

# Properties of Damped Oscillations Systems

Prepared for the  
MIT System Dynamics in Education Project  
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## 1. ABSTRACT

*Properties of Damped Oscillations Systems* is the fourth paper in a series dedicated to understanding oscillations. Please read *Generic Structures in Oscillating Systems I*<sup>1</sup>, *Oscillating Systems II: Sustained Oscillation*<sup>2</sup>, and *Generic Structures: Damped Oscillations*<sup>3</sup>, before continuing with this paper. This paper assumes knowledge of STELLA software, as well as simple system dynamics structures such as positive and negative feedback, exponential growth, S-shaped growth, and sustained oscillations.

This paper will examine the structural features of a damped oscillations system that lead to underdamped, overdamped, and critically damped behavior. System analysis will be conducted with a damped version of the Academic Performance Model from the *Oscillating Systems II: Sustained Oscillation* paper. Exercises will follow based on the and the Cleanliness of a College Dorm Room Model from the same paper.

## 2. INTRODUCTION

Previously in Road Maps, readers were introduced to the generic structure of damped oscillations systems, as well as the process to transform a sustained oscillations system into a damped oscillations system by adding a negative feedback loop. This paper will explore the properties of the damped oscillations structure and the behaviors that can result from it. The reader will gain a better understanding of the leverage points in a damped oscillations system and how policy changes can affect the system behavior.

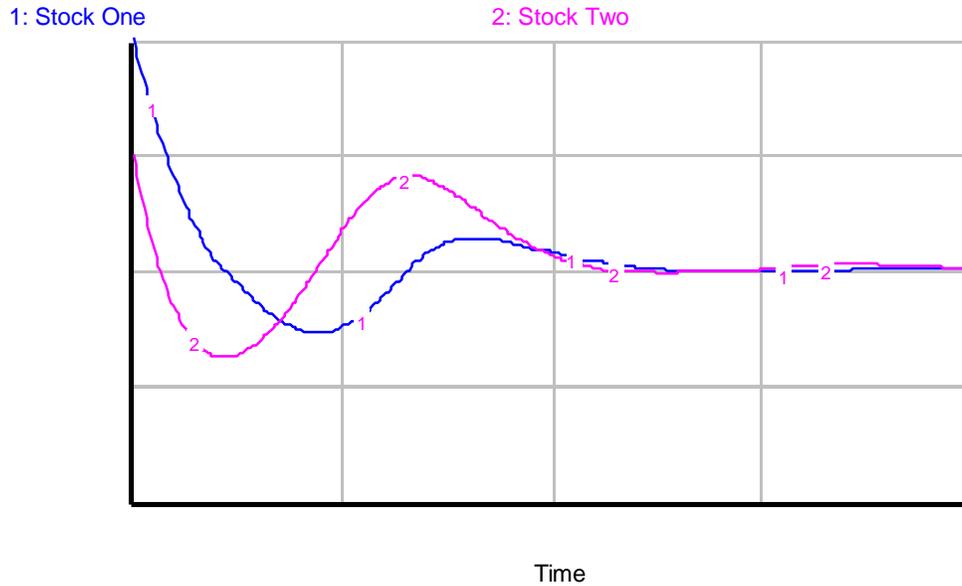
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<sup>1</sup> Celeste V. Chung, 1994. *Generic Structures in Oscillating Systems I* (D-4426-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 17, 25 p.

<sup>2</sup> Kevin A. Agatstein, 1997. *Oscillating Systems II: Sustained Oscillation* (D-4602), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 15, 31 p.

<sup>3</sup> Todd B. Kamin and Kevin M. Stange, 1999. *Generic Structures: Damped Oscillations* (D-4690), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology. Work in progress.





**Figure 2: Behavior of Damped Oscillations System**

Depending on the parameter values and initial values of the system, a damped oscillations generic structure can produce behavior of varying amplitude, degree of damping, and period of oscillations. This paper will use a damped Academic Performance Model to illustrate the following properties of damped oscillations systems.

### **Amplitude of Oscillations**

- The maximum amplitude of the damped system depends on the initial imbalance in the system, that is, the gap between the equilibrium and initial stock values.
- The maximum amplitude of the damped system is lower than that of the undamped system.
- Altering parameter values other than desired stock values does not change the equilibrium point.

### **Degree of Damping**

- The amplitude of each successive peak in the damped oscillations system is determined by the strength of the additional negative feedback loop.

