uniformly distributed in \([l_i; u_i]\) then:

\[
p_{ij}^k = \Pr(l_j \leq \tilde{w}_{k+1} \leq u_j \mid \tilde{w}_k \in I_i) = \frac{1}{\Delta_i} \left[ \int_{l_i}^{u_i} F\left( \frac{u_j - (1 - \alpha_k)y}{\alpha_k} \right) dy - \int_{l_i}^{u_i} F\left( \frac{l_j - (1 - \alpha_k)y}{\alpha_k} \right) dy \right]
\]
System behavior in steady state

Now that we know how the customers react to the decisions of the company, we can calculate the number of customers who pay the subscription fee and the service demand rate from customers to the company.

\[
\lambda_j^k = \sum_{i: I_i \in \hat{I}} p_{ij}^{k-1} \lambda_i^{k-1} \quad \text{with} \quad \hat{I} = \left\{ I_i \in I : \left[ \tilde{w} \in I_i \right] \rightarrow \left[ b^*(\tilde{w}) \geq b_{\min} \right] \right\}
\]
Customer response to a change in the service quality

Let \( \lambda \) the total service demand rate: 
\[
\lambda = \sum_{i,k} \lambda_i^k
\]

\( \lambda \) with \( W \)

BUT...

There is no such rule for the total number of customers \( N \)

( ie: \( N \) calculated with 
\[
N = \sum_{j,k} \frac{\lambda_j^k}{\eta_j}
\] )
Numerical experiment

(See Figure 4 on page 18 of the Bitran, et al. paper)
Intuitive explanation

- As the service quality decreases, the probability that the customer leaves the company increases. At the same time his number of interactions decreases.
- Let’s study the effects of this on the length of stay:

\[ E(\text{length of stay}) = E(\text{total number of interaction}) \times (\text{Average interval between interaction}) \]

- Decrease with a lower service quality
- Increase with a lower service quality

- The length of stay can thus increase with the decrease of service quality and thus make the number of customer increase.
Customer response to a change in the company pricing policy

$p_u$ and $p_s$ affect directly $w_{\text{max}}$ which in turn affects $\lambda$.

For a higher price, customers expect a better service.

Consequently:  

\[ \lambda \downarrow \text{ with } p_u \text{ and } p_s \uparrow \]

BUT...

There is no such rule for the total number of customers $N$. 
Numerical experiment

(See Figure 5 on page 21 of the Bitran, et al. paper)
Intuitive explanation

- Based on the monotonicity of $\eta^*$, the increase of $p_u$ will produce an increase in the time between customer interactions.
- But at the same time, the increase of $p_u$ will make the criteria for staying in the company more strict and thus the number of interactions will decrease.
- There is therefore no direct relation between the length of stay and $p_u$, and consequently, no direct relation between $N$ and $p_u$.
- On the other hand: $N \downarrow$ with $p_s \uparrow$. 
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Problem description

- Revenue:
  \[ R = \lambda p_u + Np_s \]

- Cost:
  \[ C(\lambda, W) \]

Thus we have the following nonlinear program to solve:

\[
\max \pi = \lambda p_u + Np_s - C(\lambda, W) \\
\text{Such that:} \quad \lambda = g(p_u, p_s, W) \\
N = h(p_u, p_s, W) \\
W \geq 0
\]
Using the Lagrange multipliers we obtain the equations:

(See equations 12, 13, and 14 on page 14 of the Bitran, et al. paper)

- Those equations equate marginal revenue with marginal costs
- Each of them can be decomposed into several intuitive parts
Conclusion

- This paper gives the analytical tools to understand the complex relation between pricing policy, service quality and customer base and behavior.
- The main originality of this paper is the introduction of this factor of relation depth: \( \eta \).
- Principal criticism: the paper does not take into account the fact that there is a difference between subscribing to the whole period and subscribing for one interaction. At the end of each period, even if the customer had been satisfied enough to make another interaction, he could decide that overall the service is not good enough to pay for another whole period.