Class outline:

• Roles of inventory
• Inventory related costs
• Types of inventory models
• Focus on EOQ model today (Newsvender model next class)
Inventory Management
Inventory Management
Inventory

• Inventory: goods that a business holds for resale and/or redistribution

• Why should a company hold inventory?
Roles of Inventory

• Anticipation Stock
• Cycle Stock
• Safety Stock
• Pipeline Stock
• Decoupling Stock
Anticipation Stock

- Stock built in *anticipation* of demand or price change
- Production capacity cannot meet demand at the time that occurs

Seasonal demand  |  Capacity Limits
Cycle Stock

- Stock created due to cyclic nature of replenishment
- Exists due to economies of scale in replenishment (e.g., a fixed cost in placing an order)
- When we reduce time between orders, cycle stock goes down
- Milk at home or in a grocery store
Safety Stock

• Exists to protect against uncertainty in demand, in lead times, and in yields

• Larger uncertainty -> larger safety stock

• Serves as a counter measure to uncertainty and disruption in the supply chain
Pipeline Stock

- Stock that has been ordered but has not arrived (in-transit stock)
- Caused by unavoidable time lags and delays
Decoupling Stock

- Stock that is positioned within a manufacturing setting or supply chain, whose purpose is to decouple the system.

- Allows the upstream segment to operate independently of the downstream segment.

- Can often be viewed analogous to a safety stock.
Roles of Inventory

• Anticipation Stock
• Cycle Stock
• Safety Stock
• Pipeline Stock
• Decoupling Stock
Types of Inventory

- Raw materials
- Work-in-process
- Finished goods
Costs of holding inventory

What are the costs of holding inventory?

- Cost of capital
- Cost of storage space and handling
- Inventory risk costs: obsolescence, damage, theft
- Inventory service costs: taxes, insurance
Inventory related costs

• Ordering Costs
  – Fixed and Variable

• Shortage Costs
  – Expedited Shipping, Reimbursement to Customers, Loss of Customer Goodwill
  – These costs are hard to estimate…
Fundamental Questions

- What items should be stocked?
- Where should items be stocked?
- How much should be ordered?
- When should an order be placed?
Types of inventory models

- **Demand**: constant, deterministic, stochastic
- **Lead times**: “0”, “>0”, stochastic
- **Horizon**: single period, finite, infinite
- **Products**: one product, multiple products
- **Capacity**: order/inventory limits, no limits
- **Service**: meet all demand, shortages allowed
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Economic Ordering Quantity (EOQ) Model
EOQ: Motivating Example

- You are managing diesel inventory of an outpost for a humanitarian organization in Uganda

- Your task is to determine when to order diesel fuel and how much to order

- Initiating an order costs $250 (transportation, road security)

- The fuel costs $1/liter

- To hold the fuel, it costs $0.50/year/liter (cost of capital – e.g. redirected from medicine)

- Your demand is constant and stable at 4,000 liters/year
EOQ: Motivating Example

• What is the key tradeoff?
  – Batch size too large (too much average inventory) versus
  – Batch size too small (too much ordering cost)

• For a fixed demand rate, the larger the order, the larger the holding cost

• The smaller the order, the larger the fixed ordering cost per unit
Other Examples

• Can you think about examples from your projects that are similar to the previous example?

• What's the order costs? What's the holding costs?
EOQ: Notation

• Decision Variables
  – Fixed order quantity: $Q$
  – Time between orders: $T$

• Inventory Costs
  – Fixed order cost: $K$
  – Variable cost/unit (purchase price): $c$
  – Inventory holding cost/unit/time: $h$

• Assume constant demand rate of $\lambda$ units/time; must meet all demand
EOQ: Key Observations

• It’s best to order only when inventory is zero. Why?
  – Zero lead time $\rightarrow$ Instantaneous replenishment
  – Therefore, no need to order inventory if can fill orders from existing stock

• The optimal ordering quantity will be constant.
  – Safety stock is no longer beneficial (constant demand, 0 lead time) and only incurs holding cost
EOQ: Graphical Representation

Given our order quantity, we know when to place the order, and vice versa.

\[ \lambda t = \text{demand up until time } t \]

\[-\lambda = \text{slope of line, rate of change in inventory}\]