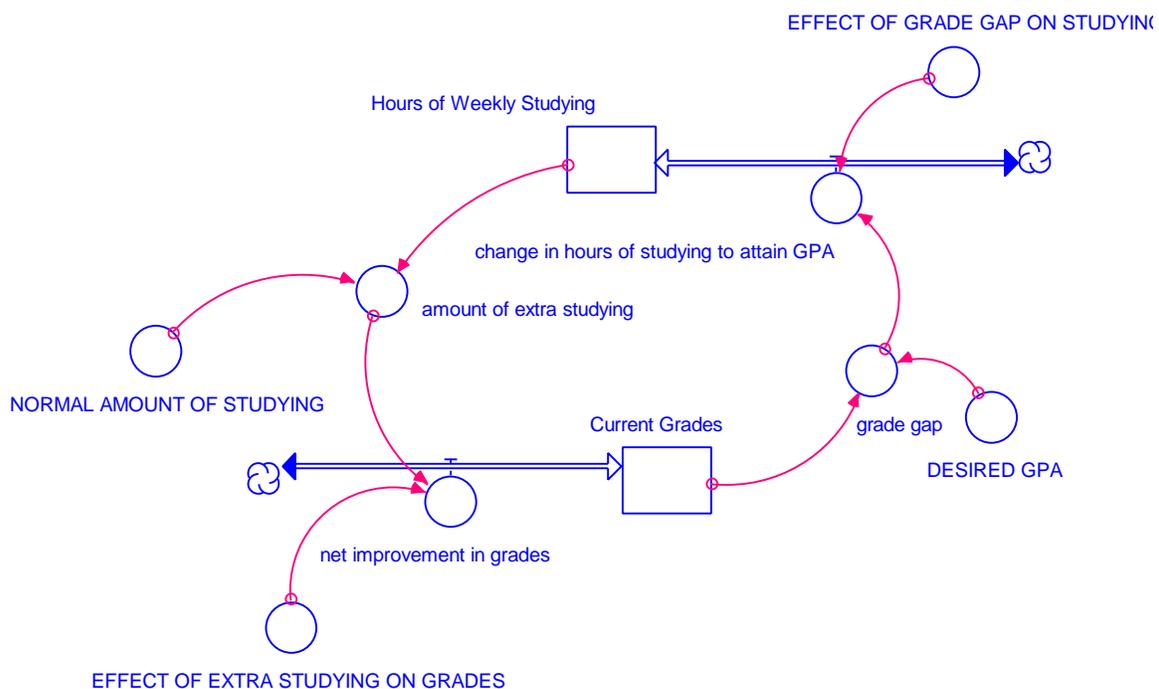
- Changing the strength of the negative feedback loop produces under-damped, critically-damped, or over-damped oscillations. A system is critically-damped when the value of its damping factor (“parameter three” in Figure 1) is the square root of four times the product of “parameter one” and “parameter two.”
- A system is under-damped when “parameter three” is smaller than the critically-damped value and over-damped when “parameter three” is larger than the critically-damped value.

### Period of Oscillations

- The period of oscillations in a damped oscillations system is greater than the period in the corresponding sustained oscillations system.

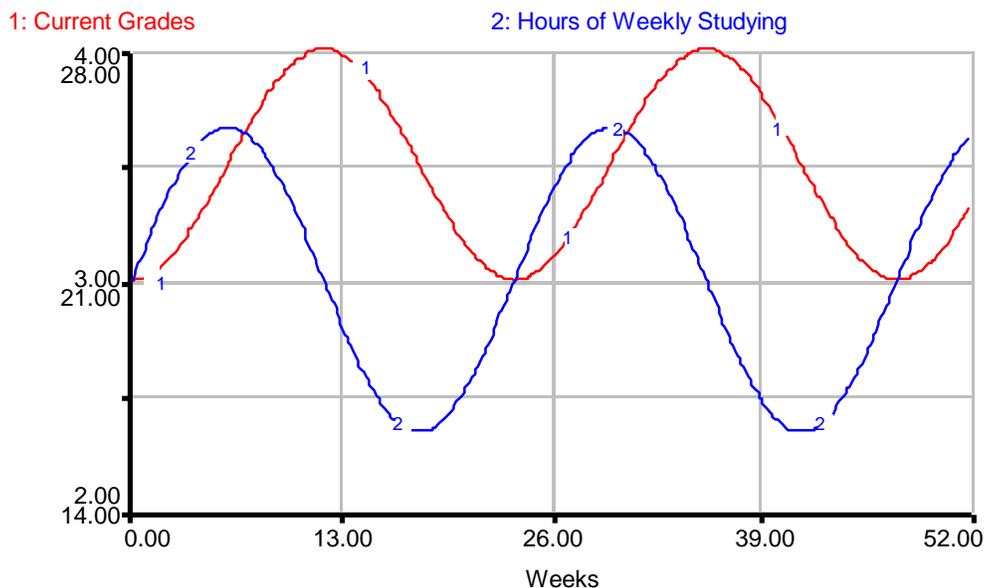
Finally, this paper will present two more damped oscillations models to demonstrate the transferability of the generic structure.

## 4. PROPERTIES OF DAMPED OSCILLATIONS SYSTEMS



### Figure 3: Undamped Academic Performance Model<sup>4</sup>

Figure 3 shows an Academic Performance Model capturing the dynamics of a student's struggle to achieve a desired Grade Point Average, also known as GPA (GPA is measured on a scale of 0 to 4.0). The model follows the generic structure for sustained oscillations. Nan, a high school student, starts out with a GPA of 3.0, represented by the stock "Current Grades." She, however, would like to maintain a 3.5 GPA. The gap between her actual and desired GPA drives Nan to change her hours of studying. Nan will study more if her "Current Grades" is lower than a 3.5 GPA, less if her "Current Grades" already exceeds 3.5. For each extra hour of studying, Nan reaps rewards in improvement in grades. Nan's "Hours of Weekly Studying" and "Current Grades" both oscillate during the entire simulation. The oscillations have the same amplitude and frequency throughout the simulation, as shown in Figure 4:



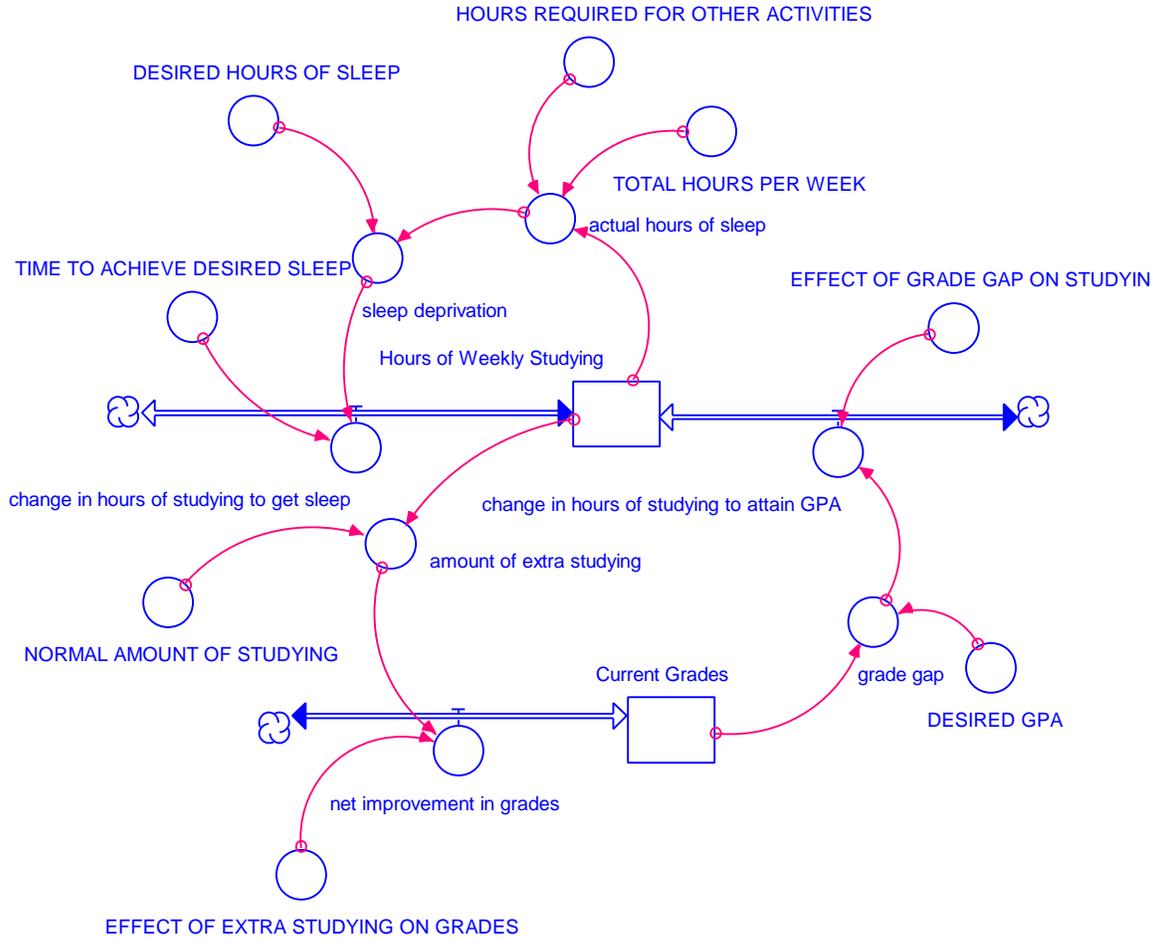
**Figure 4: Undamped Academic Performance Model Behavior**

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<sup>4</sup> For an in-depth description of the undamped Academic Performance Model, see: Kevin A. Agatstein, 1997. Oscillating Systems II: Sustained Oscillation (D-4602), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 15, 31 pp.

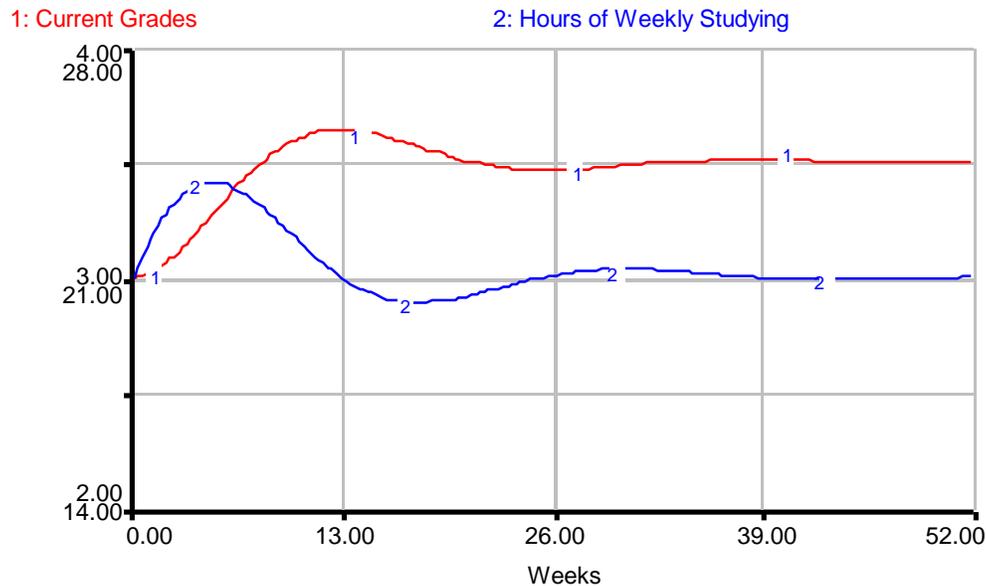
Nan realizes that it is unhealthy for her to allow her grades and lifestyle fluctuate in the long run. Instead, she would like to attain some regularity and stability in her life by using a different decision-making strategy when it comes to deciding how much to sleep and how much to study. She would like to make decisions that will help her to reach her desired grades and maintain a good lifestyle in the long term.

Nan desires not only a GPA of 3.5, but also a good lifestyle that allows her to get as close as possible to 8 hours of sleep per night, or 56 hours total in a week. Too little sleep keeps her from concentrating in class each day, but too much sleep wastes precious time. Out of each 24 hour day, Nan needs 13 hours to eat, attend classes, play in the band, sing in the chorus, help her parents with household chores, and do a bit of socializing. Nan is left with 11 hours per day (or 77 hours per week) to sleep and study. Initially, Nan has a 3.0 GPA, and she studies 3 hours a day. Although Nan is getting 8 hours of sleep per day, she needs to bring her GPA up while keeping in mind that within a certain period of time, she would like to return to her desired sleep schedule. In the beginning, Nan is not ambitious, so she decides that she will try to increase her GPA and get back on her sleep schedule within 5 weeks. Of course, if Nan is sleeping more than her desired hours of sleep, she should also adjust her habits to study more and sleep less. The modified Academic Performance Model is shown in Figure 5:



**Figure 5: Damped Academic Performance Model**

The more hours Nan studies, the fewer hours she is able to sleep. As sleep deprivation builds up, Nan studies less. The sleep-adjustment loop is a negative feedback loop that drives the stock “Hours of Weekly Studying” towards a goal of 21 hours per week. The goal-seeking nature of the sleep-adjustment loop controls the entire system and should produce damped oscillations. A simulation of the modified model results in the base case output shown in Figure 6:



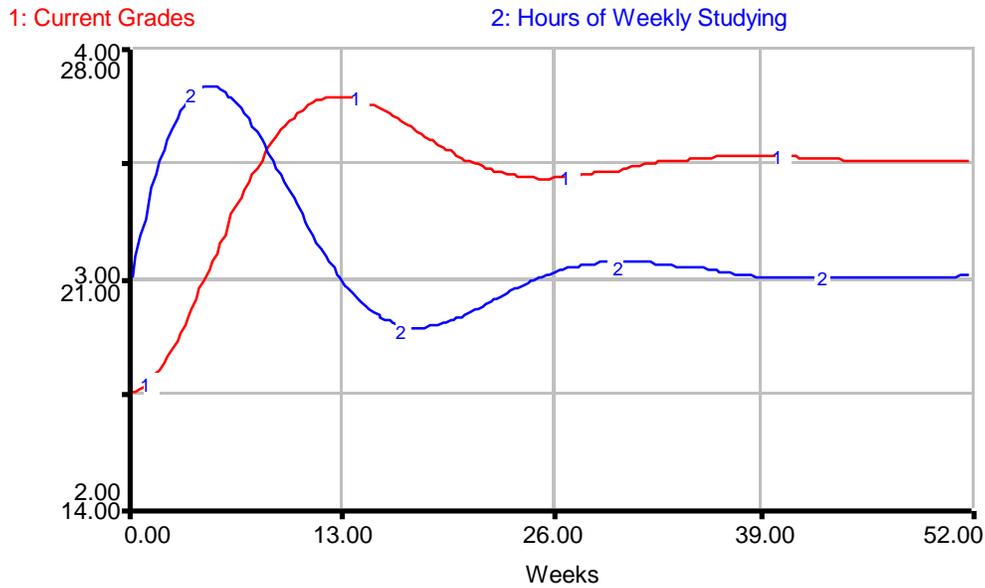
**Figure 6: Damped Academic Performance Model Behavior**

Nan initially has a 3.0 GPA. The “grade gap” forces her to study more than 21 hours per week. After a time delay, her GPA begins to rise and overshoots the desired 3.5 GPA. Nan’s first instinct is to decrease her “Hours of Weekly Studying” significantly, but then she remembers her new decision to adhere as closely to 8 hours of sleep per day as possible. To strive towards an ideal number of sleeping hours, Nan then studies more than she needs to, a decision that leads to a better GPA after a time delay, and a more regular sleep pattern in the long run. In time, Nan’s GPA and sleep pattern come to equilibrium at 3.5 and 56 hours per week, respectively. The original loop that includes both stocks drives the system to oscillate around the goal of a 3.5 GPA. The damping loop drives the “Hours of Weekly Studying” stock towards its desired value of 21 hours. One stock’s stabilizing behavior suppresses the other stock’s oscillations. Over time, both stocks approach their equilibrium values.

#### 4.1 Amplitude

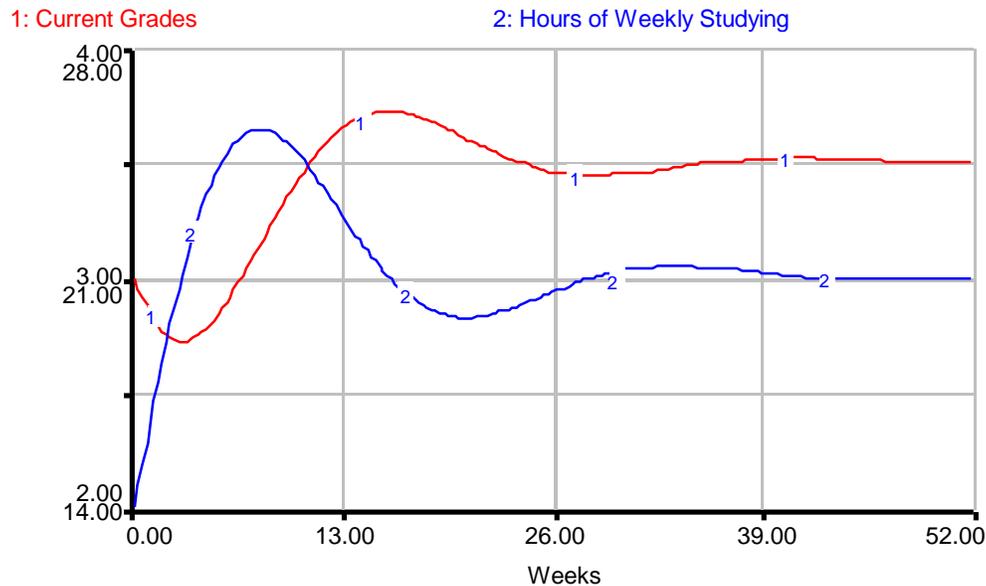
Given a particular set of parameters, the amplitude of the oscillations in the GPA system depends on only the initial imbalance in the system, that is, the difference between the desired and actual values of “Hours of Weekly Studying” and “Current Grades.” If Nan starts out with a 2.5 GPA, for example, she will need to increase her “Hours of

Weekly Studying” significantly faster in order to achieve a better GPA. The more she overshoots her desired number of studying hours, the more her grades will fluctuate. The amplitude of oscillations will be greater than in the base case, as shown in Figure 7:



**Figure 7: Lower Initial GPA (2.5 instead of 3)**

Similarly, if Nan starts out sleeping 9 hours per day (and only studying 2 hours per day, or 14 hours per week), the imbalance between desired and actual “Hours of Weekly Studying” will be larger. Nan will be motivated to immediately study many more hours. The amplitude of oscillations will again be greater than in the base case, as shown in Figure 8:



**Figure 8: Lower Initial Hours of Weekly Studying (14 instead of 21)**

In a damped oscillations system, the negative feedback loop in the system is constantly correcting the system imbalance by driving a stock towards its desired value (for example, by driving Nan towards her desired hours of sleep). As the imbalance between desired and actual amount of sleep narrows, the amplitude of oscillations lessens. Consequently, Nan's grades also approach her desired GPA. When the stock values approach their desired values, the system comes to equilibrium and the oscillations die out completely. The speed at which the system comes to equilibrium depends on the system's degree of damping, to be discussed later on in this paper.