Average inventory is \( Q/2 \)

\[ Q = \text{inventory level} \]

\[ Q - \lambda t \]

\[ t \]

\[ 2T \]

\[ 3T \]

\[ \frac{Q}{\lambda} = T \]
EOQ: Objective

- Total average cost = average holding cost + average order cost

  - Average holding cost = (holding $/unit/time) * (avg. inventory)
    = \( hQ/2 \)

  - Average order cost = order cost per replenishment cycle / length of replenishment cycle
    = \( (K + cQ)/T \)
    = \( K\lambda/Q + c\lambda \)

- Meet all demand while minimizing the total average cost ($/time)
EOQ: Optimization

- min(Total average cost) = min f(Q)
  \[= \min(hQ/2 + K\lambda/Q + c\lambda)\]
- 1\textsuperscript{st} Order Condition: \(f'(Q^*)=0\)
  \[f'(Q)=h/2 - K\lambda/Q^2\]
- 2\textsuperscript{nd} Order Condition: \(f''(Q^*)\geq0\)
  \[f''(Q)=2K\lambda/Q^3 \geq0 \text{ for any quantity}\]

\[
Q^* = \sqrt{\frac{2K\lambda}{h}}
\]

\[
T^* = \sqrt{\frac{2K}{h\lambda}}
\]
EOQ: Optimization Results

- Order Quantity/Order Time Decisions
  \[ Q^* = \sqrt{\frac{2K\lambda}{h}} \quad T^* = \sqrt{\frac{2K}{h\lambda}} \]

- Optimal Holding Cost
  \[ \frac{hQ^*}{2} = \sqrt{\frac{K\lambda h}{2}} \]

- Optimal Fixed Order Cost
  \[ \frac{K\lambda}{Q^*} = \sqrt{\frac{K\lambda h}{2}} \]

- Total Cost
  \[ c\lambda + \sqrt{2K\lambda h} \]
EOQ: Graphical View

Constructing the total cost curve
Add the two curves to one another
Total annual holding and ordering costs

Optimal order size

Average Holding Costs $\frac{hQ}{2}$

Average Fixed Order Cost $\frac{K\lambda}{Q}$
EOQ: Example Solution

• Optimal order quantity in liters =

\[ Q^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{(2)(250)(4000)}{0.5}} = 2000 \]

• Cycle time in year

\[ T^* = Q^*/\lambda = .5 \]

• Annual holding cost in dollars

\[ \frac{hQ^*}{2} = \frac{K\lambda h}{2} = \sqrt{\frac{(250)(4000)(0.5)}{2}} = 500 \]

• Annual order cost in dollars

\[ \frac{K + cQ^*}{T^*} = \frac{K\lambda h}{2} + c\lambda = \sqrt{\frac{(250)(4000)(0.5)}{2}} + 1 \times (4000) = 4500 \]

• Total annual cost in dollars

\[ 500 + 4500 = 5000 \]
EOQ: Sensitivity Analysis

What if we don’t order exactly $Q^*$?

- Fixed order cost and holding cost are affected
- Variable order cost is independent of $Q^*$
- Say we order $\gamma Q^*$ instead…how is fixed order cost + holding cost affected?
EOQ: Sensitivity Analysis

\[ C(Q^*) = \sqrt{2K\lambda h} \]
\[ C(\gamma Q^*) = \frac{1}{\gamma} \sqrt{\frac{1}{2}K\lambda h} + \gamma \sqrt{\frac{1}{2}K\lambda h} \]
\[ = \sqrt{2K\lambda h (\gamma + \frac{1}{\gamma})/2} \]

\[ \frac{C(\gamma Q^*)}{C(Q^*)} = (\gamma + \frac{1}{\gamma})/2 \]

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<th>( \gamma )</th>
<th>0.5</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
<th>1.2</th>
<th>1.5</th>
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EOQ: Key Observations

• Optimal decision (order quantity Q*) independent of variable ordering cost c

• Rather insensitive to order quantities that are close to optimal, but not optimal

• Optimal fixed order cost equals optimal inventory holding cost
Extensions

• Lead-time $L$
  – Same ordering quantity
  – Order $L$ periods in advance, when stock reaches $L/D$

• Finite production rates / Capacity restriction on orders

• Quantity discounts

• Allow shortage
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Next Class

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Newsvendor Model