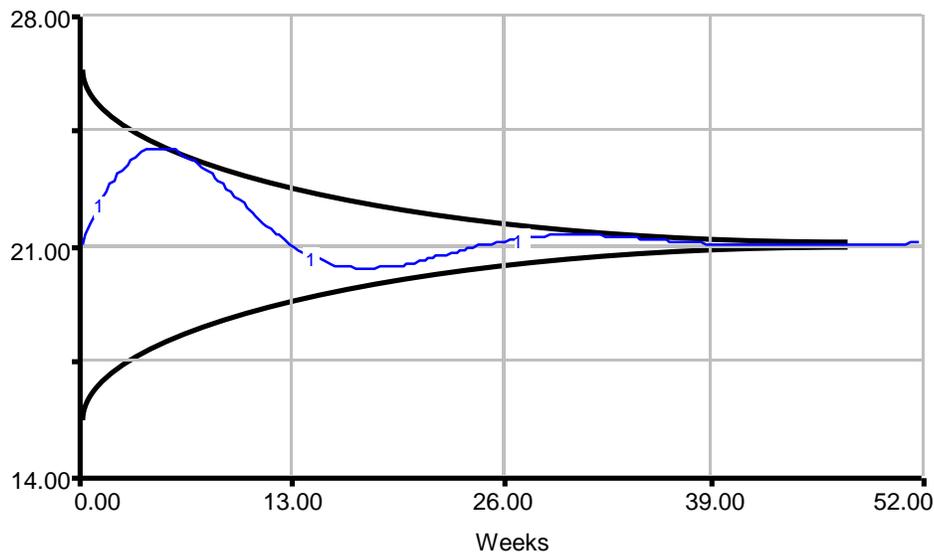
The amplitude of oscillations in a damped oscillations system can also be altered by varying parameter values because parameter changes can accentuate the initial disequilibrium in the system. For example, if Nan's "effect of grade gap on studying" is higher, then Nan's "hours of weekly studying" will grow faster. Remember, however, that parameter changes alter the system structure and policies. It is unreasonable to assume that Nan's personality can suddenly change to make her more sensitive to her grades. Similarly, Nan is unlikely to abruptly decide that she desires no less than 12 hours of sleep per night. Even if such system changes were to take place, given a certain set of initial value changes, the amplitude of oscillations will return to its original value.

The amplitude of a sustained oscillations system is always greater than the amplitude of a damped version of the same system. Because the additional negative feedback loop is constantly draining the “Hours of Weekly Studying” stock, the stock is unable to reach the amplitude of the sustained system. Consequently, “Hours of Weekly Studying” does not drive the inflow to “Current GPA” as high as in the sustained system, and the “Current GPA” stock also does not reach the amplitude of the sustained system. The exact difference between the amplitudes of the sustained and damped systems depends on the degree of damping.

## 4.2 Degree of Damping

The system is damped by the negative feedback loop representing Nan’s desire to achieve a fixed sleep schedule while targeting a certain GPA. The magnitude of oscillations decreases with each cycle, decreasing more and more slowly until the stocks approach equilibrium. The envelope around the oscillations (represented in Figure 9 by the bold lines connecting the peaks in the oscillations) takes the shape of asymptotic approach to equilibrium. The shape of the envelope is determined by the strength of the decay constant in the damped oscillations structure. Figure 9 shows the asymptotic decay effect of damping when the inverse of the decay constant, “time to achieve desired sleep,” is 5 weeks:

1: Hours of Weekly Studying



### Figure 9: Damping Envelope

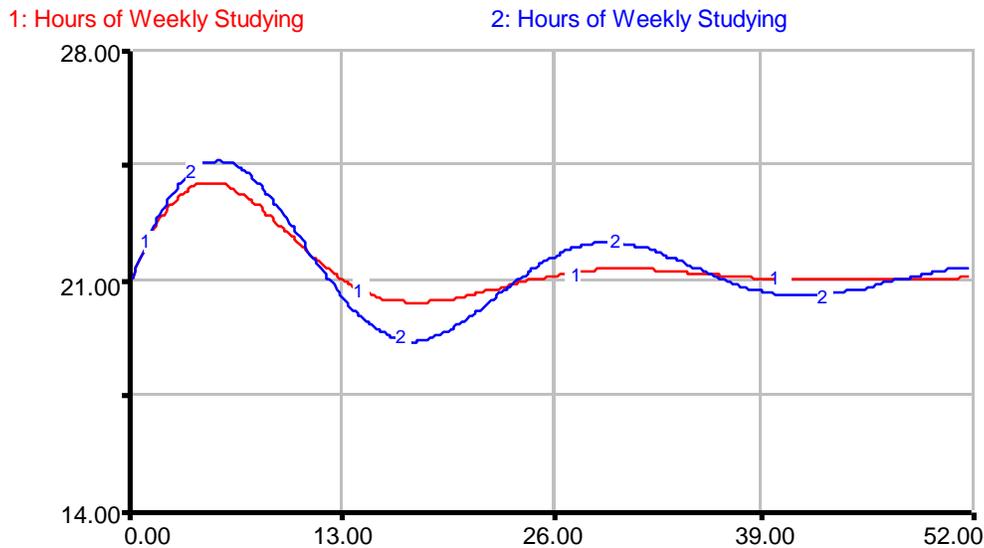
Damped oscillations behavior is essentially a combination of oscillations behavior and asymptotic behavior. Oscillations are generated by the negative feedback loop linking the two stocks via flows, while the asymptotic behavior is generated by the additional negative feedback loop linking “hours of weekly studying” and “change in hours of studying to get sleep.” The shape of the envelope in Figure 9 determines how quickly the system comes to equilibrium.

Nan decides that it takes her too long (about 40 weeks) to reach her desired sleep schedule. Instead, she would like to see her life stabilize more quickly. Nan wants to figure out what decisions she can make in order to make the system more responsive to her desires. If she can make the system damp more, the oscillations will die out faster, and her schedule will be less volatile.

For a moment, just concentrate on the bolded envelope in Figure 9. From an understanding of negative feedback, one knows that the stock approaches equilibrium faster if the time delay is smaller, or if the decay fraction (reciprocal of time delay) is greater. In the damped Academic Performance Model, then, the shape of the envelope is determined by the variables within the negative feedback loop. Most of the variables in the loop cannot be changed (such as total hours per week or hours required by other activities); Nan can only alter her “time to achieve desired sleep” in order to achieve her desired 56 hours of sleep per week as smoothly as possible. How quickly would her studying and grades reach equilibrium if she allowed herself more than five weeks to reach her desired grades and sleeping schedule? Similarly, what would happen if she allowed herself less than five weeks? Allowing herself too little time to adjust her habits would put too much inflexibility in her life, yet allowing too much time to adjust her habits would not achieve the desired effect. Nan would like to find a point at which she allows herself just enough “time to achieve desired sleep” to avoid oscillatory behavior in her life, without having to inconvenience herself any more than she has to.

#### *4.2.1 Under-Damped System*

Nan first considers the option of allowing herself more “time to achieve desired sleep” than five weeks to make her life more flexible. After she ponders for a bit, however, she realizes that increasing her “time to achieve desired sleep” would have the opposite of her desired effect. An increased “time to achieve desired sleep” decreases the “change in hours of studying to get sleep” Nan makes to remedy her sleep deprivation. Therefore, if at some time Nan’s sleep deprivation is 10 hours per week, a “time to achieve desired sleep” of 5 weeks forces Nan to work 2 hours per week less, bringing down her “Hours of Weekly Studying.” If Nan allows herself 10 weeks as her “time to achieve desired sleep,” on the other hand, a sleep deprivation of 10 hours per week forces Nan to work only 1 hour per week less, so that the “Hours of Weekly Studying” stock does not drop as steeply and quickly as before. Increasing the “time to achieve desired sleep” in effect slows the stocks’ approach to equilibrium and makes the stocks oscillate for a longer time. Figure 10 shows the “Hours of Weekly Studying” stock for “time to achieve desired sleep” of 5 weeks (curve 1) and 10 weeks (curve 2):



**Figure 10: Under-damped Oscillations**

As one can see from Figure 10, curve 2 oscillates with higher amplitude than curve 1. With a “time to achieve desired sleep” of 5 weeks, Nan’s lifestyle represented by curve 1 stabilizes at about week 40. While more desirable than curve 2, the behavior in curve 1

is still not ideal because it is still oscillating. Nan would like to discover a “time to achieve desired sleep” that drives the stock to approach but not overshoot equilibrium, thus not oscillate. Nan knows that “time to achieve desired sleep” must be shorter than 5 weeks to dampen the oscillations further. The Academic Performance Model with any value of “time to achieve desired sleep” that does not remove all oscillations is termed as an under-damped system: the system’s damping negative feedback loop is not strong enough to remove oscillatory behavior, thus oscillations decay but are still present.

#### ***4.2.2 Critically-Damped System***

The critical damping value of “time to achieve desired sleep” at which Nan’s sleeping schedule does not oscillate but rather decays to equilibrium can be derived mathematically. Given the generic structure in Figure 1, the critical value of “parameter 3” (the reciprocal of “time to achieve desired sleep” for the Academic Performance Model) is:

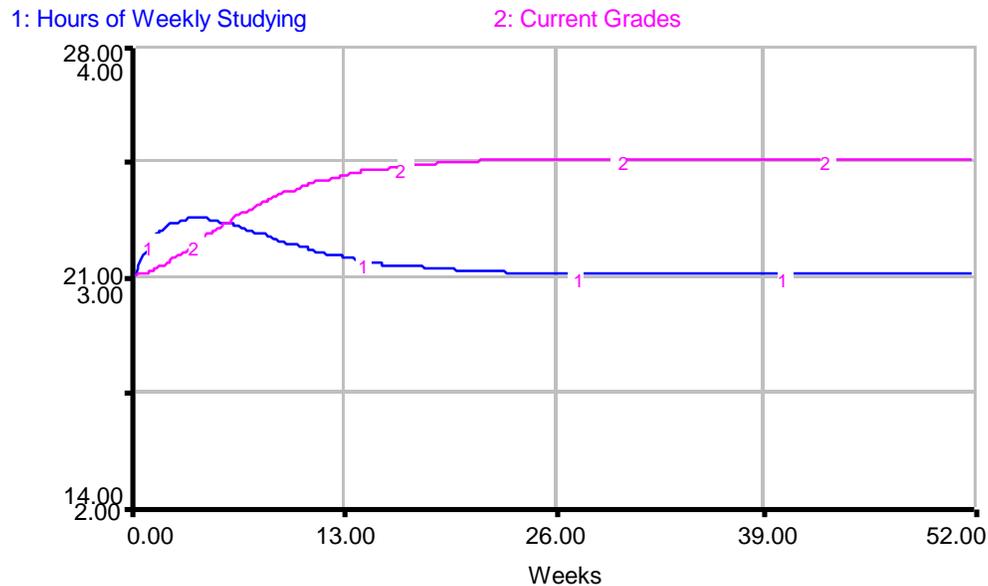
$$\text{parameter three (critical)} = \sqrt{4 * \text{parameter one} * \text{parameter two}}$$

$$\text{parameter three (critical)} = \sqrt{4 * 2.45 * 0.0285}$$

$$\text{time to achieve desired sleep} = 1 / \text{parameter three} = 1 / \sqrt{4 * 2.45 * 0.0285}$$

$$\text{time to achieve desired sleep} = 1.89 \text{ weeks}$$

Figure 11 shows the stock behavior when “time to achieve desired sleep” is 1.89 weeks, and thus the system is critically-damped:

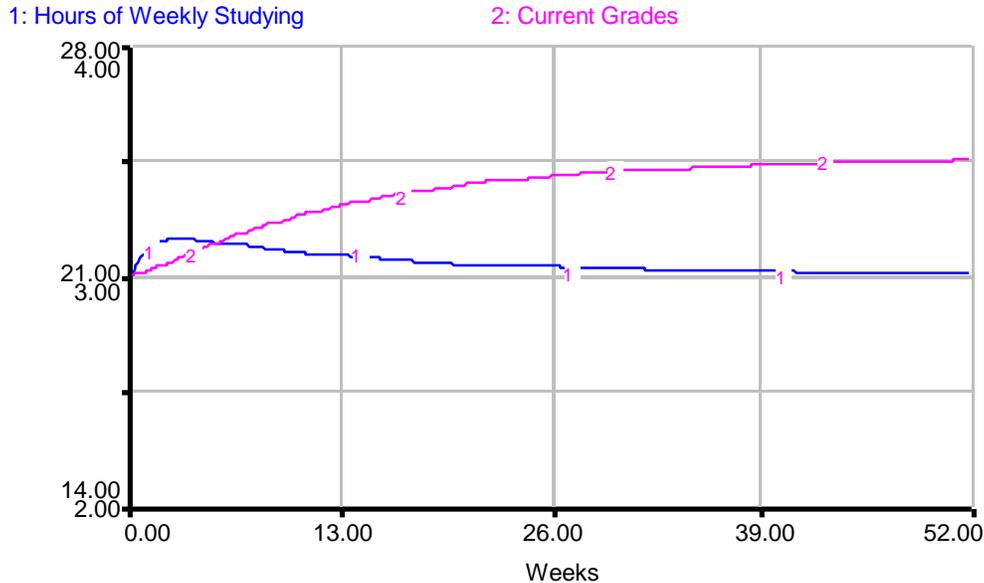


**Figure 11: Critically-Damped Oscillations**

Because the “time to achieve desired sleep” is sufficiently low, Nan forces herself to adjust her studying schedule more quickly. As a result, Nan’s response time is low enough that “Hours of Weekly Studying” no longer oscillates. If Nan allows herself any more than 1.89 weeks to achieve her desired sleep, the system will be under-damped and show some oscillatory behavior (although the oscillations will not be visible on the graph pad unless the time constant is significantly different from the critical damping point). If Nan allows herself less than 1.89 weeks to achieve her desired sleep, the system will still not oscillate but will be over-damped.

#### **4.2.3 Over-Damped System**

The system is considered over-damped when “time to achieve desired sleep” (reciprocal of the damping constant), is shorter than 1.89 weeks. That is, the system is more than sufficiently damped to suppress oscillatory behavior in the system. Figure 12 shows the over-damped system behavior when “time to achieve desired sleep” is only 1 week:



**Figure 12: Over-Damped Oscillations**

Compared to the critically-damped behavior, Figure 12 shows “Hours of Weekly Studying” grow but not quite reach its peak value attained in Figure 11. The curve then is able to approach equilibrium as in a critically-damped system.

In the Academic Performance Model, Nan would like to adjust her lifestyle and study habits. She wants to adjust quickly enough that her sleeping schedule does not oscillate, but also not adjust so quickly that her flexibility is sacrificed. In other words, the ideal value for Nan’s “time to achieve desired sleep” is the critical damping point of 1.89 weeks.

#### **4.2.4 Understanding Damping**

Understanding how to control the degree of damping in a damped oscillations system can allow one to achieve optimal system behavior. For example, when designing the suspension system in a car, one would not want the system to be under-damped because the car would bounce up and down every time it hit a bump. On the other hand, to make the suspension system over-damped would require a larger shock absorber that could be significantly more expensive in price. An overly large shock absorber will also

transmit more of the road bumps to the car instead of cushioning the bumps. The perfect compromise would be to use a shock absorber that achieves critical damping.

Other systems exist, however, in which the ideal behavior may not be critical damping. A playground swing, for example, should oscillate back and forth quite effortlessly with minimal frictional resistance. The engineer who designs the swing, therefore, would try to make the system as under-damped as possible, by oiling the hinges. Over-damping may be desirable at times as well, when one wishes to stabilize a system and the cost of stabilization is low or zero, or the cost of oscillation is extremely high. For example, if a doctor is trying to operate on a violent patient who is having periodic seizures, he may wish to give the patient a high enough dose of tranquilizer to knock the patient out immediately before any more harm can be done. In conclusion, understanding the degree of damping in a damped oscillations system is useful for controlling the system.

### **4.3 Period**

In order to examine the effect of damping on the period of oscillations, a comparison must be made between the undamped and damped behaviors. The undamped model behavior shown in Figure 4 has a period of oscillation of approximately 23.5 weeks. In comparison, the damped model behavior shown in Figure 6 has a period of oscillation of approximately 24 weeks. Damping the system increased the period of oscillations. Why?

As mentioned earlier in this paper, damping a sustained oscillations system keeps the stock that is affected by the additional negative feedback loop from achieving the amplitude observed in the undamped system. The greater the damping factor of the additional loop, the less steep the upward and downward slope of the stock (remember, the slope of the stock depends on the magnitude of its flow). The more slowly the stock moves upward and downward, the greater the period of the oscillations. In other words, as the damping factor of the system increases, so does the period of oscillations. Because the system damping factor is the inverse of the “time to achieve desired sleep,” as “time to achieve desired sleep” decreases, period of oscillations increases.

Although damping a sustained oscillations system increases its period, the effect is most often not observable. If the damping factor of the system is fairly small, the effect on the period is negligible and difficult to see. If the damping factor of the system is greater, it has a more significant effect on increasing the period of oscillations. At the same time, however, a high damping factor often puts the system into critical or over-damped states where oscillations die right away, and one can no longer measure the period of oscillations.

A second-order system oscillates when one stock is driving the other stock's flow. As the flow fluctuates, the stock it feeds into fluctuates also, in turn driving the flow to the first stock. The fact that a system may be damped with an additional negative feedback loop does not change the basic dynamics of the oscillations system. The parameter values in the system determine the period of oscillations that applies to both stocks and does not change throughout the simulation in a sustained or damped system.

## 5. CONCLUSION

Adding a negative feedback loop to the generic structure for a sustained oscillations system results in a damped oscillations system. The damped system has a smaller amplitude and longer period than the undamped system. The stocks of a damped oscillations system gradually decay in amplitude until the imbalance between the actual and desired stock values disappears. The damping constant of the damping negative feedback loop determines the rate of decay and the degree of damping (under-damped, critically-damped, or over-damped). Understanding the leverage points in a system can allow one to better control the system to attain a desired amplitude, period, or degree of damping.

## 6. APPENDIX: ACADEMIC PERFORMANCE MODEL EQUATIONS

$$\text{Current\_Grades}(t) = \text{Current\_Grades}(t - dt) + (\text{net\_improvement\_in\_grades}) * dt$$

INIT Current\_Grades = 3.0

DOCUMENT: Nan's academic performance at any time, measured by her grade point average.

Units: GPA

**INFLOWS:**

net\_improvement\_in\_grades = amount\_of\_extra\_studying \*

EFFECT\_OF\_EXTRA\_STUDYING\_ON\_GRADES

DOCUMENT: The change in Nan's grades as a result of studying more or less.

Units: GPA/week

Hours\_of\_Weekly\_Studying(t) = Hours\_of\_Weekly\_Studying(t - dt) +

(change\_in\_hours\_of\_studying\_to\_attain\_GPA -

change\_in\_hours\_of\_studying\_to\_get\_sleep) \* dt

INIT Hours\_of\_Weekly\_Studying = 21

DOCUMENT: The number of hours Nan spends studying per week. The initial value of 14 hours corresponds to 2 hours per night.

Units: hours/week

**INFLOWS:**

change\_in\_hours\_of\_studying\_to\_attain\_GPA = grade\_gap \*

EFFECT\_OF\_GRADE\_GAP\_ON\_STUDYING

DOCUMENT: The number of hours by which Nan increases or decreases her amount of studying, based on her desire to attain a certain GPA.

Units: (hours/week)/week

**OUTFLOWS:**

change\_in\_hours\_of\_studying\_to\_get\_sleep = sleep\_deprivation / TIME\_TO\_ADJUST

DOCUMENT: The increase or decrease in Nan's hours of weekly studying in order to attain her desired amount of sleep.

Units: (hours/week)/week

actual\_hours\_of\_sleep = TOTAL\_HOURS\_PER\_WEEK -  
 HOURS\_REQUIRED\_FOR\_OTHER\_ACTIVITIES -  
 Hours\_of\_Weekly\_Studying

DOCUMENT: The number of hours Nan sleeps per week.

Units: hours/week

amount\_of\_extra\_studying = Hours\_of\_Weekly\_Studying -  
 NORMAL\_AMOUNT\_OF\_STUDYING

DOCUMENT: The number of hours in a week Nan spends studying above her normal amount.

Units: hours/week

DESIRED\_GPA = 3.5

DOCUMENT: The GPA Nan would like to attain.

Units: GPA

DESIRED\_HOURS\_OF\_SLEEP = 7 \* 8

DOCUMENT: The number of hours per week that Nan would like to sleep in order to be alert and happy.

units: hours/week

EFFECT\_OF\_EXTRA\_STUDYING\_ON\_GRADES = 0.0285

DOCUMENT: The amount of GPA improvement achieved when Nan studies one extra hour.

Units: GPA/hour

EFFECT\_OF\_GRADE\_GAP\_ON\_STUDYING = 2.45

DOCUMENT: The amount by which Nan increases her weekly studying when she is below her desired GPA by one point.

Units: ((hours/week)/week)/GPA

$\text{grade\_gap} = \text{DESIRED\_GPA} - \text{Current\_Grades}$

DOCUMENT: The difference between Nan's desired and actual GPA.

Units: GPA

$\text{HOURS\_REQUIRED\_FOR\_OTHER\_ACTIVITIES} = 7 * 13$

DOCUMENT: Hours required by other activities includes eating, attending class, socializing, etc.

Units: hours/week

$\text{NORMAL\_AMOUNT\_OF\_STUDYING} = 21$

DOCUMENT: The number of hours Nan normally spends studying in a week.

Units: hours/week

$\text{sleep\_deprivation} = \text{DESIRED\_HOURS\_OF\_SLEEP} - \text{actual\_hours\_of\_sleep}$

DOCUMENT: The discrepancy between desired and actual hours of sleep per week.

Units: hours/week

$\text{TIME\_TO\_ACHIEVE\_DESIRED\_SLEEP} = 5$

DOCUMENT: The amount of time within which Nan would like to attain her desired amount of sleep.

Units: week

$\text{TOTAL\_HOURS\_PER\_WEEK} = 7 * 24$

DOCUMENT: The total number of hours in a week.

Units: hours/week